SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12

MATHEMATICS EXTENSION 1

ASSESSMENT TASK TWO

2003

Time allowed: 70 minutes

Instructions

*Attempt all questions.
*Answers to be written on the paper provided.
*Start each question on a new page.
*Marks may not be awarded for careless or badly arranged working.

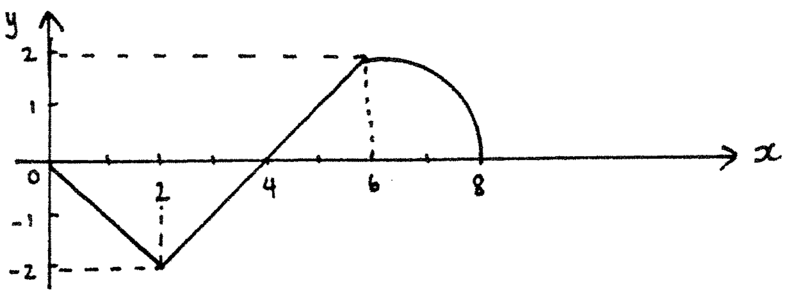
Q1	Q2	Q3	Q4	Q5	Q6	TOTAL
/9	/8	/9	/8	/9	/8	

Question 1 **(9 Marks)**

a. Find i.. $\int 4(1-2x)^3 dx$ ii. $\int (x^2-1)^2 dx$ **(3)**

b. Evaluate $\int_{-1}^{4} \frac{x^2+x}{2x} dx$ **(3)**

c. The graph below shows the curve $y = f(x)$ between $x = 0$ and $x = 8$



Find the value of $\int_0^8 f(x)dx$ **(1)**

d. Evaluate $\int_{-4}^{4} \sqrt{16-x^2}\, dx$ **(2)**

Question 2 **(8 Marks)**

a. Evaluate $\int_{-2}^{2} \frac{x^3}{1+x^2} dx$, giving a reason to support your answer **(2)**

b Find the area bounded by the curve $y = x - x^2$, the x axis, and the lines $x = 1$ and $x = -1$ **(3)**

c. Find the value of $\int_0^2 \frac{dx}{\sqrt{1+2x}}$ **(3)**

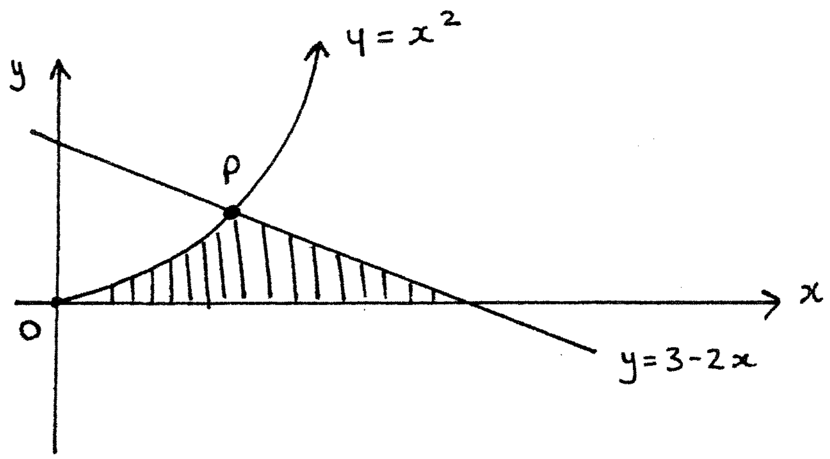
Question 3 **(9 Marks)**

a. By use of the substitution $t = x + 1$ (4)

i.. Show $\int \frac{x}{\sqrt{x+1}}\,dx = \int (\sqrt{t} - \frac{1}{\sqrt{t}})\,dt$

ii. Find the exact value of $\int_3^8 \frac{x}{\sqrt{x+1}}\,dx$

b. (5)



The diagram shows the parabola $y = x^2$ and the line $y = 3 - 2x$ intersecting at the point P, in the first quadrant.

i. Show that P is the point (1, 1)

ii The shaded region is rotated about the x – *axis*. Find the volume of the solid formed.

Question 4 **(8 Marks)**

a. i.. Estimate the area between the function $y = \sqrt{x},$ the *x-axis* and the ordinates $x=0$ and $x=1$, correct to 3 decimal places by using Simpson's rule with 3 function values. (4)

ii Find the percentage error in (i) correct to two decimal places

b. The graphs $y = 16 - x^2$ and y = $6x$ intersect at P and Q. (4)

i. Find the x-values of P and Q

ii. Hence, find the area between the two curves

Questions 5. **(9 Marks)**

a. Find $\int \frac{2x}{\sqrt{x^2-4}}dx$ using the substitution **(3)**

$u = x^2 - 4$.

b. Consider the function $y = x + 2 + \frac{4}{x-1}$ **(6)**

i. For what values of x is the function underfined ?

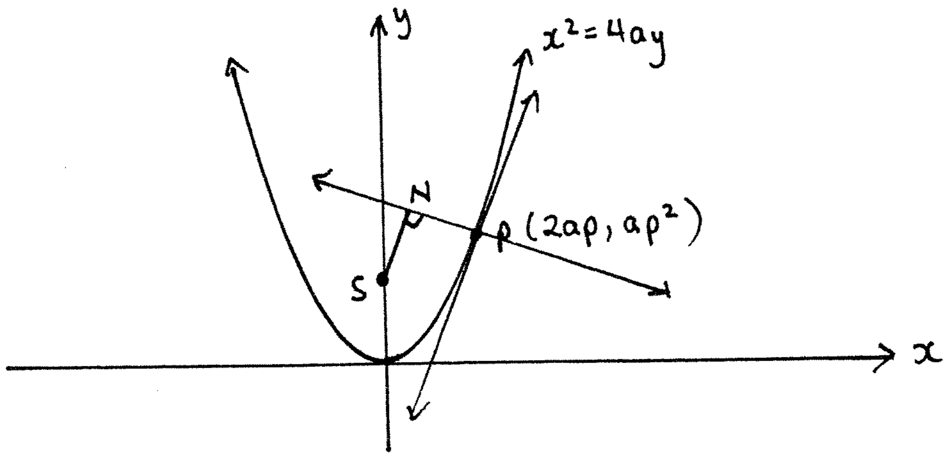
ii. What is the equation of the oblique asymptote ?

iii Find the co-ordinates of any stationary points and determine their nature

iv . Sketch the curve $y = x + 2 + \frac{4}{x-1}$.

Question 6 **(8 Marks)**

P $(2ap, ap^2)$ is a point on the parabola at $x^2 = 4ay$. SN is perpendicular to the normal at P, where S is the focus of the parabola and N the foot of the perpendicular from SN to the normal



a. Show that the equation of the normal at P is

$$x + py = 2ap + ap^3$$

b. Find the equation of SN

c. Show that the co-ordinates of the point N are $(ap, ap^2 + a)$

d. Find the locus of N as P moves on the parabola

Question 1

i. $\int 4(1-2x)^3\,dx$

$= \frac{4}{-2}\int -2(1-2x)^3\,dx$

$= \frac{-2(1-2x)^4}{4}$

$= -\frac{1}{2}(1-2x)^4 + c$ ✓

ii. $\int (x^2-1)^2\,dx$

$= \int x^4 - 2x^2 + 1\,dx$ ✓

$= \frac{x^5}{5} - \frac{2}{3}x^3 + x + c$ ✓

* No mark unless +c attached

$\int_{-1}^{4} \frac{x^2+x}{2x}\,dx$

$= \frac{1}{2}\int_{-1}^{4} \frac{x^2}{x} + \frac{x}{x}\,dx$ ✓

$= \frac{1}{2}\left[\frac{x^2}{2} + x\right]_{-1}^{4}$ ✓

$= \frac{1}{2}\left[\frac{16}{2} + 4 - \left(\frac{1}{2} - 1\right)\right]$

$= 6\frac{1}{4}$ ✓

$\int_0^8 f(x)\,dx$

$= -\frac{1}{2}\times 2\times 4 + \frac{1}{2}\times 2\times 2 + \frac{1}{4}\times\pi\times 2^2$

$= -4 + 2 + \pi$

$= \pi - 2$ (incorrect if units such as u^2 written) ✓

$\int_{-4}^{4} \sqrt{16-x^2}\,dx$

$= \frac{1}{2}$ circle (radius 4)

$= \frac{1}{2}\times\pi\times 4^2$ ✓

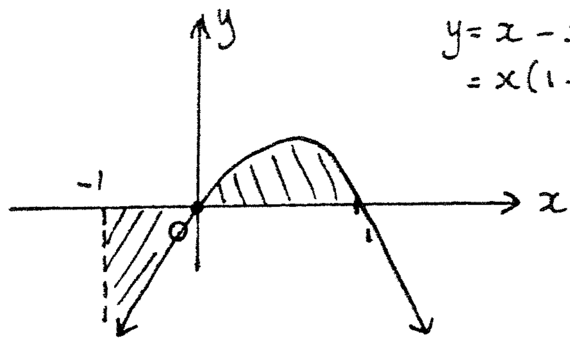
$= 8\pi$ ✓

Question 2

a. $\int_{-2}^{2} \frac{x^3}{1+x^2}\,dx = 0$ ✓

odd fnc between symmetrical limits. ✓

b.



$y = x - x^2$
$= x(1-x)$

$A_1 = \int_0^1 x - x^2\,dx$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$= \frac{1}{2} - \frac{1}{3} - 0$

$= \frac{1}{6}$ ✓

$A_2 = \left|\int_{-1}^{0} x - x^2\,dx\right|$ or $\int_0^{-1}$

$= \left|\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{0}\right|$

$= \left|0 - \left(\frac{1}{2} - -\frac{1}{3}\right)\right|$

$= \left|-\frac{5}{6}\right|$ ✓

$= \frac{5}{6}$

$\therefore$ area $= 1u^2$ ✓

c. $\int_0^2 (1+2x)^{-1/2}\,dx$

$= \frac{(1+2x)^{1/2}}{\frac{1}{2}\times 2}$ ✓

$= \left[\sqrt{1+2x}\right]_0^2$ ✓

$= \sqrt{5} - \sqrt{1}$

$= \sqrt{5} - 1$ ✓

Question 3

) $\int \frac{x}{\sqrt{x+1}}\, dx$ $\quad t = x+1$

$\frac{dt}{dx} = 1$

$\int \frac{t-1}{\sqrt{t}}\, dt$ $\quad dt = dx$ ✓

$\int \frac{t}{\sqrt{t}} - \frac{1}{\sqrt{t}}\, dt$ ✓

$\int \sqrt{t} - 1/\sqrt{t}\ \ dt$

$\int_3^8 \frac{x}{\sqrt{x+1}}\, dx$ $\quad x=8\ t=9$

$x=3\quad t=4$

$\int_4^9 \sqrt{t} - 1/\sqrt{t}\ dt$ ✓

$= \left.\frac{2}{3} t^{3/2} - 2t^{1/2}\right]_4^9$

$\frac{2}{3}\sqrt{9}^3 - 2\sqrt{9} - \left(\frac{2}{3}\sqrt{4}^3 - 2\sqrt{4}\right)$ ✓

$18 - 6 - (1\frac{1}{3})$

$= 10\,^2/_3$

ii. $x^2 = 3 - 2x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = 1\quad y = 1$

$\therefore P(1,1)$ ✓

(ii) $V_x = \pi \int y^2\, dx$

$V_1 = \pi \int_0^1 x^4\, dx$ ✓

$= \pi \left[\frac{x^5}{5}\right]_0^1$

$= \pi\left[1/5 - 0\right]$

$= \pi/5$

$V_2 = \pi \int_1^{3/2} (3-2x)^2\, dx$

$= \pi\left[\frac{(3-2x)^3}{-6}\right]_1^{3/2}$ ✓

$= -\frac{\pi}{6}\left[0 - (3-2)^3\right]$

$= -\frac{\pi}{6} \times -1$

$= \pi/6$ ✓

$\therefore$ Volume $= \frac{11\pi}{30}\ u^3$ ✓

Question 4.

x	0	$\frac{1}{2}$	1
$\sqrt{x}$	0	$\frac{1}{\sqrt{2}}$	1

$$I \doteqdot \frac{h}{3}\left(f + L + 4m\right)$$

$$= \frac{1}{6}\left(0 + 1 + \frac{4}{\sqrt{2}}\right) \quad ✓$$

$$= 0.63807\ldots$$

$$= 0.6381 \quad (4 \text{ sig fig}) \quad ✓$$

$$\int_0^1 \sqrt{x}\ dx$$

$$= \frac{2}{3} x^{3/2} \Big]_0^1$$

$$= 2/3 - 0$$

$$= 2/3 \quad ✓$$

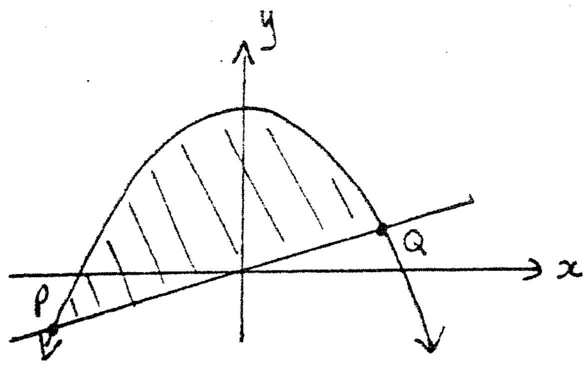
$$. \ \% \text{ error} = \frac{\text{diff}}{\text{exact}} \times 100\%$$

$$= \frac{0.02859}{2/3} \times 100\%$$

$$= 4.289\ldots$$

$$= 4.29\ \% \quad ✓$$

b.



i. $6x = 16 - x^2$

$$x^2 + 6x - 16 = 0$$

$$(x + 8)(x - 2) = 0$$

$$x = -8 \qquad x = 2 \quad ✓$$

ii.

$$A = \int_{-8}^{2} (16 - x^2) - 6x \ dx \quad ✓$$

$$= \left[16x - \frac{x^3}{3} - 3x^2\right]_{-8}^{2} \quad ✓$$

$$= 32 - \frac{8}{3} - 12 - \left(-128 + \frac{8^3}{3} - 3 \times 64\right)$$

$$= 17\tfrac{1}{3} - \left(-149\tfrac{1}{3}\right)$$

$$= 166\tfrac{2}{3} \quad ✓$$

Question 5

.. $\int \frac{2x}{\sqrt{x^2-4}}\,dx$ $\qquad u = x^2 - 4$

$\frac{du}{dx} = 2x$

$\int \frac{1}{\sqrt{x^2-4}}\,2x\,dx$ $\qquad du = 2x\,dx$

$= \int \frac{1}{\sqrt{u}}\,du$

$= \int u^{-1/2}\,du$

$= 2u^{1/2}$

$= 2\sqrt{x^2-4} + c$

o.

i. undefined at $x = 1$

ii. $y = x + 2$

..i. $dy/dx = 0$ stat pts

$\frac{dy}{dx} = 1 - 4(x-1)^{-2}$

$0 = 1 - \frac{4}{(x-1)^2}$

$\frac{4}{(x-1)^2} = 1$

$4 = (x-1)^2$

$x - 1 = 2$ $\qquad x - 1 = -2$

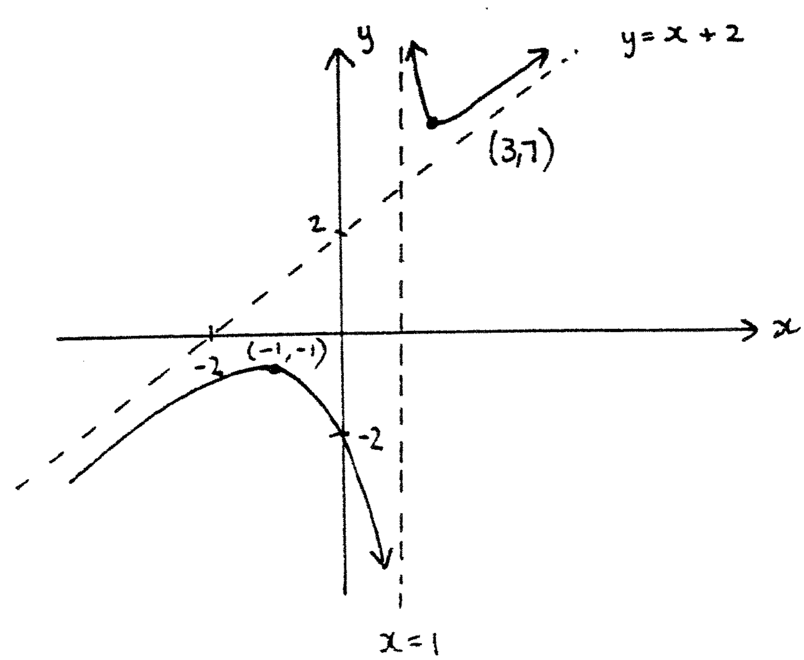
$x = 3$ $\qquad x = -1$

$y = 7$ $\qquad y = -1$

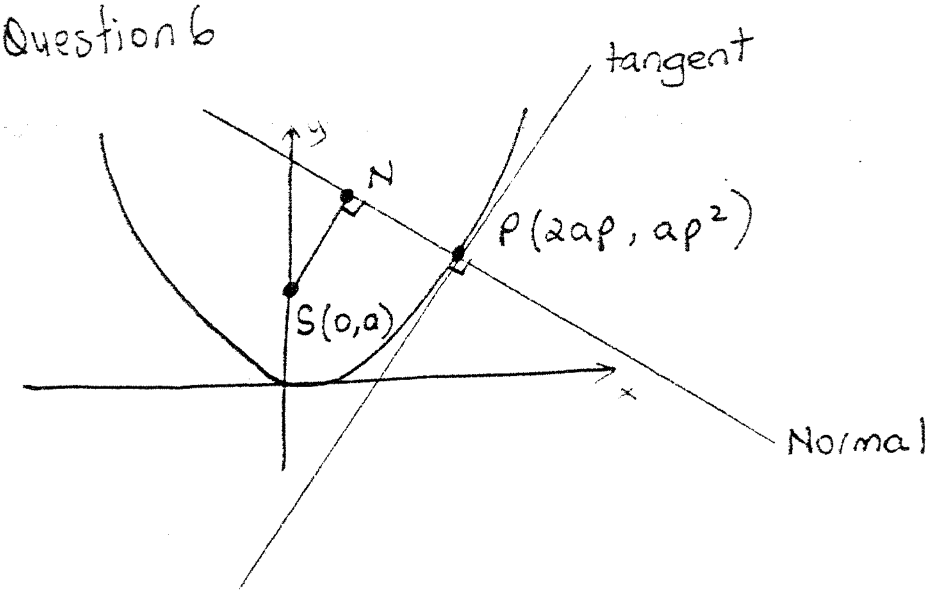
$\frac{d^2y}{dx^2} = \frac{8}{(x-1)^3}$

at $x = 3$ $\quad y'' > 0 \therefore$ Min $(3,7)$

at $x = -1$ $\quad y'' < 0 \therefore$ MAX $(-1,-1)$



Question 6



$x = ap$

& $y = p(ap) + a$
$= ap^2 + a$ ✓ ✓

ie $N(ap, ap^2 + a)$

i. $x^2 = 4ay$

$\frac{x^2}{4a} = y$

$y' = \frac{2x}{4a}$ at $x = 2ap$

$m_T = p$

$\therefore m_N = -1/p$ ✓

ie $y - ap^2 = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = -x + 2ap$

$x + py = 2ap + ap^3$ ✓

ii. SN $\quad m_{SN} = p$ ✓

$\therefore \quad y - a = p(x - 0)$ ✓

$y = px + a$

iii. $x + py = 2ap + ap^3$

$y = px + a$

$\therefore \quad x + p(px + a) = 2ap + ap^3$

$x + p^2x + ap = 2ap + ap^3$

$x(1 + p^2) = ap + ap^3$

$x(1 + p^2) = ap(1 + p^2)$

iv Locus

$x = ap$ ✓

$\frac{x}{a} = p$ ✓ $\therefore \quad y = ap^2 + a$

✓ $y = a\left(\frac{x}{a}\right)^2 + a$

any pt from here on collect

$y = a.\frac{x^2}{a^2} + a$

$y = \frac{x^2}{a} + a$

$ay = x^2 + a^2$

$x^2 = ay - a^2$

$x^2 = a(y - a)$