Name: ______________________________ Class: ________________

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 12

Mathematics

Extension 1

HSC ASSESSMENT 2

March 2004

Time Allowed: **70 minutes**

Instructions:

- Write your name and class at the top of this page.
- **Start each question on a new page**
- At the end of the examination this examination paper must be attached to the front of your answers.
- The marks for each question are indicated on the question sheet
- **ALL** questions should be attempted
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary..

Q1	Q2	Q3	Q4	Q5	Q6	Total
/8	/9	/8	/9	/7	/9	/50

Question 1 (8 marks)

a) Solve for x $\dfrac{1}{\sqrt{3}+\sqrt{x}}+\dfrac{1}{\sqrt{3}-\sqrt{x}}=2\sqrt{3}$ **2**

b) Find all real x such that $x^2+4x>5$ **2**

c) By drawing a suitable sketch or otherwise solve $|x-1| > |x-2|$ **2**

d) Find all positive values of x for which $\dfrac{6}{x}>x-1$ **2**

Question 2 (9 marks) Start a new page

a) Find $\displaystyle\int_2^8 [f(x)-3]\,dx$ *given* $\displaystyle\int_2^8 f(x)\,dx = 24$ **3**

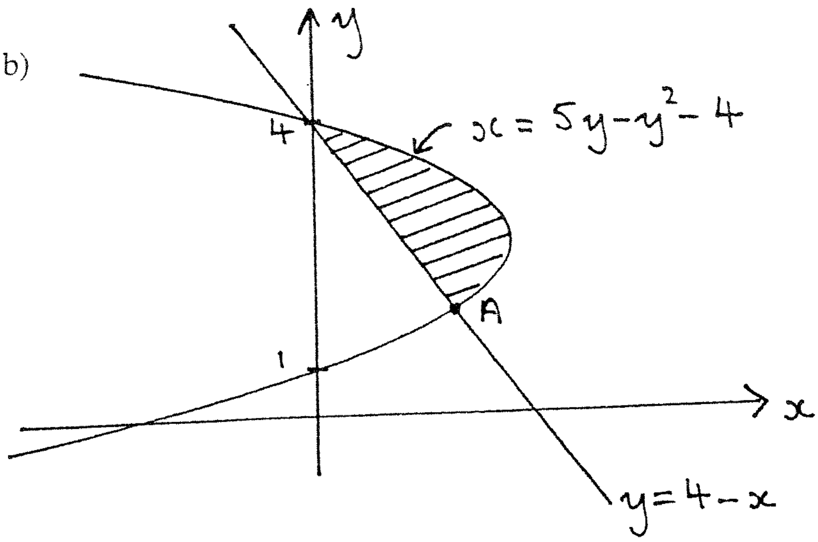
b) Find $\dfrac{d}{dx}\left(\dfrac{1}{x^2+1}\right)$ hence find $\displaystyle\int\frac{2x}{(x^2+1)^2}\,dx$ **3**

c) i) On one set of axes sketch $y=x^2$ and $y=\sqrt{x}$ **1**

ii) <u>Without integrating</u> explain why $0<\displaystyle\int_0^1 x^2dx < \int_0^1\sqrt{x}\,dx < 1$ **2**

Question 3 (8 marks) Start a new page

a) Find any stationary point(s) and determine their nature for the curve $y=x^3+3x^2+3x-2$. There is no need to sketch the curve. **4** b)



i) Find the coordinates of A **1**

ii) Hence find the shaded area **3**

Question 4 (9 marks) Start a new page

a) Evaluate $\int_{-1}^{2} |x-1| \, dx$ **2**

b) Find $\int \frac{x}{\sqrt{x-2}} dx$ using the substitution $u^2 = x - 2$ or otherwise **3**

c) i) Copy and complete the table of values for $f(x) = \sqrt{1-x^2}$ (correct to 4 dec.pl) **1**

x	0	.25	.5	.75	1
$f(x)$	1				0

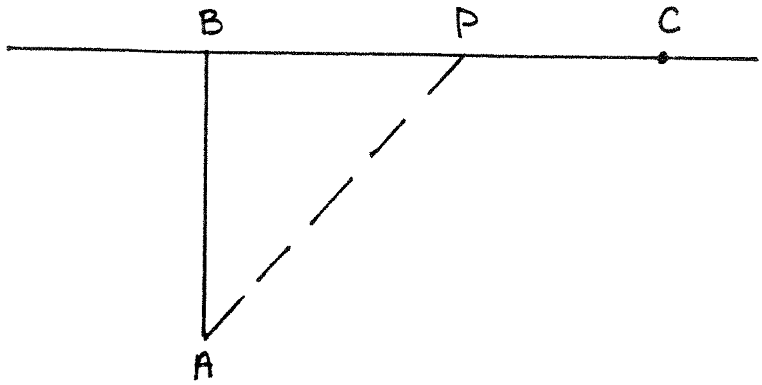
ii) Use Simpsons Rule and the above table to find $\int_0^1 \sqrt{1-x^2} \; dx$ **2**

Use all 5 function values.

Write your answer correct to 4 dec. pl.

iii) Use part ii) to estimate π to 3 dec. pl. **1**

Question 5 **(7 marks)** **Start a new page**



The diagram shows a straight road BC running due East. A four-wheel drive ambulance is in open country at A, 3km due South of B. It must reach C, 9 km due East of B, as quickly as possible.

The driver knows that she can travel at 80km per hour in open country and at 100km per hour along the road. She intends to proceed in a straight line to some point P on the road and then to continue along the road to C. She wishes to choose P so that total time for the journey APC is a minimum.

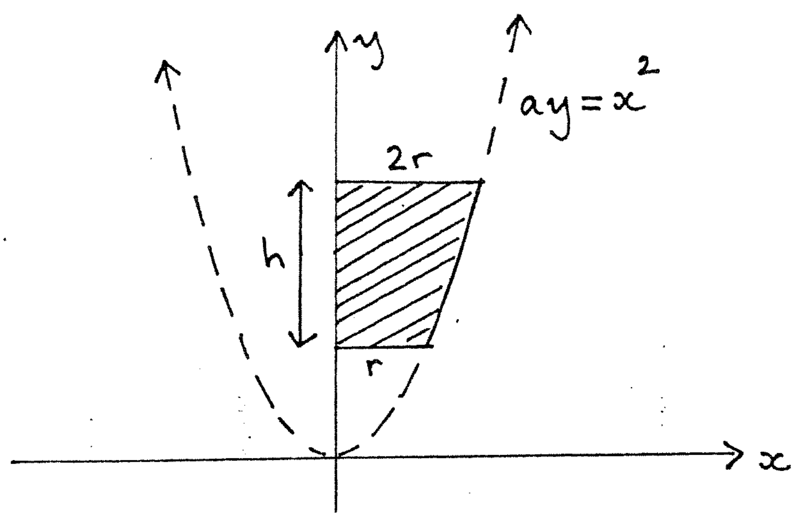
a) If the distance BP is x km, show that an expression for T (the total time for the journey from A to C via P) is given by $T = \frac{\sqrt{9+x^2}}{80} + \frac{9-x}{100}$ (hours) **2**

b) Show the minimum time for the total journey APC is $6\frac{3}{4}$ minutes. 5

Question 6 (9 marks) Start a new page

a) Evaluate $\int_{-1}^{1} x^2\sqrt{1+x^3}\ dx$ using the substitution $u = 1 + x^3$ or otherwise. Leave your answer in simplest <u>exact</u> form. **4**

b) The curved surface of an open bowl with a flat circular base may be traced out by the complete revolution of the portion of the curve $ay = x^2$ around the y axis. **5**



The radius of the top rim of the bowl is twice that of the base that's 2r and r respectively

The volume of the bowl is $\frac{5}{6}\pi a^3$ $unit^3$

Find the height (h) of the bowl.

Suggested Marks.

H.S.C. Asses Task 2 MARCH 2004 – EXT 1

Question 1 (8 marks)

a) $\frac{1}{\sqrt{3}+\sqrt{x}} + \frac{1}{\sqrt{3}-\sqrt{x}} = 2\sqrt{3}$

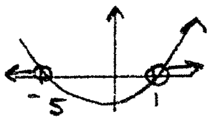
$\frac{\sqrt{3}-\sqrt{x}+\sqrt{3}+\sqrt{x}}{(3-x)} = 2\sqrt{3}$ — ①

$\frac{2\sqrt{3}}{3-x} = \frac{2\sqrt{3}}{1}$

$\therefore\ x = 2$ — ①

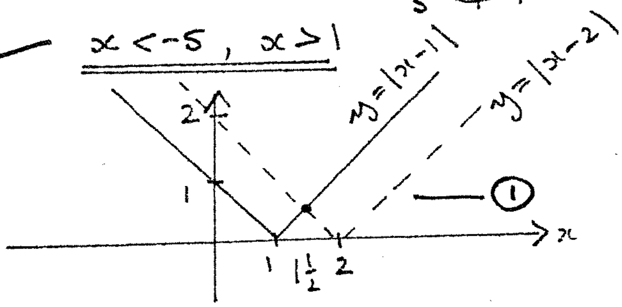
b) $x^2+4x-5>0$

$(x+5)(x-1)>0$



② — $x<-5,\ x>1$

c)



— ①

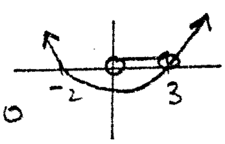
$x > 1\tfrac{1}{2}$ — ①

d) $\frac{6}{x} > x-1$

$\therefore\ 6 > x^2 - x$ since $x>0$

$x^2-x-6<0$

$(x-3)(x+2)<0$



$0<x<3$ — ②

Question 2 (9 marks)

a) $\int_2^8 f(x)-3\,dx = \int_2^8 f(x)\,dx - \int_2^8 3\,dx$ — ①

$= 24 - \left[3x\right]_2^8$ — ①

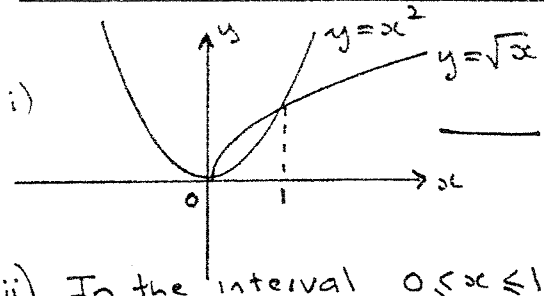
$= 24 - \left[24-6\right]$

① — $= 6$

b) $\frac{d}{dx}\left(\frac{1}{x^2+1}\right) = \frac{-2x}{(x^2+1)^2}$ — ①

$\int \frac{2x}{(x^2+1)^2}\,dx = \frac{-1}{x^2+1} + c$ — ②

c) i)



— ①

ii) In the interval $0 \leq x \leq 1$ $y=x^2$ lies below $y=\sqrt{x}$ and above the x axis — ②

Question 3 (8 marks)

a) $y = x^3+3x^2+3x-2$

$\frac{dy}{dx} = 3x^2+6x+3$

$\frac{d^2y}{dx^2} = 6x+6$

• st pts $\frac{dy}{dx}=0$ — ①

$3x^2+6x+3=0$

$x^2+2x+1=0$

$(x+1)^2=0$

• test pt $(-1,-3)$ $y''=0$ — ①

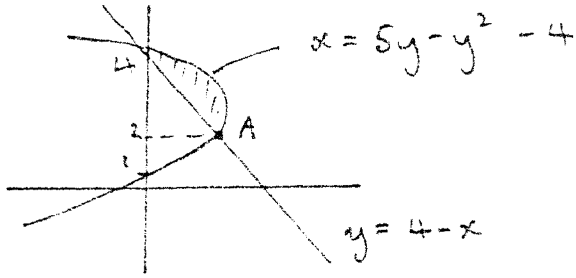
• test concavity for hory pt. inf. (or test gradient on either side) — ①

x	-2	-1	0
y''	−ve	0	+ve

concavity changes

$\therefore\ (-1,-3)$ is a horizontal point of inflexion — ①

b)



i) $4 - y = 5y - y^2 - 4$

$y^2 - 6y + 8 = 0$

$(y-4)(y-2) = 0$

$y = 4 \quad y = 2$

$\therefore A(2, 2)$ —— (1)

ii) $A_y = \int_2^4 (5y - y^2 - 4) - (4 - y)\,dy$ —— (1)

$= \int_2^4 (5y - y^2 - 4 - 4 + y)\,dy$

$= \int_2^4 6y - y^2 - 8\,dy$

$= \left[3y^2 - \frac{y^3}{3} - 8y\right]_2^4$ —— (1)

$= \left(48 - \frac{64}{3} - 32\right) - \left(12 - \frac{8}{3} - 16\right)$

$= 1\frac{1}{3}$ unit^2 —— (1)

b) $u^2 = x - 2 \qquad u = \sqrt{x-2}$

$u^2 + 2 = x$

$2u = \frac{dx}{du}$

$2u\,du = dx$

$\int \frac{x}{\sqrt{x-2}}\,dx = \int \frac{u^2+2}{\sqrt{u^2}\,u} \cdot 2u\,du$

(1) —— $= 2\int (u^2 + 2)\,du$

(1) —— $= 2\left[\frac{u^3}{3} + 2u\right] + c$

(1) —— $= \frac{2}{3}\left(\sqrt{x-2}\right)^3 + 4\sqrt{x-2} + c$

c) i)

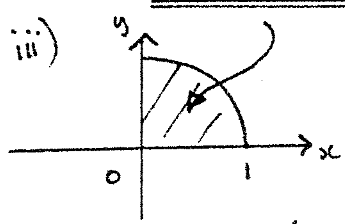
x	0	·25	·5	·75	1
$f(x)$	1	·9682	·8660	·6614	0
	F	y_1	y_2	y_3	L

(1)

ii) $\int_0^1 \sqrt{1-x^2}\,dx = \frac{1}{3} \times \cdot 25\left[1 + 0 + 4(\cdot 9682 + \cdot 6614) + 2(\cdot 8660)\right]$

$= \cdot 7709$ —— (2)

iii)



Area of unit circle $= 4 \times \cdot 7709$

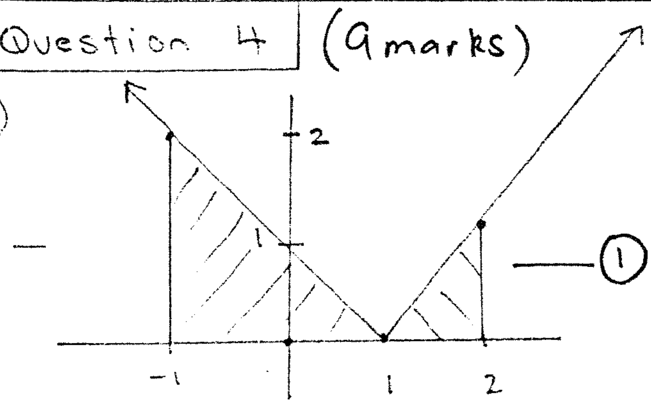
$\therefore \pi = 4 \times \cdot 7709$

$\pi \doteqdot 3{\cdot}084$ (3 dec pl)

(1)

Question 4 (9 marks)

a)

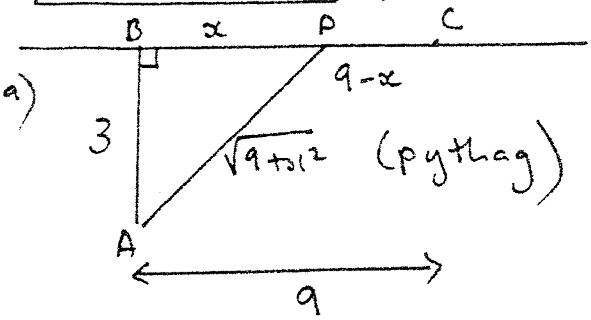


—— (1)

$\int_{-1}^{2} |x - 1|\,dx = \left(\frac{2 \times 2}{2}\right) + \frac{1}{2}$

Question 5 (7 marks)

a)



$D = ST \quad \therefore \quad T = \frac{D}{S}$

to travel A to P $\quad T_1 = \frac{\sqrt{9+x^2}}{80}$

to travel P to C $\quad T_2 = \frac{9-x}{100}$ ①

∴ total time $T = \frac{\sqrt{9+x^2}}{80} + \frac{9-x}{100}$ ①

b) $\frac{dT}{dx} = \frac{1}{2} \times \frac{2x(9+x^2)^{-1/2}}{80} + \frac{-1}{100}$

$= \frac{x}{80\sqrt{9+x^2}} - \frac{1}{100}$ ①

st pt $\frac{dT}{dx} = 0$

$\frac{x}{80\sqrt{9+x^2}} = \frac{1}{100}$

$100x = 80\sqrt{9+x^2}$

$100x^2 = 64(9+x^2)$

$100x^2 = 576 + 64x^2$

$36x^2 = 576$

$x^2 = 16$

$x = 4 \qquad x > 0$ ②

test max/min

x	3	4	5
$\frac{dT}{dx}$	−ve	0	+ve

min

∴ min time if $x = 4$ ①

$T = \left(\frac{\sqrt{9+16}}{80} + \frac{5}{100}\right)$ hrs

$T = \frac{9}{80}$ hrs ①

$T = 6\,^3/_4$ minutes

Question 6 (9 marks)

a) $u = 1 + x^3 \qquad \frac{du}{dx} = 3x^2$

① $\begin{cases} x = 1 & u = 2 \\ x = -1 & u = 0 \end{cases}$

$du = 3x^2\,dx$

$\frac{du}{3x^2} = dx$

$\therefore \int_0^2 x^2\sqrt{u}\,.\,\frac{du}{3x^2}$

$\frac{1}{3}\int_0^2 u^{1/2}\,du$ ①

$= \frac{1}{3}\left[\frac{2u^{3/2}}{3}\right]_0^2$ ①

$= \frac{2}{9}\left[\sqrt{8}\right]$

$= \frac{2}{9} \times 2\sqrt{2}$

$= \frac{4\sqrt{2}}{9}$ ①

b) $x = r \quad y = \frac{r^2}{a}$
$x = 2r \quad y = \frac{4r^2}{a}$ } limits (1)

$$V_y = \pi \int_{\frac{r^2}{a}}^{\frac{4r^2}{a}} (ay)\, dy$$

$$= \pi \left[\frac{ay^2}{2} \right]_{\frac{r^2}{a}}^{\frac{4r^2}{a}} \quad \text{— (1)}$$

$$= \frac{a\pi}{2} \left[\frac{16r^4}{a^2} - \frac{r^4}{a^2} \right]$$

$$= \frac{a\pi}{2} \left[\frac{15r^4}{a^2} \right]$$

$$= \frac{15\pi r^4}{2a} \quad \text{— (1)}$$

since $V = \frac{5}{6}\pi a^3$

and $h = \frac{4r^2}{a} - \frac{r^2}{a}$

$\therefore \quad h = \frac{3r^2}{a}$ — (1)

$$\frac{5}{6}\pi a^3 = \frac{15\pi r^4}{2a}$$

$$10\pi a^4 = 90\pi r^4$$

$$a^4 = 9r^4$$

$$\frac{a^4}{9} = r^4$$

$$\therefore r^2 = \frac{a^2}{3}$$

sub into $h = \frac{3r^2}{a}$

$$h = \frac{3}{a}\left(\frac{a^2}{3}\right)$$

$\underline{\underline{h = a}}$ — (1)