# SYDNEY TECHNICAL HIGH SCHOOL

# YEAR 12 HSC ASSESSMENT TASK 2

# MARCH 2005

# MATHEMATICS

## Extension 1

**Time Allowed:** 70 minutes

**Instructions:**

- Attempt all questions
- Start each question on a new page
- Show all necessary working
- The marks for each question are indicated next to the question
- Marks may be deducted for careless or badly arranged work
- Approved calculators may be used
- Marks indicated are a guide only and may be varied if necessary

Name: _____ Teacher: _____

| Question 1 | Question 2 | Question 3 | Question 4 | Question 5 | Question 6 | Total |
|---|---|---|---|---|---|---|
| | | | | | | |

**QUESTION 1**

a) Find $\int (3x+5)^6 \, dx$ **(1)**

b) (i) Differentiate $(x^3-1)^6$ **(1)**

(ii) Hence find $\int x^2(x^3-1)^5 \, dx$ **(2)**

c) Show why the curve $y = -x^3 + 6x^2$ has a non-horizontal point of inflexion when $x = 2$ **(2)**

d) Find the equation of the tangent in general form, to the curve $y = x^3 - 6x^2 + 3x + 2$ at the point where $\frac{d^2y}{dx^2} = 0$ **(4)**

**QUESTION 2 (Start a new page)**

a) For the curve $f(x) = \frac{x}{x^2-4}$

(i) Show that it is an odd function **(1)**

(ii) Show that $f(x)$ has no stationary points **(2)**

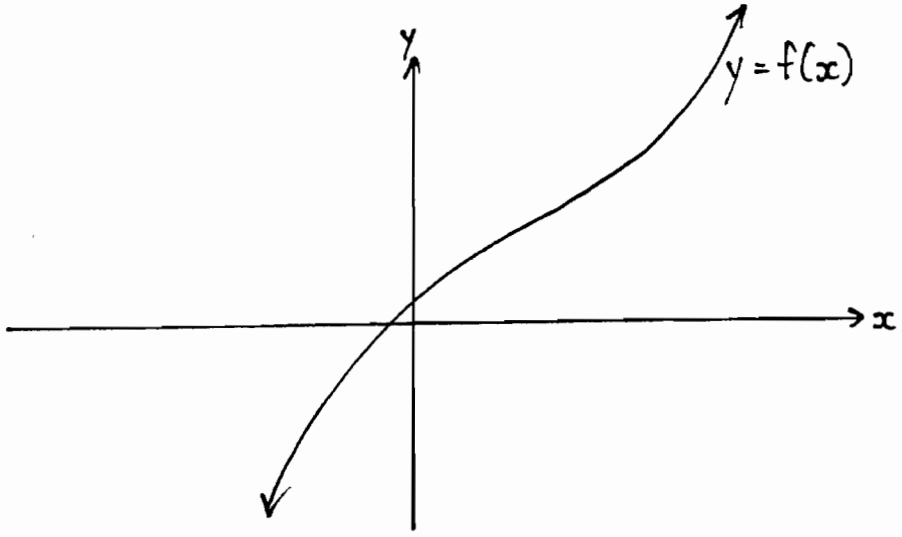
(iii) Find the equations of the vertical asymptotes **(1)**

(iv) Hence sketch the curve **(2)**

**QUESTION 2 (cont)**

b) Copy the graph of $y = f(x)$ below onto your answer sheet.

Sketch a graph of $y = f^1(x)$ on the same number plane, given $f^1(0) = 4$ **(2)**




c) For what values of $x$ is $f(x) = x^3 - 3x^2$ concave down? **(2)**

**QUESTION 3 (Start a new page)**

a) In the diagram, $PQRS$ is a rectangle with $PQ = 40cm$ and $SP = 10cm$.

The shaded portions are cut away, leaving the parallelogram $KLMN$

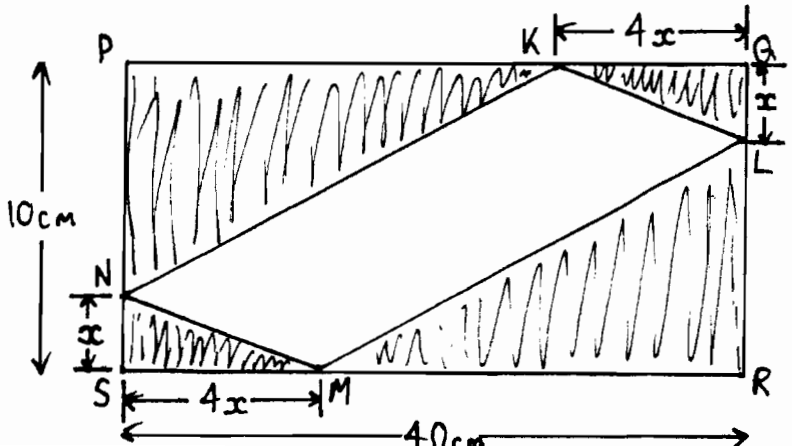
$QL = SN = x\,cm$ and $QK = SM = 4x\,cm$

(i) Show that the area of the parallelogram $KLMN$ is given by **(2)**

$A = 80x - 8x^2$

(ii) Find the allowable values of $x$ **(1)**

(iii) Find the value of $x$ for which $A$ is a maximum **(2)**



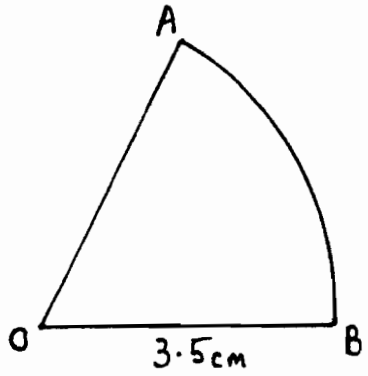
**Question 3 cont**

b) *A* sector *AOB* of a circle has a radius of 3.5cm. Its perimeter is 9.5cm

(i) Find the size of $\angle AOB$ in radians correct to one decimal place **(2)**

(ii) Find the length of the arc *AB* correct to one decimal place **(1)**

(iii) Find the area of the sector *AOB* correct to one decimal place **(2)**

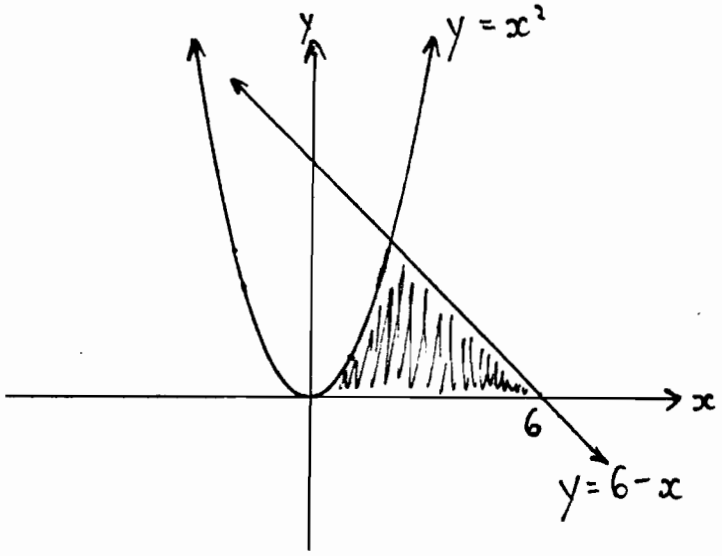


**QUESTION 4 (Start a new page)**

a) Find the equation of $\frac{dy}{dx}$ given

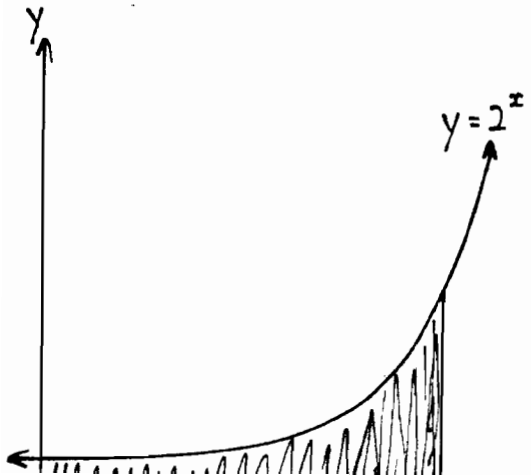
$\frac{d^2y}{dx^2} = 2x$ and when $x = 1$, $\frac{dy}{dx} = 2$. **(2)**

b) (i) Find the $x$ value of the point of intersection in the 1st quadrant of the 2 functions below **(2)**




(ii) Hence find the exact value of the shaded area **(3)**

c) Use the function values in the table below to estimate the shaded area using the Trapezoidal Rule (leave your answer in exact form)

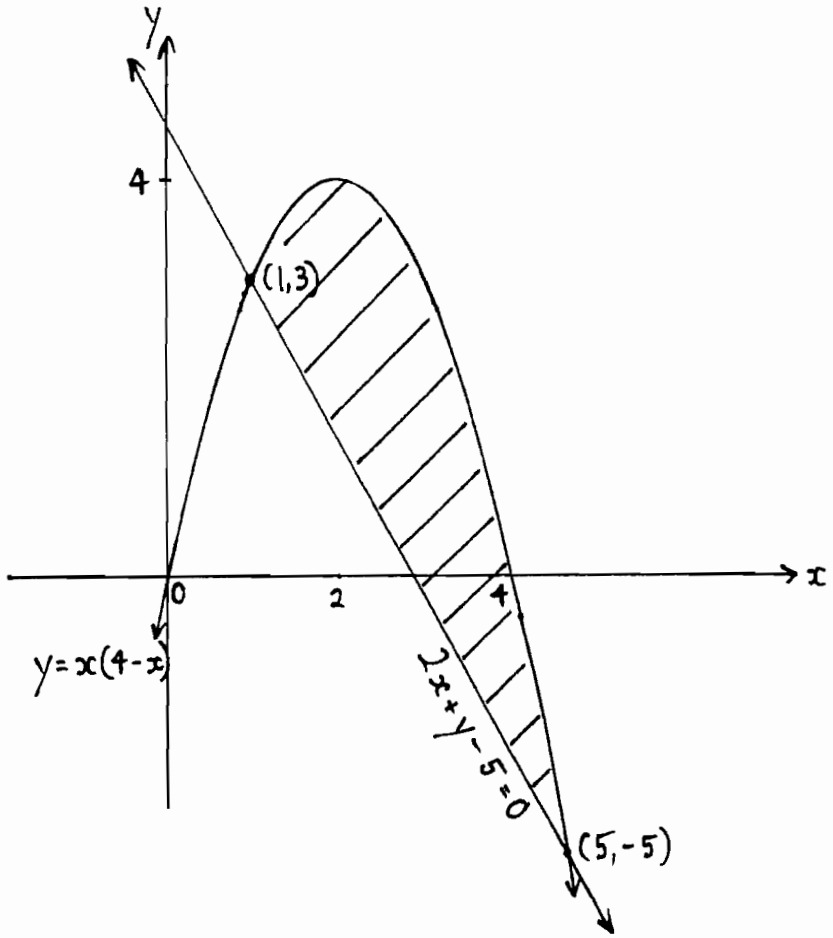



| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $y$ | 1 | $\sqrt{2}$ | 1 | $2\sqrt{2}$ | 4 |

**QUESTION 5 (Start a new page)**

a) Evaluate $\lim_{x \to 0} \frac{\cos 2x - 1}{x^2}$ **(2)**

b) On the graph below, the functions $2x + y - 5 = 0$ and $y = x(4 - x)$ intersect at the points indicated. Find the shaded area. **(4)**




c) Find $\int_0^3 \frac{x}{\sqrt{1+x}}\,dx$ using the substitution $x = u^2 - 1$ where $u > 0$ **(4)**

**QUESTION 6 (Start a new page)**

a) The curve $y = \frac{4}{x}$ is rotated around the $x-$ axis between $x = 1$ and $x = 3$ to form a solid.

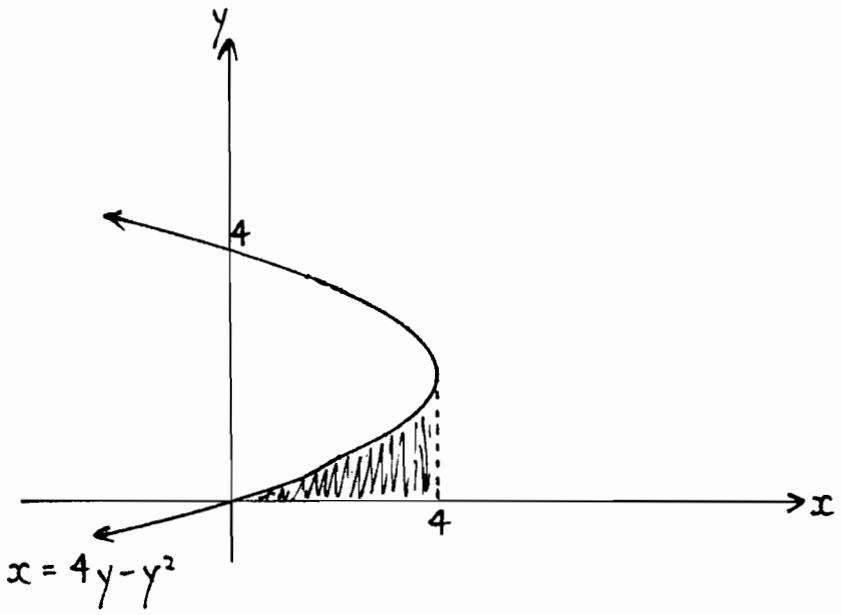
(i) Write down the integral which gives this volume **(1)**

(ii) Using your answer to part (i), complete a table with 5 function values for $x = 1, 1.5, 2, 2.5, 3$ **(1)**

(iii) Use Simpsons Rule to estimate the volume correct to one decimal place **(2)**

b) Sketch the curve $y = 2\sin(2x + \pi)$ for $-\pi \leq x \leq \pi$ **(3)**

c) Find the exact value of the shaded area below **(3)**



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## Question 1

i) $\int (3x+5)^6\,dx$

$= \frac{(3x+5)^7}{7\times 3}$

$= \frac{(3x+5)^7}{21} + C$ (1) give if they forget +C

ii) $\frac{d}{dx}(x^3-1)^6$

$= 6(x^3-1)^5 \times 3x^2$

$= 18x^2(x^3-1)^5$ (1)

iii) $\int x^2(x^3-1)^5\,dx$

$\frac{d}{dx}(x^3-1)^6 = 18x^2(x^3-1)^5$

$\frac{1}{18}\frac{d}{dx}(x^3-1)^6 = x^2(x^3-1)^5$ (1)

$\int \frac{1}{18}\frac{d}{dx}(x^3-1)^6\,dx = \int x^2(x^3-1)^5\,dx$

$= \frac{1}{18}(x^3-1)^6 + C$ (1)

c) $y = -x^3 + 6x^2$

$y' = -3x^2 + 12x$

$y'' = -6x + 12 = 0$ for a pt. of inflexion

$6x = 12$

$x = 2$ (1)

Since $y' \neq 0$ at $x=2$ it must be a non-horizontal pt. of inflexion (1)

OR

| x | 1 | 2 | 3 |
|---|---|---|---|
| y' | + | 0 | + |

| x | 1 | 2 | 3 |
|---|---|---|---|
| y'' | − | 0 | + |

d) $y = x^3 - 6x^2 + 3x + 2$

$y' = 3x^2 - 12x + 3$

$y'' = 6x - 12 = 0$

$\Rightarrow x = 2 \quad \therefore y = -8$ (1)

At $x=2$, $y' = m = -9$ (1)

$y - y_1 = m(x - x_1)$ (1)

$\rightarrow y + 8 = -9x + 18$

$9x + y - 10 = 0$ (1)

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## Question 2

a) (i) Odd if $f(-x) = -f(x)$

$\frac{-x}{(-x)^2-4} = -\frac{x}{x^2-4}$

$\frac{-x}{x^2-4} = -\frac{x}{x^2-4}$ ✓ (1)

(ii) Stat. pts. where $y' = 0$

$y' = \frac{x^2-4\times 1 - x\times 2x}{(x^2-4)^2}$

$= \frac{-x^2-4}{(x^2-4)^2} = 0$ (1)

if $-x^2 - 4 = 0$

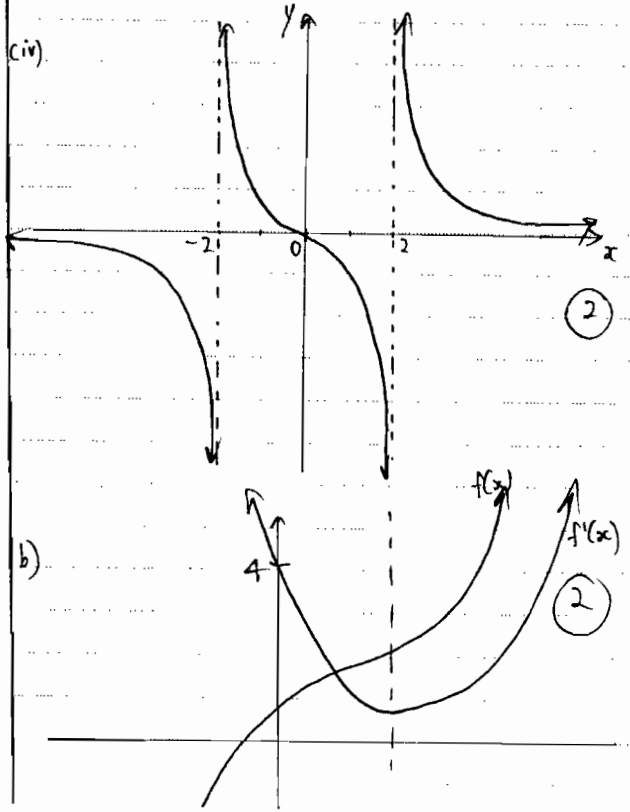
$x^2 = -4$ (1)

No soln.

(iii) Vertical Asymptotes where $x^2 - 4 = 0$

ie: $x = 2, \quad x = -2$ (1)

(iv)




(2)

-1 if $y=0$ not an asymptote

-1 if outside branches OK but between $x = \pm 2$ wrong

b)




(2)

-1 if parabola vertex not in line with inflexion

-1 if parabola touches or cuts x-axis

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c) $f(x) = x^3 - 3x^2$ is concave down where $f''(x) < 0$

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$ ①

$6x - 6 < 0$

$x < 1$ ①

## Question 3

a)(i) $A = 40 \times 10 - 2 \times \frac{1}{2} \times 4x \times x - 2 \times \frac{1}{2}(40 - 4x) \times (10 - x)$ ①

$= 400 - 4x^2 - (40 - 4x)(10 - x)$

$= 400 - 4x^2 - (400 - 40x - 40x + 4x^2)$

$= 400 - 4x^2 - 400 + 40x + 40x - 4x^2$

$= 80x - 8x^2$ as required ①

(ii) $0 < x < 10$ ①

$x < 10$ OK for ①

(iii) $\frac{dA}{dx} = 80 - 16x = 0$ for a maximum

$80 = 16x$

$x = 5$ ①

| $x$ | 4 | 5 | 6 |
|---|---|---|---|
| $\frac{dA}{dx}$ | + | 0 | − |

$\therefore$ $x = 5$ gives a maximum ①

b)(i) $P = 9 \cdot 5 = 3 \cdot 5 \times 2 + r \times \theta$ ①

$9 \cdot 5 = 7 + 3 \cdot 5 \times \theta$

$2 \cdot 5 = 3 \cdot 5 \times \theta$

$\theta = 0 \cdot 7$ radians ①

(ii) $l = r \times \theta$

$l = 3 \cdot 5 \times 0 \cdot 7$

$= 2 \cdot 5$ cm ①

(iii) $A = \frac{1}{2} r^2 \theta$ ①

$= \frac{1}{2} \times 3 \cdot 5^2 \times 0 \cdot 7$

$= 4 \cdot 3 \text{ cm}^2$ ①

Deduct 1 mark once only if answers not given correct to one d.p.

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## Question 4

a) $\frac{d^2y}{dx^2} = 2x$

$\frac{dy}{dx} = x^2 + C$ ①

When $x = 1$, $\frac{dy}{dx} = 2$

$2 = 1^2 + C$

$\therefore C = 1$

$\therefore \frac{dy}{dx} = x^2 + 1$ ①

b)(i) Solve $\begin{cases} y = x^2 \\ y = 6 - x \end{cases}$ simultaneously

$\therefore x^2 = 6 - x$

$x^2 + x - 6 = 0$ ①

$(x - 2)(x + 3) = 0$

$x = 2$ in 1st quadrant

(ii) $A = \int_0^2 x^2 \, dx + \int_2^6 6 - x \, dx$ ①

$= \left[\frac{x^3}{3}\right]_0^2 + \left[6x - \frac{x^2}{2}\right]_2^6$ ①

$= \frac{8}{3} + 36 - \frac{36}{2} - (12 - \frac{4}{2})$

$= \frac{8}{3} + 18 - 10$

$= 10\frac{2}{3}$ units$^2$ ①

c) $A \doteqdot \frac{h}{2}(y_0 + y_n + 2\Sigma y_{0 \text{ thers}})$ ①

$= \frac{0 \cdot 5}{2}(1 + 4 + 2(\sqrt{2} + 1 + 2\sqrt{2}))$

$= \frac{1}{4}(7 + 6\sqrt{2})$ ①

## Question 5

a) $\lim_{x \to 0} \frac{\cos 2x - 1}{x^2}$

$= \lim_{x \to 0} \frac{1 - 2\sin^2 x - 1}{x^2}$ ①

$= -2 \lim_{x \to 0} \frac{\sin^2 x}{x^2}$

$= -2 \times 1^2$

$= -2$ ①

b) $A = \int_1^5 x(4 - x) - (-2x + 5) \, dx$

$= \int_1^5 4x - x^2 + 2x - 5 \, dx$

$= \int_1^5 6x - x^2 - 5 \, dx$

$= \left[3x^2 - \frac{x^3}{3} - 5x\right]_1^5$

$= 75 - \frac{125}{3} - 25 - (3 - \frac{1}{3} -$

$- 10\frac{2}{3}$ units$^2$ ①

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) $\int_0^3 \frac{x}{\sqrt{1+x}}\,dx$

$x = u^2 - 1 \quad \therefore 1 + x = u^2$

$\frac{dx}{du} = 2u$

$dx = 2u\,du$

When $x = 0$, $u = 1$ $(u > 0)$

When $x = 3$, $u = 2$ $(u > 0)$

$\int_1^2 \frac{u^2 - 1}{u} 2u\,du$ ①

$2\int_1^2 u^2 - 1\,du$

$2\left[\frac{u^3}{3} - u\right]_1^2$ ①

$= 2\left(\frac{8}{3} - 2 - \left(\frac{1}{3} - 1\right)\right)$

$= \frac{8}{3}$ units$^2$ ①

Question 6

a)(i) $V = \pi \int y^2\,dx$

$= \pi \int_1^3 \left(\frac{4}{x}\right)^2 dx$ ①

$= \pi \int_1^3 \frac{16}{x^2}\,dx$ or

(ii)

| $x$ | 1 | 1·5 | 2 | 2·5 | 3 |
|---|---|---|---|---|---|
| $y$ | 16 | 7$\frac{1}{9}$ | 4 | 2·56 | $\frac{16}{9}$ |

$y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4$ ①

(iii) $V \doteq \pi \times \frac{h}{3}\left[y_0 + y_n + 4(y_{odds}) + 2(y_{evens})\right]$ ← OR ①

$= \pi \times \frac{0.5}{3}\left[16 + \frac{16}{9} + 4(7\frac{1}{9} + 2{\cdot}56) + 2 \times 4\right]$

$= 33{\cdot}8$ ①

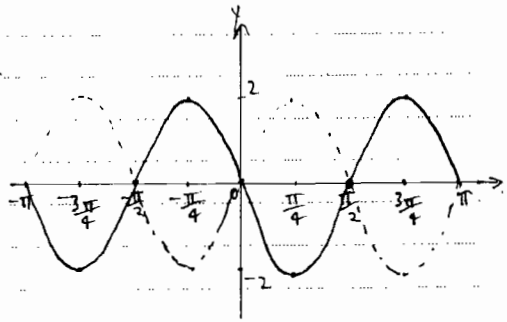
b) $y = 2\sin(2x + \pi)$

Amplitude 2

Period $\frac{2\pi}{2} = \pi$

Phase shift $\frac{\pi}{2}$ left

} -1 for each error

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c) $A = \int x\,dy$

When $x = 4$,

$4 = 4y - y^2$

$y^2 - 4y + 4 = 0$

$(y - 2)^2 = 0$

$y = 2$ ①

Area = Rectangle $- \int_0^2 x\,dy$

$= 2 \times 4 - \int_0^2 4y - y^2\,dy$

$= 8 - \left[2y^2 - \frac{y^3}{3}\right]_0^2$ ①

$= 8 - \left(8 - \frac{8}{3} - 0\right)$

$= \frac{8}{3}$ or $2\frac{2}{3}$ units$^2$ ①