SYDNEY TECHNICAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 2

MARCH 2013

Mathematics Extension 1

General Instuctions

- Working time - 70 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in questions 6 to 13
- Start each question on a new page
- A table of standard integrals is provided at the back of this paper

Total marks - 47

Section 1 - 5 marks

Attempt Questions 1 – 5.
Allow about 7 minutes for this section.

Section 2 - 42 marks

Attempt Questions 6 – 11.
Allow about 63 minutes for this section.

Name :____________________________

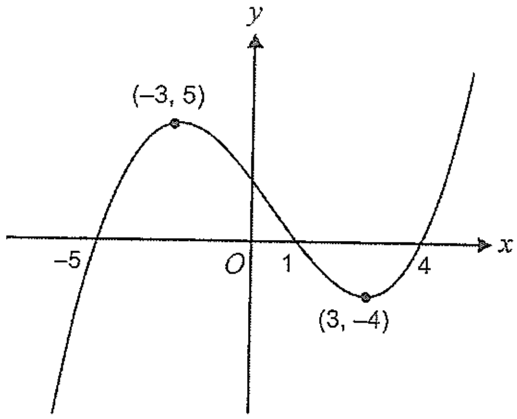
Teacher :____________________________

Section 1

5 marks
Attempt Questions 1 – 5
Allow about 7 minutes for this section

Use the multiple-choice answer sheet in your answer booklet for Questions 1 – 5.
Do not remove the multiple-choice answer sheet from your answer booklet.

1



For the graph of $y = f(x)$ shown above, $f'(x)$ is negative when

(A) $-3 < x < 3$

(B) $x < -3 \ \ or \ \ x > 3$

(C) $1 < x < 4$

(D) $-5 < x < 1 \ \ or \ \ x > 4$

2 The volume of the solid of revolution formed by rotating the graph of

$y = \sqrt{9 - (x-1)^2}$ about the x-axis is given by

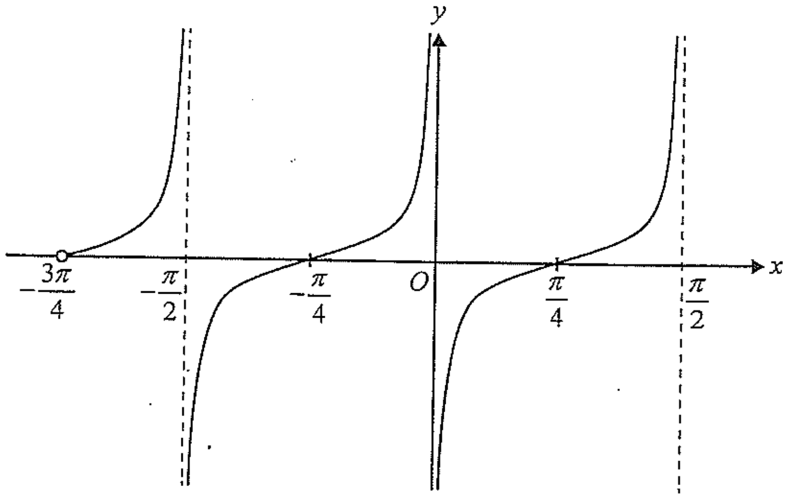
(A) $\pi \int_{-3}^{3} (9 - (x-1)^2)\, dx$

(B) $\pi \int_{-2}^{4} \sqrt{9 - (x-1)^2}\, dx$

(C) $\pi \int_{-2}^{4} (9 - (x-1)^2)\, dx$

(D) $\pi \int_{-2}^{4} (9 - (x-1)^2)^2\, dx$

3 A section of the graph of f is shown below.



The rule of f could be

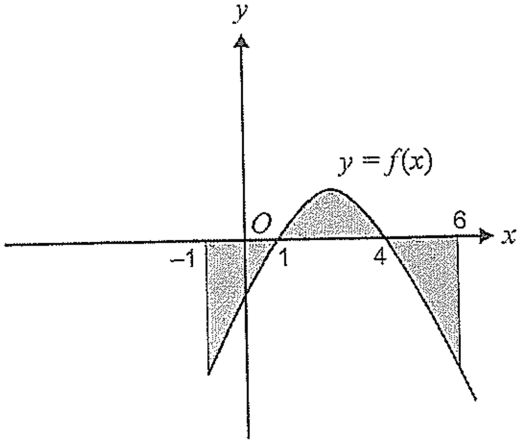
(A) $f(x) = \tan\left(x - \frac{\pi}{4}\right)$

(B) $f(x) = \tan\left(2\left(x - \frac{\pi}{4}\right)\right)$

(C) $f(x) = \tan\left(2\left(x - \frac{\pi}{2}\right)\right)$

(D) $f(x) = \tan\left(\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right)$

4



The total area of the shaded regions in the diagram is given by

(A) $\int_{-1}^{6} f(x)\,dx$

(B) $\int_{1}^{4} f(x)dx + 2\int_{4}^{6} f(x)dx$

(C) $-\int_{-1}^{1} f(x)dx + \int_{1}^{4} f(x)dx - \int_{4}^{6} f(x)dx$

(D) $-\int_{1}^{-1} f(x)dx + \int_{1}^{4} f(x)dx - \int_{6}^{4} f(x)dx$

5 If $\frac{d^2y}{dx^2} = x^2 - x$ and $\frac{dy}{dx} = 0$ at $= 0$, then the graph of y will have

(A) A maximum turning point at $x = 0$ and a minimum turning point at $x = 1$

(B) A horizontal point of inflexion at $x = 0$ and $x = 1$, and a minimum turning point at $x = \frac{3}{2}$

(C) A horizontal point of inflection at $x = 0$, no other points of inflection and a minimum turning point at $x = \frac{3}{2}$

(D) A horizontal point of inflexion at $x = 0$, a minimum turning point at $x = \frac{3}{2}$ and a point of inflexion at $x = 1$

Section 2

42 marks
Attempt Questions 6 – 11
Allow about 63 minutes for this section
Start each question on a new page

Question 6 (7 marks)

a) Evaluate i) $\log_9 3$ 1

ii) $\lim_{x \to 0} \frac{4x}{\tan 2x}$ 1

b) Differentiate $\frac{\sin 2x}{x}$ 2

c) Find the area bounded by the curves $y = x^2$ and $y = 8x - x^2$. 3

Question 7 (7 marks) Start a new page

a) Use the substitution $u = 3x - 1$ 3

to find $\int \frac{x}{(3x-1)^3}\,dx$

b) Let $f(x) = \frac{(x+10)^3}{x}$.

i) Find any stationary points of $y = f(x)$ and determine their nature. 3

ii) Sketch $y = f(x)$ clearly labelling any important features. 1

Question 8 (7 marks) Start a new page

a) Solve $2\log(x-1) - \log(x+3) = \log 2$ 3

b) i) Show that $\frac{d}{d\theta}(tan^3\theta) = 3\,sec^2\theta\,(\,sec^2\theta - 1)$ 1

ii) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{4}} \sec^4\theta \, d\theta$ 3

Question 9 (7 marks) Start a new page

a) Find a primitive of $\frac{1}{(2x-1)^3}$ 1

b) i) Draw a neat sketch of the curves $y = \cos x$ and $y = \sin 2x$ 2

for $0 \leq x \leq \pi$ on the same diagram.

ii) Find the x coordinates of the points of intersection of $y = \cos x$ 2

and $y = \sin 2x$ for $0 \leq x \leq \pi$.

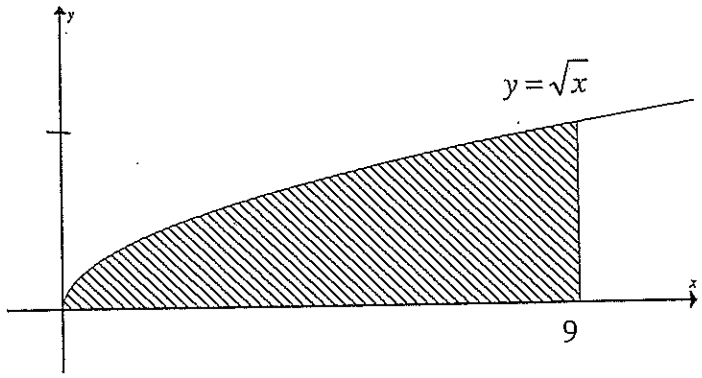
iii) Find the area bound by $y = \cos x$, $y = \sin 2x$ and the $x - axis$ 2

for $0 \leq x \leq \pi$.

Question 10 (7 marks) Start a new page

a) Find $\int \sin^2 4x\, dx$ 2

b) Find the volume of the solid formed when the shaded area under the curve $y = \sqrt{x}$, shown below, is rotated about the y axis. 3

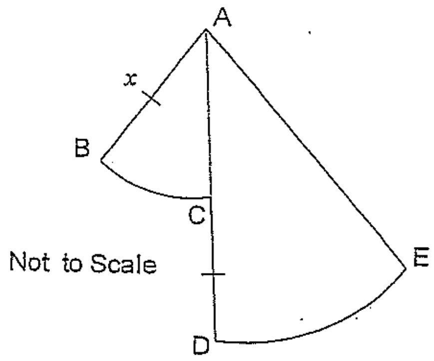


c) Use Simpson's rule with 3 function values 2

to approximate $\int_0^4 \frac{3}{1+\sqrt{x}}\, dx$ correct to 2 decimal places.

Question 11 (7 marks) Start a new page

a) Two sectors make up a company logo as shown below.



Both sectors have centre A, AB=CD, AB= x and AC bisects angle BAE.

Let angle BAC= θ radians.

i) If the area of the logo is 8π square units, show that $\theta = \frac{16\pi}{5x^2}$ 1

ii) Find an expression for the perimeter (P) of the logo in terms of x . 2

b) Evaluate $\int_0^{\frac{\pi}{3}} \frac{\sin 2x}{\left(1+\sin^2 x\right)^2}\,dx$ using the substitution $u = sin^2x$ 4

End of paper

Student Name: ______ Teacher Name: ______

SOLUTIONS – EXT 1 MARCH 2013

1. A
2. C
3. B
4. C
5. D

QUESTION 6

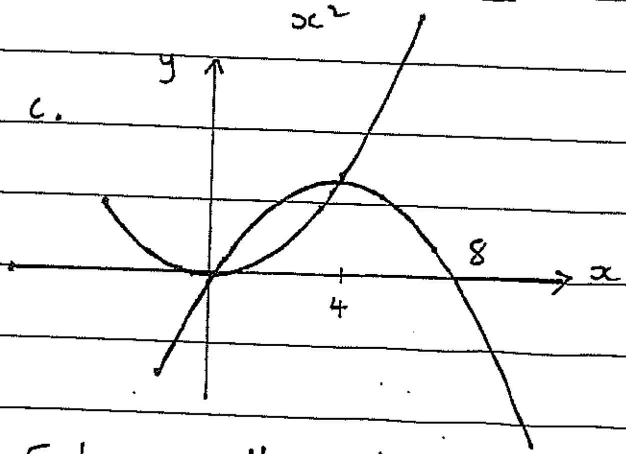
a. i) $\frac{1}{2}$

ii) 2

b. $\frac{d}{dx}\left(\frac{\sin 2x}{x}\right)$

$= \frac{2x\cos 2x - \sin 2x}{x^2}$

c.



Solve simultaneously.

$x^2 = 8x - x^2$

$2x(x-4) = 0$

$x = 0, 4$

$\therefore A = \int_0^4 8x - x^2 - x^2 \, dx$

$= \int_0^4 8x - 2x^2 \, dx$

$= \left[4x^2 - \frac{2}{3}x^3\right]_0^4$

$= 21\frac{1}{3}$ sq. units

QUESTION 7

a) $\int \frac{x}{(3x-1)^3} dx$ $\quad u = 3x-1$

$du = 3\,dx$

$x = \frac{1}{3}(u+1)$

$= \frac{1}{9}\int \frac{u+1}{u^3} du$

$= \frac{1}{9}\int u^{-2} + u^{-3} \, du$

$= \frac{1}{9}\left[-u^{-1} - \frac{1}{2}u^{-2}\right]$

$= -\frac{1}{9}\left[\frac{1}{3x-1} + \frac{1}{2(3x-1)^2}\right] + c$

b) $f(x) = \frac{(x+10)^3}{x}$

i) $f'(x) = \frac{3x(x+10)^2 - (x+10)^3}{x^2}$

$= \frac{(x+10)^2(2x-10)}{x^2}$

st. pts when $y' = 0$

i.e. $x = -10, 5$

test $x = 5$

x	4	5	6
y'	–ve	0	+ve

\ _ /

$\therefore$ min at $(5, 675)$

Student Name: ______________________ Teacher Name: ______________________

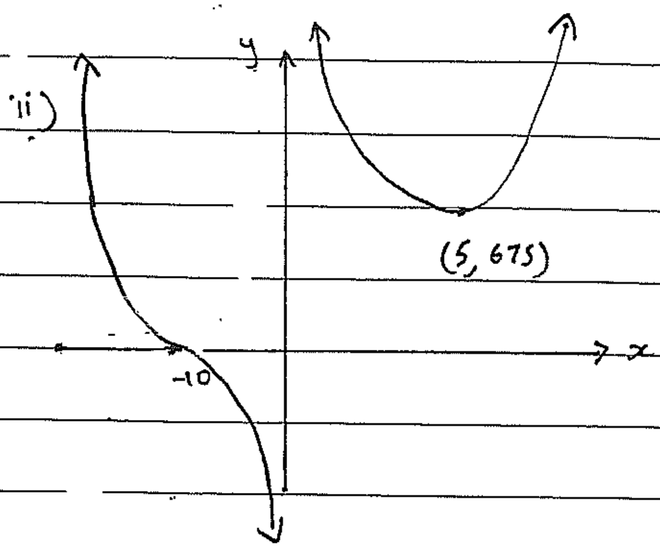
$x = -10$

x	-11	-10	-9
y'	$-ve$	0	$-ve$

\ — \

$\therefore$ horizontal pt. of inflexion at $(-10, 0)$

ii)



QUESTION 8

a. $2\log(x-1) - \log(x+3) = \log 2$

$$\log \frac{(x-1)^2}{x+3} = \log 2$$

$$\frac{(x-1)^2}{x+3} = 2$$

$$x^2 - 2x + 1 = 2x + 6$$

$$x^2 - 4x - 5 = 0$$

$$(x-5)(x+1) = 0$$

$x = 5, -1$ but $x = -1$ does not satisfy equation

$\therefore \quad x = 5$

b. i) $\frac{d}{d\theta}(\tan^3\theta)$

$$= 3\tan^2\theta \sec^2\theta$$

$$= 3(\sec^2\theta - 1)\sec^2\theta$$

$$= 3\sec^2\theta(\sec^2\theta - 1)$$

ii)

$$\int_0^{\frac{\pi}{4}} \sec^4\theta \, d\theta$$

$$= \frac{1}{3}\left[\int_0^{\frac{\pi}{4}} 3\sec^2\theta + \frac{d}{d\theta}(\tan^3\theta)\, d\theta\right]$$

$$= \left[\tan\theta + \frac{1}{3}\tan^3\theta\right]_0^{\frac{\pi}{4}}$$

$$= \left(\tan\frac{\pi}{4} + \frac{1}{3}\tan^3\frac{\pi}{4}\right) - (0)$$

$$= \frac{4}{3}$$

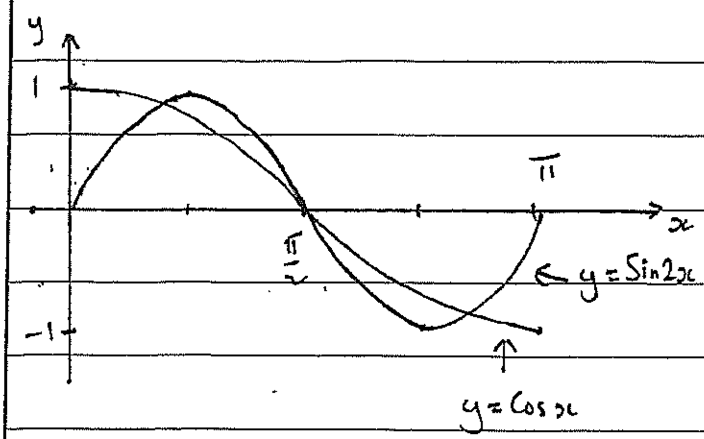
QUESTION 9

a. $\dfrac{-(2x-1)^{-2}}{4} + c$

or $\dfrac{-1}{4(2x-1)^2} + c$

Student Name: ______________________ Teacher Name: ______________________

b. i)



ii) $\cos x = \sin 2x$

$\cos x - 2\sin x \cos x = 0$

$\cos x (1 - 2\sin x) = 0$

$\cos x = 0 \quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

iii) $A = 2\int_0^{\frac{\pi}{6}} \sin 2x \, dx +$

$2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos x \, dx$

$= 2\left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}} + 2\left[\sin x\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$

$= 2\left(-\frac{1}{2}\cos\frac{\pi}{3} + \frac{1}{2}\cos 0\right)$

$+ 2\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{6}\right)$

$= 2\left(-\frac{1}{4} + \frac{1}{2}\right) + 2\left(1 - \frac{1}{2}\right)$

$= \frac{3}{2}$ sq units

QUESTION 10

a) $\cos 2A = 1 - 2\sin^2 A$

$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$

$\therefore \int \sin^2 4x \, dx$

$= \frac{1}{2}\int 1 - \cos 8x \, dx$

$= \frac{1}{2}\left[x - \frac{1}{8}\sin 8x\right] + C$

b) $V = \pi \times 9^2 \times 3 - \pi\int_0^3 y^4 \, dy$

$= 243\pi - \pi\left[\frac{1}{5}y^5\right]_0^3$

$= 243\pi - \pi\left[\frac{1}{5}\cdot 3^5 - 0\right]$

$= \frac{972\pi}{5}$ cubic units

c)

x	0	2	4
y	3	1.243	1

$\int_0^4 \frac{3\,dx}{1+\sqrt{x}}$

$\approx \frac{2}{3}\left[3 + 1 + 4 \times 1.243\right]$

$= 5.98$

Student Name: ______________________ Teacher Name: ______________________

QUESTION 11

a. i) $A = \frac{1}{2}r^2\theta$

$8\pi = \frac{1}{2}x^2\theta + \frac{1}{2}(2x)^2\theta$

$8\pi = \frac{1}{2}x^2\theta + 2x^2\theta$

$16\pi = x^2\theta + 4x^2\theta$

$16\pi = 5x^2\theta$

$\theta = \frac{16\pi}{5x^2}$

ii) $P = 4x + x\theta + 2x\theta$

$= 4x + 3x\theta$

$= 4x + 3x\left(\frac{16\pi}{5x^2}\right)$

$= 4x + \frac{48\pi}{5x}$

b. $\int_0^{\frac{\pi}{3}} \frac{\sin 2x}{(1+\sin^2 x)^2}\,dx$

$u = \sin^2 x$

$du = 2\sin x\cos x\,dx$

$= \sin 2x\,dx$

$x=0 \quad u=0$

$x=\frac{\pi}{3} \quad u=\frac{3}{4}$

$= \int_0^{\frac{3}{4}} \frac{du}{(1+u)^2}$

$= \int_0^{\frac{3}{4}} (1+u)^{-2}\,du$

$= \left[-(1+u)^{-1}\right]_0^{\frac{3}{4}}$

$= \left(\frac{-1}{1+\frac{3}{4}}\right) - \left(\frac{-1}{1}\right)$

$= \frac{3}{7}$