Name: ______________________

Teachers Name: ______________________

SYDNEY TECHNICAL HIGH SCHOOL

Year 12

Mathematics
Extension 1

Assessment Task 2

March 2014

Instructions

- Attempt all questions.
- Answers to be written on the paper provided.
- Start each question on a new page.
- All working must be shown. Full marks may not be awarded for poorly set out work.
- Indicated marks are a guide and may be changed slightly if necessary during the marking process.
- Approved calculators may be used.
- These questions must be handed in on top of your solutions.

SECTION 1: MULTIPLE CHOICE:

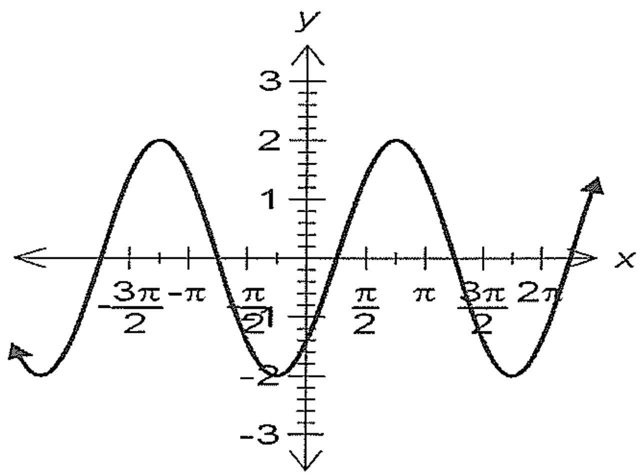
Answer on the Multiple Choice Answer Sheet provided.

1. For what values of x is the curve $y = 4x^2 - x^3$ concave upwards?

   A. $0 < x < \frac{8}{3}$ B. $x < \frac{4}{3}$

   C. $x > \frac{4}{3}$ D. $x < 0, x > \frac{8}{3}$

2. Below is the graph of the function $y = f(x)$.



   A possible equation for this function is:

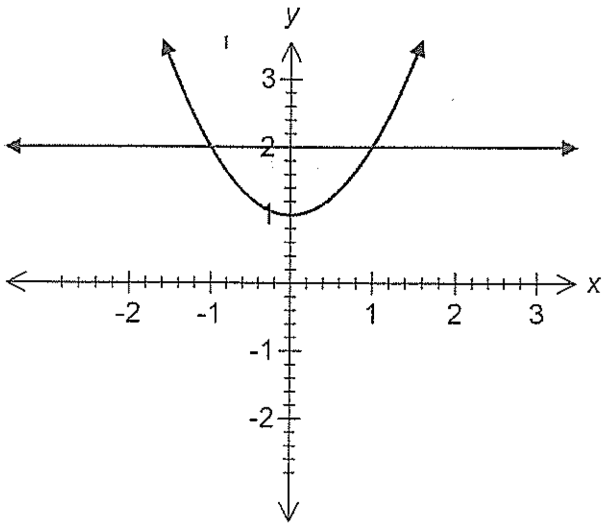
   A. $y = 2\sin(x + \frac{\pi}{4})$ B. $y = \sin 2(x - \frac{\pi}{4})$

   C. $y = -2\cos(x + \frac{\pi}{4})$ D. $y = 2\sin(x - \frac{\pi}{4})$

3. $\lim_{x \to 0} \frac{\sin x}{7x} =$

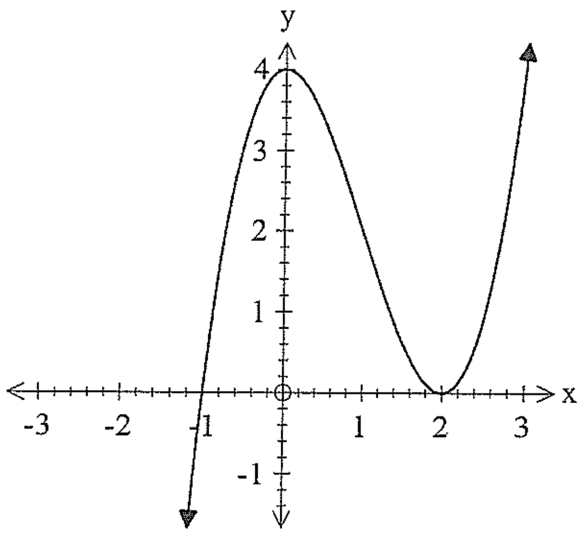
   A. 7 B. 0 C. $\frac{1}{7}$ D. ∞

4. The shaded area in square units is:



A. $\frac{4}{3}$ B. $\frac{10}{3}$ C. $\frac{2}{3}$ D. $\frac{20}{3}$

5. Below is a graph of $y = f(x)$.



Which of the following is true about $y = f'(x)$?

a) There is a stationary point at $x = -1$.

b) There is an inflexion point at $x = 1$.

c) The x intercepts are -1 and 2.

d) The y intercept is 0

SECTION 2: Write your solutions in the answer booklet provided.

QUESTION 6 (8 marks) **MARKS**

a) Differentiate tan^2x with respect to x. 1

b) Find the exact value of the gradient of the tangent to the curve $y = xsinx$ at the point where $x = \frac{\pi}{3}$. 2

c) Find i) $\int \frac{x^2+3x+2}{x+1}\, dx$ 2

ii) $\int_1^2 \frac{1}{2\sqrt{3x-2}}\, dx$ 3

QUESTION 7 **Start a new page** (8 marks)

a) The normal to the curve $y = 3tan2x$ at the point P $(\frac{\pi}{8}, 3)$ cuts the y axis at Q. Find the coordinates of Q in exact form. 3

b) A rectangle is cut from a circular disc of radius 3 cm.

i) Show that the area of the rectangle is given by $A = l\sqrt{36 - l^2}$, where l is the length of the rectangle. 2

ii) Find the area of the largest rectangle that can be produced. 3

QUESTION 8 **Start a new page** (8 marks) **MARKS**

a) Calculate the area bounded by the curve $y = x^2 - 4$ and the x axis, between $x = 1$ and $x = 3$. 3

b) i) Sketch $y = 3sin2x$ for $0 \leq x \leq \pi$. 1

ii) State the period and amplitude of $y = 3sin2x$. 2

iii) On the same set of axes, sketch $y = cosx$ for $0 \leq x \leq \pi$. 1

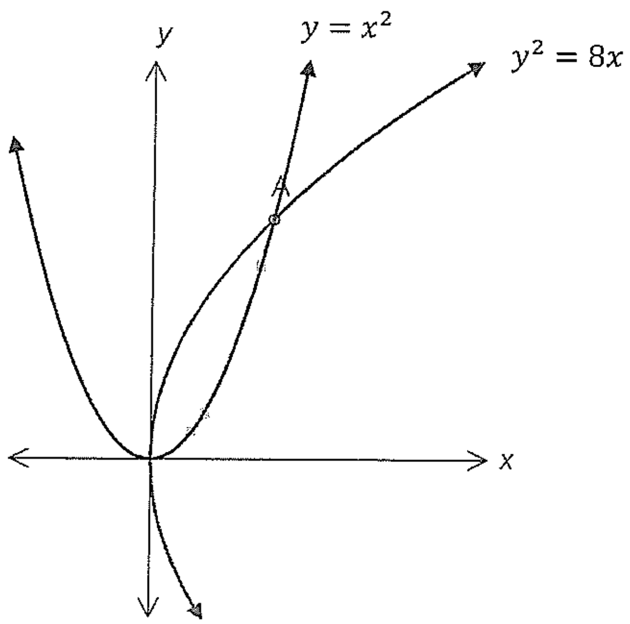
iv) State how many solutions exist for $3sin2x - cosx = 0$ for $0 \leq x \leq \pi$. 1

QUESTION 9 **Start a new page** (8 marks)

a) $\int 5\sin(3x + 2)\, dx$ 1

b) Using the substitution $u = 2 - x$, or otherwise, evaluate $\int_1^2 x\sqrt{2-x}\,dx$. 3

c) The curves $y^2 = 8x$ and $y = x^2$ intersect at A.



i) Find the coordinates of A. 1

ii) Calculate the volume of the solid generated when the shaded region is rotated about the y **axis**. 3

QUESTION 10 **Start a new page** (8 marks) **MARKS**

a) i) $\int_{-5}^{5} \sqrt{25 - x^2}dx$ represents the area of a semi-circle. 1
Calculate the exact area of this semi-circle.

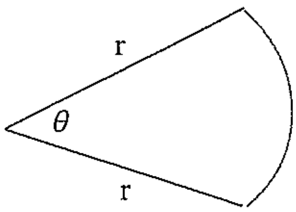
ii) Use Simpson's Rule with 5 function values to find an approximation to $\int_{-5}^{5} \sqrt{25 - x^2}dx$, correct to 3 decimal places. 2

iii) By comparing your results from parts i) and ii), find the percentage error in the use of the Simpson's Rule for the approximation of the actual area. 2

b) The area enclosed between the curve $y = 2sinx$ and the x axis for $0 \leq x \leq \pi$ is rotated about the x axis. Find the volume of the solid formed. 3

QUESTION 11 **Start a new page** (8 marks)

a)



The diagram shows the section of a circle of radius r metres and angle θ radians.

If the area of the sector is $50m^2$, find an expression for the perimeter of the sector in terms of r only. 2

b) For the curve $y = x^4 - 2x^3$,

i) Find the stationary points and determine their nature. 3

ii) Find any inflexion points. 2

iii) Sketch its graph, showing all important features. 1

END OF TEST

YEAR 12 MATHEMATICS EXTENSION 2 TASK 2 2014

SECTION 1

1. D
2. D
3. C
4. A
5. D

SECTION 2

QUESTION 6 8 marks

a) $\frac{d}{dx}\tan^2 x = 2\tan x\sec^2 x$ (1)

b) $y = x\sin x$

$\frac{dy}{dx} = \sin x + x\cos x$ (1)

at $x = \frac{\pi}{3}$, $\frac{dy}{dx} = \sin\frac{\pi}{3} + \frac{\pi}{3}\cos\frac{\pi}{3}$

$= \frac{\sqrt{3}}{2} + \frac{\pi}{3}\times\frac{1}{2}$

$= \frac{3\sqrt{3}+\pi}{6}$ (1)

c) i) $\int \frac{x^2+3x+2}{x+1}\,dx$

$= \int \frac{(x+1)(x+2)}{x+1}\,dx$

$= \frac{1}{2}x^2 + 2x + c$ (2) (must have 'c')

ii) $\int_1^2 \frac{1}{2\sqrt{3x-2}}\,dx$

$= \frac{1}{2}\int_1^2 (3x-2)^{-\frac{1}{2}}\,dx$ (1)

$= \frac{1}{2}\left[\frac{2}{3}(3x-2)^{\frac{1}{2}}\right]_1^2$ (1)

$= \frac{1}{3}\left(4^{\frac{1}{2}} - 1^{\frac{1}{2}}\right)$

$= \frac{1}{3}$ (1)

QUESTION 7 8 marks

a) $y = 3\tan 2x$

$\frac{dy}{dx} = 6\sec^2 2x$

at $x = \frac{\pi}{8}$, $\frac{dy}{dx} = 6\sec^2\frac{\pi}{4}$ (1)

$= 12$

$\therefore$ m of tangent $= 12$

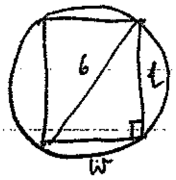
m of normal $= -\frac{1}{12}$ (for perpendicular lines $m_1m_2 = -1$)

$y - 3 = -\frac{1}{12}\left(x - \frac{\pi}{8}\right)$ (1)

$12y + x - 36 - \frac{\pi}{8} = 0$

at $x = 0$ $y = 3 + \frac{\pi}{96}$

$\therefore$ Co-ords of Q are $\left(0, 3+\frac{\pi}{96}\right)$ (1)

b) 

i) $6^2 = l^2 + w^2$ (Pythagoras Theorem)

$w = \sqrt{36 - l^2}$ (1)

$A = lw$

$= l\sqrt{36-l^2}$ (1)

ii) $\frac{dA}{dl} = (36-l^2)^{\frac{1}{2}} + l \times \frac{1}{2}(36-l^2)^{-\frac{1}{2}} \times -2l$

$= (36-l^2)^{-\frac{1}{2}}\left((36-l^2) - l^2\right)$

$= \frac{36-2l^2}{\sqrt{36-l^2}}$ (1)

$\frac{d^2A}{dl^2} = \frac{-4l\sqrt{36-l^2} - \frac{1}{2}(36-l^2)^{-\frac{1}{2}} \times -2l\,(36-2l^2)}{36-l^2}$

$= \frac{l(36-l^2)^{\frac{1}{2}}\left(-4(36-l^2)+36-2l^2\right)}{36-l^2}$

$= \frac{l\,(-108+2l^2)}{36-l^2}$ (1)

stat pts at $\frac{dA}{dx} = 0$

$36 - 2l^2 = 0$

$l = \pm 3\sqrt{2}$ but l is +ve

$A = 3\sqrt{2}\sqrt{36-18}$

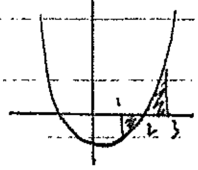
$= 18$

at $l = 3\sqrt{2}$, $\frac{d^2A}{dx^2} < 0$ $\therefore$ Maximum (1)

$\therefore$ Maximum area is 18 units^2

QUESTION 8   8 marks

a)



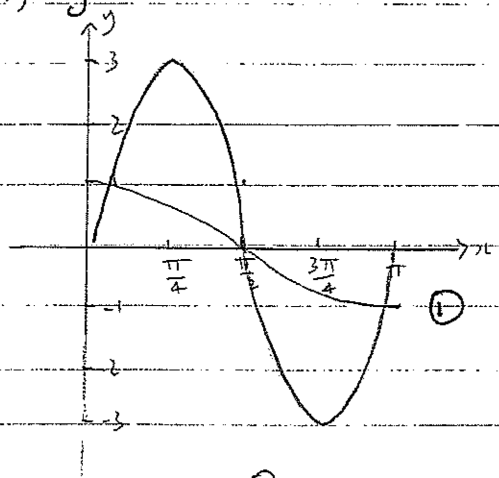
$$A = \left| \int_1^2 (x^2-4)\,dx \right| + \int_2^3 (x^2-4)\,dx$$ (1)

$$= \left| \left[\tfrac{1}{3}x^3 - 4x \right]_1^2 \right| + \left[\tfrac{1}{3}x^3 - 4x \right]_2^3$$ (1)

$$= \left| \left(\tfrac{8}{3} - 8\right) - \left(\tfrac{1}{3} - 4\right) \right| + (9-12) - \left(\tfrac{8}{3} - 8\right)$$

$= 4$ units^2 (1)

b) i) $y = 3\sin 2x$



ii) Period $= \pi$ (1), amplitude $= 3$ (1)

iii) on graph

iv) $3\sin 2x = \cos x$

3 solutions (1)

QUESTION 9   8 marks

a) $-\frac{5}{3}\cos(3x+2) + c$ (1)

(Don't deduct for no 'c')

b) $u = 2 - x \rightarrow x = 2 - u$

$du = -dx$ at $x = 1$, $u = 1$; $x = 2$, $u = 0$ (1)

$$\int_1^2 x\sqrt{2-x}\,dx = \int_1^0 (2-u)\sqrt{u}\,.-du$$ (1)

$$= \int_0^1 \left(2u^{\frac{1}{2}} - u^{\frac{3}{2}}\right) du$$

$$= \left[\tfrac{4}{3}u^{\frac{3}{2}} - \tfrac{2}{5}u^{\frac{5}{2}} \right]_0^1$$

$$= \frac{4}{3} - \frac{2}{5}$$

$$= \frac{14}{15}$$ (1)

c) i) $x^4 = 8x$

$x(x^3 - 8) = 0$

$x = 0,\ x = 2$

$y = 0,\ y = 4$

$\therefore$ Co-ords of A are $(2, 4)$ (1)

ii) $$V = \pi \int_0^4 \left(y - \frac{y^4}{64} \right) dy$$ (1)

$$= \pi \left[\tfrac{1}{2}y^2 - \frac{y^5}{320} \right]_0^4$$ (1)

$$= \pi \left(8 - \frac{16}{5} \right)$$

$$= \frac{24\pi}{5} \text{ units}^3$$ (1)

QUESTION 10   8 marks

a) i) $A = \frac{1}{2} \times \pi \times 5^2$ (1)

$= \frac{25\pi}{2}$ units^2

ii)

x	-5	$-2\frac{1}{2}$	0	$2\frac{1}{2}$	5
y	0	$\frac{5\sqrt{3}}{2}$	5	$\frac{5\sqrt{3}}{2}$	0

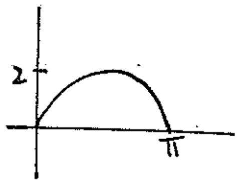
$\int_{-5}^{5} \sqrt{25 - x^2}\, dx$

$\doteqdot \frac{2\frac{1}{2}}{3}\left(0 + 0 + 2(5) + 4\left(\frac{5\sqrt{3}}{2} + \frac{5\sqrt{3}}{2}\right)\right)$ (1)

$\doteqdot 37.201$ units^2 (3 dp) (1)

iii) $\frac{\text{ii)} - \text{i)}}{\text{i)}} \times 100 = 5.269\%$ (3 dp) (2)

b)



$V = \pi \int_0^{\pi} 4\sin^2 x\, dx$ (1)

$= 4\pi \int_0^{\pi} \frac{1}{2}(1 - \cos 2x)\, dx$

$= 2\pi \left[(x - \frac{1}{2}\sin 2x \right]_0^{\pi}$ (1)

$= 2\pi(\pi - \frac{1}{2}\sin 2\pi - 0)$

$= 2\pi^2$ units^3 (1)

QUESTION 11   8 marks

a) $A = \frac{1}{2}r^2\theta = 50$

$\theta = \frac{100}{r^2}$

$l = r\theta$

$= r \times \frac{100}{r^2}$ (1)

$= \frac{100}{r}$

$P = 2r + \frac{100}{r}$ (1)

b) i) $y = x^4 - 2x^3$

$\frac{dy}{dx} = 4x^3 - 6x^2$

$\frac{d^2y}{dx^2} = 12x^2 - 12x$

stat pts at $\frac{dy}{dx} = 0$

$4x^3 - 6x^2 = 0$

$2x^2(2x - 3) = 0$

$x = 0 \quad x = \frac{3}{2}$ (1)

$y = 0 \quad y = -\frac{27}{16}$

at $x = 0$, $\frac{d^2y}{dx^2} = 0$

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
$\frac{d^2y}{dx^2}$	>0	0	<0

(1)

$\therefore$ Concavity changes

$\therefore$ Horizontal inflexion pt at $(0,0)$

at $x = \frac{3}{2}$, $\frac{d^2y}{dx^2} > 0$ (1)

$\therefore$ Concave upwards

$\therefore$ Minimum turning point at $\left(\frac{3}{2}, -\frac{27}{16}\right)$

iii) For inflexion pts $\frac{d^2y}{dx^2} = 0$

$12x^2 - 12x = 0$

$12x(x - 1) = 0$

$x = 0, \quad x = 1$

$y = 0 \quad y = -1$ (1)

See ii)

at $x = 1$:

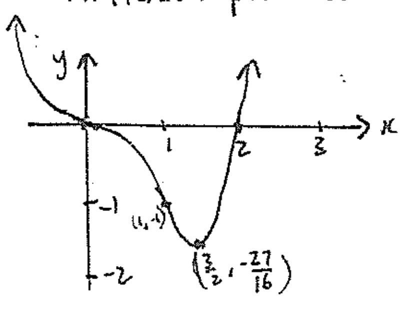
x	$\frac{1}{2}$	1	1.1
$\frac{d^2y}{dx^2}$	<0	0	>0

(1)

$\therefore$ Concavity changes

$\therefore$ Inflexion point at $(1, -1)$

iv)



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