TRINITY GRAMMAR SCHOOL
MATHEMATICS DEPARTMENT

YEAR 12 2010 ASSESSMENT TASK 3

MATHEMATICS

(EXTENSION 1)

Time Allowed - *two hours*

WEIGHTING 20% towards final result

Outcomes referred to: P2, P4, P5, P6, P7, P8, H1, H2, H4, H5, H6, H7, H9

INSTRUCTIONS:

1. Attempt **ALL** questions.
2. Show all necessary working.
3. **Begin** each question on a **new page**.
4. Each question is of equal value. Mark values are shown beside each part.
5.Non-programmable silent Board of Studies approved calculators are permitted.
6..If requested, additional writing sheets may be obtained from the examinations supervisor upon request.
7.A double sided A4 page of notes is permitted to be referred to throughout this task.

Question 1.(12marks) (Start this question on a New page) **Marks**

(a) Find the acute angle between the lines $y = 5x - 1$ and $6x - 3y + 1 = 0$. Give your answer correct to the nearest minute. **3**

(b) Solve $\frac{3}{x-2} > 4$. **3**

(c) Solve $x^2 \leq 9x$ and draw your answer on a number line. **3**

(d) The angle between $y = mx + 1$ and $x - 2y = 0$ is 45^0.

Find the possible value(s) of m. **3**

Question 2.(12marks) (Start this question on a New page)

~~(a) Find the coordinates of the point that divides the join of (-3,2) and (5,7) externally in~~ the ratio 1:3. **2**

(b) The position of a particle moving along the x axis is given by:

$x = \frac{t^3}{3} - 3t^2 + 8t$ where x and t are measured in centimetres and seconds respectively. Find:

(i) expressions for the velocity and acceleration of the particle. **2**

(ii) when and where the particle first comes to rest. **2**

(iii) the distance travelled by the particle in the first 3 seconds. **2**

(c) Evaluate $\sin^{-1}(\frac{1}{2}) + \cos^{-1}(1)$ **2**

(d) Find $\int \frac{2}{\sqrt{4-25x^2}} dx$ **2**

Question 3.(12marks) (Start this question on a New page)

(a) Solve for $0^0 \le \theta \le 360^0$

(i) $2\sin 2\theta = 1$ **2**

(ii) $8\sin^2\theta - 2\cos^2\theta = \cos 2\theta$ (to nearest minute) **4**

(b) (i) Show that $\dfrac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$ **2**

(ii) Hence find the value of $\tan 22\frac{1}{2}^0$ in simplest exact value. **2**

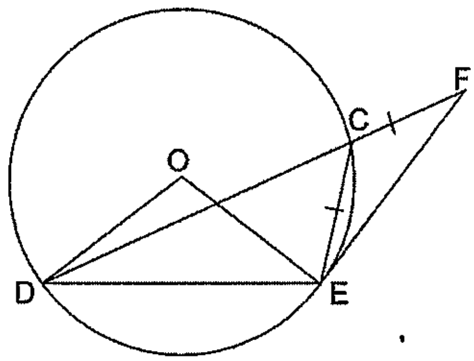
(c) Find the derivative of $\cos^{-1}(\frac{x}{3})$ **2**

Question 4.(12marks) (Start this question on a New page)

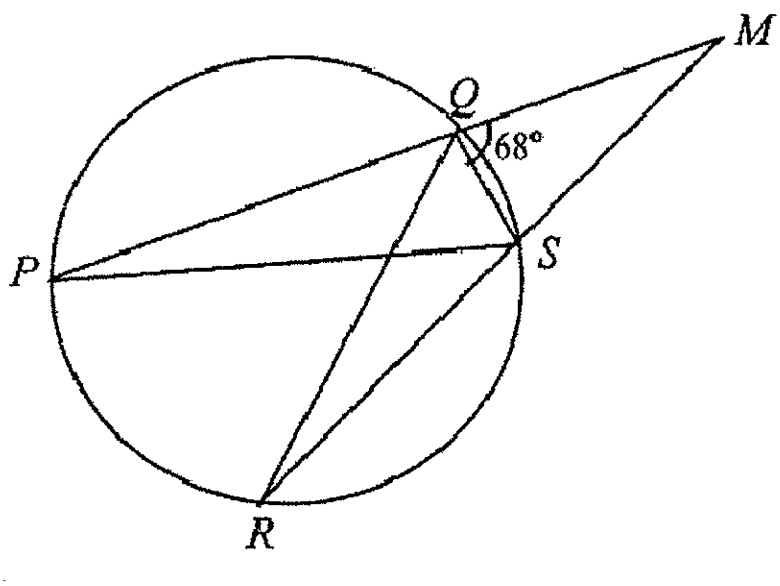
(a) In the figure, FE is a tangent to the circle, centre O.
D and F are joined so that EC = CF.

(i) If $\angle CEF = x$, prove that DE = EF. **2**

(ii) Express $\angle DOE$ in terms of x. **2**



(b) In the diagram, $MQ = MS$ and $\angle MQS = 68^0$. Copy or trace this diagram and prove that $MP = MR$. **4**



(c) (i) Show that the point P(-2,0) lies on the line joining A(7,-3) and B(-5,1)

(ii) Find the ratio in which it divides the line. **4**

Question 5.(12marks) (Start this question on a New page)

(a) Forty five percent of a population are of blood group O, 40% are of blood group A and the remainder are of neither group O nor group A.

Three people are chosen at random from the population

By drawing a tree diagram, or otherwise, find the probability that

(i) all three people are of blood group A; **1**

(ii) two of the people are of blood group A and the other is group O; **1**

(iii) there is one person each of group O, group A, and neither group O nor group A. **2**

(b) The amount A grams of a given carbon isotope in a dead tree trunk is given by $A = A_0 e^{-kt}$

where A_0 and k are positive constants, and where time t is measured in years from the death of the tree.

(i) Show that A satisfies the equation $\frac{dA}{dt} = -kA$, **1**

(ii) Find the value of k if the amount of isotope present is halved every 5500 years. **2**

(iii) For a particular dead tree trunk the amount of isotope is only 15% of the original amount in the living tree. How long ago did the tree die? **2**

(c) Sketch the graph of $y = 2\sin^{-1}(\frac{x}{3})$ indicating clearly its domain and range. **3**

Question 6.(12marks) (Start this question on a New page)

(a) If $\tan\frac{\theta}{2}=t$, show that

$$\frac{\sin\theta+\sin\frac{\theta}{2}}{1+\cos\theta+\cos\frac{\theta}{2}}=t$$ **4**

(b) For the function $y=x^2-2x+1$,

(i) find a suitable domain such that this function has an inverse.

(ii) find the equation of this inverse and state its range. **4**

(c) Prove by Mathematical induction that $3^{2n}-1$ is divisible by 8, where n is a positive integer. **4**

Question 7.(12marks) (Start this question on a New page)

(a) Find $\frac{d}{dx}(x\tan^{-1}x)$ and hence show that

$$\int_0^1 \tan^{-1} x\,dx = \frac{\pi}{4} - \frac{1}{2}\log_e 2$$ **3**

(b) Find k if $\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{5} = \tan^{-1} k$ **3**

(c) The points $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ lie on the parabola $x^2 = 4ay$.

(i) Derive the equation of the chord joining P and Q. **1**

(ii) If PQ passes through $(2a,0)$ show that $p+q=pq$. **1**

(iii) Hence show that the locus of M, the mid point of PQ is a parabola. **2**

(iv) Find the vertex and focus of the locus of M. **2**

END OF PAPER

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Year 12 Ext 1.

Question 1

a) $m_1 = 5$ $\quad m_2 = 2$ $\quad$ 1 (one correct)

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{5-2}{1+5\times2}\right| = \frac{3}{11}$ $\quad$ 1 (correct fraction from above)

$\theta = 15^\circ 15'$ $\quad$ 1 (correct angle from above)

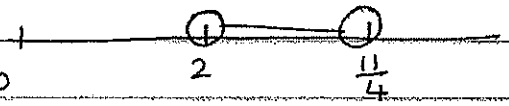
b) $\frac{3}{x-2} > 4$ $\quad$ $x \neq 2$ $\quad$ 1 (must have)

using zero $-\frac{3}{2} \not> 4$ check.

$3 = 4x - 8$

$11 = 4x$

$x = \frac{11}{4}$ $\quad$ 1 (or equival) $\quad$ $2 < x < \frac{11}{4}$ $\quad$ 1 (soln)



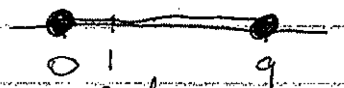
c) $x^2 \leq 9x$

$x^2 - 9x \leq 0$ $\quad$ assume $= 0$

$x(x-9) = 0$ $\quad$ 1 (or equivalent)

$x = 0$ or 9 $\quad$ 1 (soln).

$0 \leq x \leq 9$ $\quad$ 1 soln.



check 1 $\quad 1^2 \leq 9$ works.

d) $m_1 = m$ $\quad m_2 = \frac{1}{2}$ $\quad \theta = 45$

$\tan 45 = \left|\frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}\right|$ $\quad$ 1 (subst)

$1 = \frac{m - \frac{1}{2}}{1 + \frac{1}{2}m}$ $\quad$ or $\quad$ $1 = \frac{-(m - \frac{1}{2})}{1 + \frac{1}{2}m}$

$1 + \frac{1}{2}m = m - \frac{1}{2}.$ $\quad\quad$ $1 + \frac{1}{2}m = -m + \frac{1}{2}$

$1\frac{1}{2} = \frac{1}{2}m$ $\quad\quad$ $\frac{3}{2}m = -\frac{1}{2}$

$m = 3$ $\quad$ 1 soln $\quad\quad$ $m = -\frac{1}{3}$. $\quad$ 1 soln.

Question 2.

a) $(-3, 2)$ $(5, 7)$ $-1 : 3$

$x_3 = \dfrac{3\times -3 + -1\times 5}{-1+3}$ $y_3 = \dfrac{3\times 2 + -1\times 7}{2}$ ① (correct sub in x or y)

$= \dfrac{-14}{2}$ $= -\dfrac{1}{2}$ ① (Soln)

$= -7$

$(-7, -\frac{1}{2})$.

b) $x = \dfrac{t^3}{3} - 3t^2 + 8t$

i) $\dfrac{dx}{dt} = v = t^2 - 6t + 8$ ①

$\dfrac{d^2x}{dt^2} = a = 2t - 6$ ①

ii) rest $v = 0$

$t^2 - 6t + 8 = 0$

$(t-4)(t-2) = 0$

when $t = 4$ or 2 ①

where $x_4 = \frac{64}{3} - 48 + 32$ $x_2 = \frac{8}{3} - 12 + 16$

$= 21\frac{1}{3} - 48 + 32$ $= 2\frac{2}{3} - 12 + 16$

$= 5\frac{1}{3}$ $= 6\frac{2}{3}$ ①

(if only one answer for t & correct x ①)

iii) $t = 0$ $x = 0$

$t = 3$ $x = 9 - 27 + 24 = 6$ ① (for 6.)

$t = 2$ $x = 6\frac{2}{3}$.

Distance $= \underset{t=0 \text{ to } 2}{6\frac{2}{3}} + \underset{t=2 \text{ to } t=3}{\frac{2}{3}} = 7\frac{1}{3}$ ② (for $7\frac{1}{3}$)

c) $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}(1) = \frac{\pi}{6} + 0 = \frac{\pi}{6}$. ① (for either correct).

① (Answer)

d) $\displaystyle\int \frac{2\,dx}{\sqrt{4 - 25x^2}} = 2\int \frac{dx}{\sqrt{4-25x^2}}$

$= 2\sin^{-1}\left(\frac{5x}{2}\right) \times \frac{1}{5}$ ① (without $\frac{1}{5}$)

$= \frac{2}{5}\sin^{-1}\left(\frac{5x}{2}\right)$. ① (Ans)

Question 3

a) Solve $0 \le \theta \le 360$

$2\sin 2\theta = 1$

$\sin 2\theta = \frac{1}{2}$

$2\theta = \sin^{-1}(\frac{1}{2}) = 30, 150, 390, 510$ 1 (2 Answers)

$\theta = 15, 75, 195, 255$ 1 (Ans)

a) ii) $8\sin^2\theta - 2\cos^2\theta = \cos 2\theta$

$= \cos^2\theta - \sin^2\theta$ 1 (or equiv).

$9\sin^2\theta - 3\cos^2\theta = 0$

$9\sin^2\theta = 3\cos^2\theta$ $\div \sin^2\theta$. 1

$3\tan^2\theta = 1$

$\tan^2\theta = \frac{1}{3}$

$\tan\theta = \pm\frac{1}{\sqrt{3}}$ 1 ($\pm$)

$\theta = 30, 150, 210, 330.$ 1 (four soln)

(only 2 soln 3 marks).

b) i) $\frac{1-\cos 2x}{1+\cos 2x} = \tan^2 x$

$\frac{1-(\cos^2 x - \sin^2 x)}{1+(\cos^2 x - \sin^2 x)}$ 1 $\quad 1-\sin^2 x = \cos^2 x$

$\frac{\sin^2 x + \sin^2 x}{\cos^2 x + \cos^2 x}$ $\quad 1-\cos^2 x = \sin^2 x.$

$\frac{\cancel{2}\sin^2 x}{\cancel{2}\cos^2 x} = \tan^2 x$ 1 (soln) $\quad \tan x = \frac{\sin x}{\cos x}.$

ii) $\tan^2(22\tfrac{1}{2}°) = \frac{1-\cos 45}{1+\cos 45}$

$= \frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}+1}$ 1 (simplify)

$\tan(22\tfrac{1}{2}) = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}.$ 1 soln

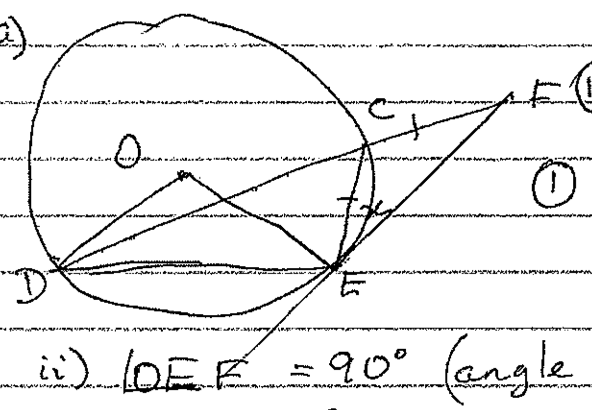
(ignore if went further)

c) $\frac{d}{dx}\left(\cos^{-1}\left(\frac{x}{3}\right)\right) = \frac{-1}{\sqrt{9-x^2}}$ 2 soln

1 (no negative sign)

Question 4

a)



i) $\angle CEF = x$ Prove $DE = EF$

① $\angle CFE = x$ (isosceles Δ).

① $\angle CDE = \angle CEF$ (angles in alternate segment from tangent)

$\therefore DE = EF$ (Base angles are equal)

ii) $\angle OEF = 90°$ (angle from centre to tangent $= 90°$)

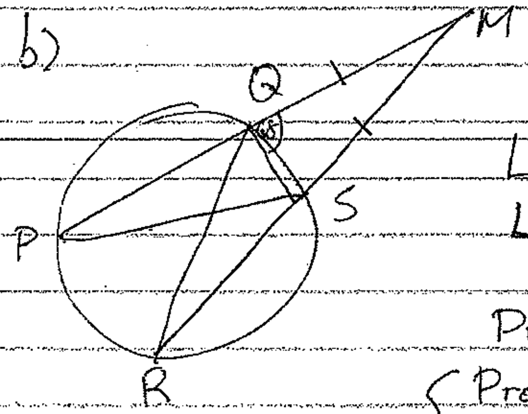
$\angle OEC = 90 - x$ ①

$\angle DCE = 2x$ (external angle of Δ) ′ ″

$\therefore \angle DOE = 4x$ (angle at centre is twice angle at circumfa)

(② for equivalent solution).

b)



Prove $MP = MR$.

$\angle MQS = 68 = \angle MSQ$ (Isosceles Δ)

Let $\angle PQR = x = \angle RSP$ (angles on same arc are equal)

$\angle PQS = 180 - 68$ (st lines).

$= 112° = \angle RSQ$ ①

Prove $\Delta PQS \equiv \Delta RSQ$

② Proof 1. $\angle PQS = \angle RSQ = 112°$ (as above)

2. QS is common

3. $\angle P = \angle R$ (angles standing on same arc are equal)

$\therefore \Delta PQS \equiv \Delta RSQ$ (AAS).

$PQ = RS$ (matching sides).

① $\therefore PM = QM + PQ = MS + SR = MR$

c) i) $M_{AB} = \frac{1 - -3}{-5-7} = \frac{4}{-12} = -\frac{1}{3}$

line AB $\quad y - -3 = -\frac{1}{3}(x-7)$

$3y + 9 = -x + 7$

$x + 3y + 2 = 0$ ① (line)

check $P(-2,0)$ lies on $\quad -2 + 0 + 2 = 0$ ✓ ① (subst)

$\therefore P$ lies on AB.

Q4c(ii) $x_3 = \frac{mx_2 + nx_1}{m+n}$ $y_3 = \frac{my_2 + ny_1}{m+n}$

$-2 = \frac{-5m+7n}{m+n}$ $0 = \frac{m-3n}{m+n}$

$-2m - 2n = -5m + 7n$ $0 = m - 3n$

$3m = 9n$ $m = 3n$ ① (eqn)

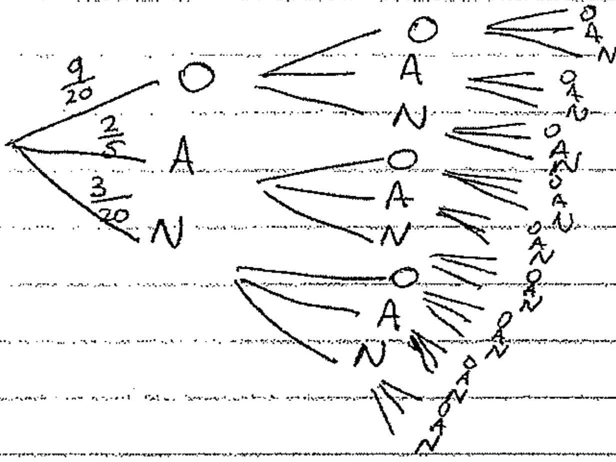
$m = 3n$

$\therefore$ m is 3 times bigger

Ratio 3:1 ① (correct ratio)

Question 5.

a) 45% is O 40% is A 15% neither



i) $P(3A) = \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$
$= \frac{8}{125}$ (1)

ii) $P(2A, O) = \frac{2}{5} \times \frac{2}{5} \times \frac{9}{20} \times 3$
$= \frac{108}{500} = \frac{27}{125}$ (1)

iii) $P(\text{1 of each}) = \frac{2}{5} \times \frac{9}{20} \times \frac{3}{20} \times 6$
$= \frac{81}{500}$

(1) (3 numbers mult) (1) for 6 ways.

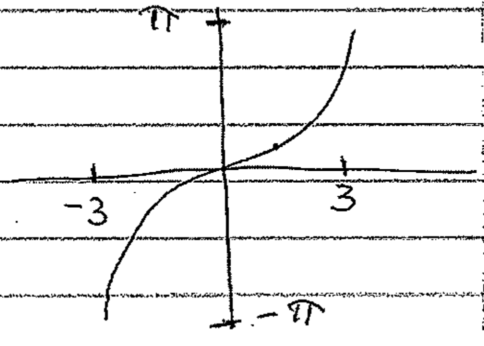
b) $A = A_0 e^{-kt}$ ---- (1)

i) $\frac{dA}{dt} = -k A_0 e^{-kt}$ from (1). (1)
$= -kA$.

ii) $t = 5500$ $A = \frac{1}{2} A_0$
$\frac{1}{2} A_0 = A_0 e^{-k \times 5500}$
$\frac{1}{2} = e^{-5500k}$ log both sides (1)
$\ln \frac{1}{2} = \ln e^{-5500k} = -5500k$
$k = \frac{\ln \frac{1}{2}}{-5500} = 1.260267 \times 10^{-4}$ (1)
$\approx 1.26 \times 10^{-4}$ or 0.000126.

iii) $\frac{3}{20} \cancel{A_0} = \cancel{A_0} e^{-kt}$
$\frac{3}{20} = e^{-kt}$
$\ln(\frac{3}{20}) = \ln e^{-kt} = -kt$ (1)
$t = \ln(\frac{3}{20}) \div (-k) = 15053$ yrs (1)
≈ 15000 or 15100

c) $y = 2 \sin^{-1} (\frac{x}{3})$
(1) showing x values
(1) showing y values.



Question 6

a) $\tan\frac{\theta}{2} = t$.

$\sin\theta = \sin\left(\frac{\theta}{2}+\frac{\theta}{2}\right) = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$ (1)

$\cos\theta = \cos\left(\frac{\theta}{2}+\frac{\theta}{2}\right) = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$ (1)

$$\frac{\sin\theta + \sin\frac{\theta}{2}}{1+\cos\theta+\cos\frac{\theta}{2}} = \frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \sin\frac{\theta}{2}}{1+\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2} + \cos\frac{\theta}{2}}$$

$$= \frac{\sin\frac{\theta}{2}\left(2\cos\frac{\theta}{2}+1\right)}{\cos^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+\cos\frac{\theta}{2}} \qquad 1-\sin^2\frac{\theta}{2} = \cos^2\frac{\theta}{2}$$

$$= \frac{\sin\frac{\theta}{2}\cancel{\left(2\cos\frac{\theta}{2}+1\right)}}{\cos\frac{\theta}{2}\cancel{\left(2\cos\frac{\theta}{2}+1\right)}} \quad (1)$$

$$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2} = t. \quad (1)$$

(or equivalent).

b) $y = x^2 - 2x + 1 = (x-1)^2$

i) Vertex $(1, 0)$ (1) Vertex

Domain $x \geq 1$ (1) or $x \leq 1$

ii) Inverse $x = y^2 - 2y + 1$

$x = (y-1)^2$

$\pm\sqrt{x} = y - 1$

$y = 1 \pm \sqrt{x}$ (1) (Must show both)

if $x \geq 1$ then $y = 1 + \sqrt{x}$ (1)

c) Prove $8 / 3^{2n} - 1$

Prove true for $n = 1$

$8 / 3^2 - 1 = 9 - 1 = 8$

$\therefore 8/8 = 1$ true for $n = 1$

Assume true for $n = k$.

$8 / 3^{2k} - 1$

$8M = 3^{2k} - 1$

$3^{2k} = 8M + 1$

Assume answer true for some positive integer M

Q6 c) Prove true for $n = k+1$

$8 \,/\, 3^{2(k+1)} - 1$

$3^{2k+2} - 1$

$= 3^2 \cdot 3^{2k} - 1$

now $3^{2k} = 8M + 1$

$\therefore \; 8 \,/\, 9(8M+1) - 1$

$72M + 9 - 1$

$72M + 8$

$8 \,/\, 8(9M+1)$

$\therefore$ true for $n = k+1$

$\therefore$ true for $n = 1, 2, \ldots$ true forall positive integer of $n \geq 1$.

Question 7

a) $\frac{d}{dx}(x\tan^{-1}x) = x \cdot \frac{1}{1+x^2} + 1 \cdot \tan^{-1}x$

$= \frac{x}{1+x^2} + \tan^{-1}x$ (1)

Now integrate both sides.

Now $\int_0^1 \frac{d}{dx}(x\tan^{-1}x)\,dx = \int_0^1 \frac{x}{1+x^2}\,dx + \int_0^1 \tan^{-1}x\,dx$

$\left[x\tan^{-1}x\right]_0^1 = \frac{1}{2}\left[\ln(1+x^2)\right]_0^1 + \int_0^1 \tan^{-1}x\,dx$ (1)

$\tan^{-1}1 - 0 = \frac{1}{2}(\ln 2 - \ln 1)$

$\tan^{-1}(1) = \frac{1}{2}\ln 2 + \int_0^1 \tan^{-1}x\,dx$

$\int_0^1 \tan^{-1}x\,dx = \tan^{-1}(1) - \frac{1}{2}\ln 2 = \frac{\pi}{4} - \frac{1}{2}\ln 2.$ (1)

b) $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}(k)$

$\tan(\underbrace{\tan^{-1}(\tfrac{1}{3})}_{\alpha} + \underbrace{\tan^{-1}(\tfrac{1}{5})}_{\beta}) = k$ (1)

$\tan(\alpha+\beta) = k$

$k = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} = \frac{\tan(\tan^{-1}\frac{1}{3}) + \tan(\tan^{-1}(\frac{1}{5}))}{1-\tan(\tan^{-1}(\frac{1}{3}))\tan(\tan^{-1}(\frac{1}{5}))}$ (1)

$= \frac{\frac{1}{3}+\frac{1}{5}}{1-\frac{1}{3}\times\frac{1}{5}} = \frac{\frac{8}{15}}{1-\frac{1}{15}} = \frac{8}{14} = \frac{4}{7}$ (1)

c) $P(2ap, ap^2)$ $\quad Q(2aq, aq^2)$

i) $m_{PQ} = \frac{aq^2-ap^2}{2aq-2ap} = \frac{a(q-p)(q+p)}{2a(q-p)} = \frac{q+p}{2}$

$y - ap^2 = \left(\frac{q+p}{2}\right)(x-2ap)$ (1)

$2y - \cancel{2ap^2} = (q+p)x - 2apq - \cancel{2ap^2}$

$(q+p)x - 2y - 2apq = 0$

ii) $(2a, 0)$

$(q+p)\times 2a - 0 - 2apq = 0$

$2aq + 2ap = 2apq \quad \div 2a$

$q + p = pq$ (1)

Q7 c iii) Mid Pt. $\left(\frac{2ap+2aq}{2} , \frac{ap^2+aq^2}{2} \right)$

$x = a(p+q)$ $\qquad y = \frac{ap^2+aq^2}{2}$

$\frac{x}{a} = p+q$ $\qquad = \frac{a}{2}(p^2+q^2)$

$y = \frac{a}{2}\left[(p+q)^2 - 2pq\right]$ 1 (getting eqn)

since $pq = p+q$

$y = \frac{a}{2}\left[(p+q)^2 - 2(p+q)\right]$

$y = \frac{a}{2}\left[\left(\frac{x}{a}\right)^2 - 2\left(\frac{x}{a}\right)\right]$ 1 soln.

iv) $\qquad y = \frac{a}{2}\left[\frac{x^2}{a^2} - \frac{2x}{a}\right]$

$y = \frac{x^2}{2a} - x$

$2ay = x^2 - 2ax$

$2ay = (x-a)^2 - a^2$

$2ay + a^2 = (x-a)^2$

$2a(y + \frac{a}{2}) = (x-a)^2$

Vertex $(a, -\frac{a}{2})$ 1 $\cancel{fo}$. Equation

$x^2 = 4ay$

but only $2a$ in eqn.

focal length $\frac{a}{2}$ not a

Focus $(a, 0)$. 1