TRINITY GRAMMAR SCHOOL

Mathematics Department

2012

HALF YEARLY EXAMINATION

HSC ASSESSMENT TASK 3

Year 12

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11 – 14
- Write your Board of Studies Student Number **and** Class Teacher on the writing booklet(s) **or** sheet(s) submitted
- **Weighting**: 30%

Total marks – 70

Section I Pages 3 – 6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 7 – 14

60 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 45 minutes for this section

Section I 10 marks

- Circle the correct response on the answer sheet provided
- Each question is worth 1 mark

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1 The exact value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is:

(A) $\frac{\pi}{3}$

(B) $-\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $-\frac{2\pi}{3}$

2 State the domain for which the function $y = 4x - x^2$ is an increasing, one-to-one function.

(A) $x \leq 2$

(B) $x < 2$

(C) $x > 0$

(D) $x < 4$

3 What is the value of $\lim_{x \to 0} \frac{\sin 3x}{x}$?

(A) $\frac{1}{3}$

(B) 1

(C) 3

(D) undefined

4 The domain of the function $y = \ln\left(x^2 + e\right)$ is:

(A) $x \geq 0$

(B) $x \leq 0$

(C) all real values of x

(D) all real x, $x \neq 0$

5 The range of the function $y = \ln\left(x^2 + e\right)$ is:

(A) $y \geq 0$

(B) $y \geq 1$

(C) $y \geq e$

(D) all real y, $y \neq 0$

6 The area under the curve $y = \sin x$ between $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$ and the x - axis is:

(A) -1

(B) 0

(C) 1

(D) 2

7 To find $\int_0^1 x\sqrt{x^2 + 1}\ dx$ using the substitution $u = x^2 + 1$, in terms of u the correct expression is:

(A) $\int_0^1 \sqrt{u}\ du$

(B) $\frac{1}{2}\int_0^1 \sqrt{u}\ du$

(C) $\int_1^2 \sqrt{u}\ du$

(D) $\frac{1}{2}\int_1^2 \sqrt{u}\ du$

8 The exact value of $\sin\left(\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(-\frac{4}{3}\right)\right)$ is:

(A) -1

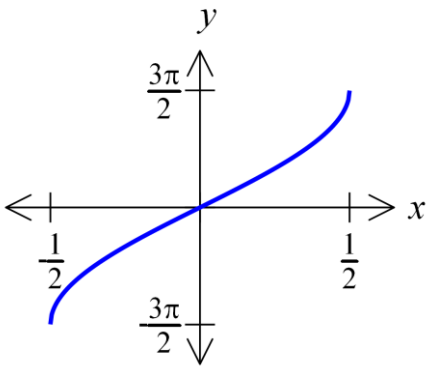
(B) $-\frac{7}{25}$

(C) $\frac{7}{25}$

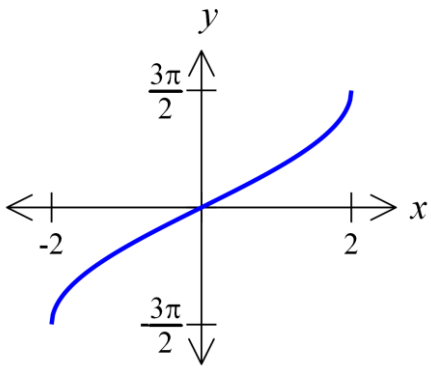
(D) 1

9 Which of the graphs below represents $y = 3\sin^{-1}\frac{x}{2}$?

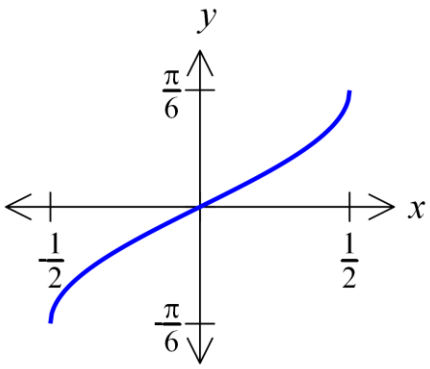
(A)



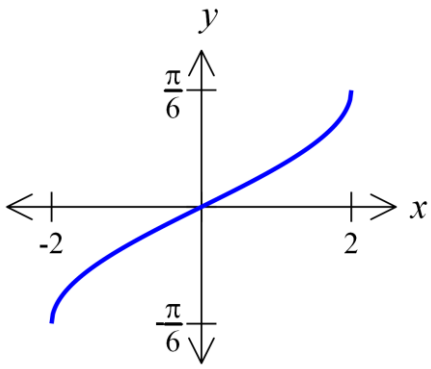
(C)



(B)



(D)



10 Below is the solution to the integral $\int_0^{\frac{\pi}{4}} \cos^2 x \, dx$.

$$\int_0^{\frac{\pi}{4}} \cos^2 x \, dx$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} (1 + \cos 2x) \, dx \qquad \textit{LINE I}$$

$$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x \right]_0^{\frac{\pi}{4}} \qquad \textit{LINE II}$$

$$= \frac{1}{2}\left[\left(\frac{\pi}{4} - \frac{1}{2}\sin \frac{\pi}{2} \right) - \left(0 - \frac{1}{2}\sin 0 \right) \right] \qquad \textit{LINE III}$$

$$= \frac{\pi}{8} - \frac{1}{4} \qquad \textit{LINE IV}$$

The line containing the FIRST mistake is:

(A) LINE I

(B) LINE II

(C) LINE III

(D) LINE IV

End of Section I

Section II 60 marks

- Begin each question in a new writing booklet or on a new answer sheet
- Show all necessary working
- Each question is worth 15 marks

Question 11 (15 marks)

a) Find:

i) $\frac{d}{dx}\sin(3x-2)$ **1**

ii) $\int \sec^2 3x\,dx$ **1**

iii) $\int \sin^2 2x\,dx$. **2**

b) Find the value of $\lim\limits_{x \to 0} \frac{1-\cos 2x}{x^2}$ **2**

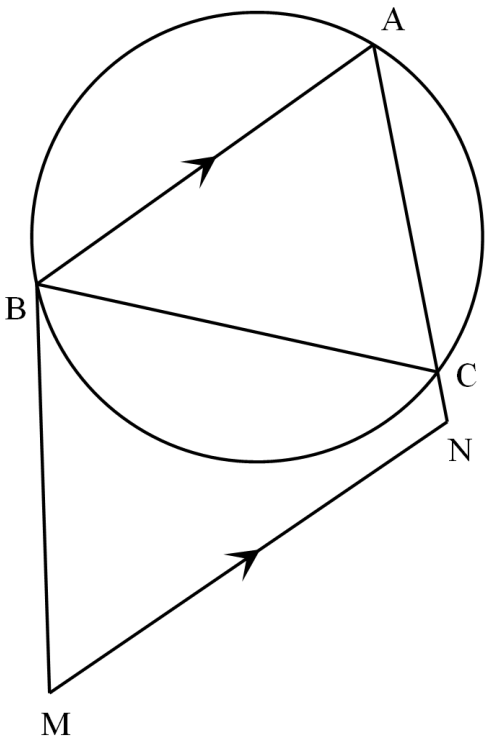
c)

i) Show that $\frac{1+\cos 2x}{\sin 2x} = \cot x$. **2**

ii) Hence, find the exact value of $\cot 15^\circ$. **1**

d) Use the principle of mathematical induction to show that $9^{n+2} - 4^n$ is divisible by 5 for all positive integers of n. **3**

e) *ABC* is a triangle inscribed in a circle. *M* is a point on the tangent to the circle at *B* and *N* is a point on *AC* produced so that *MN* is parallel to *BA*.



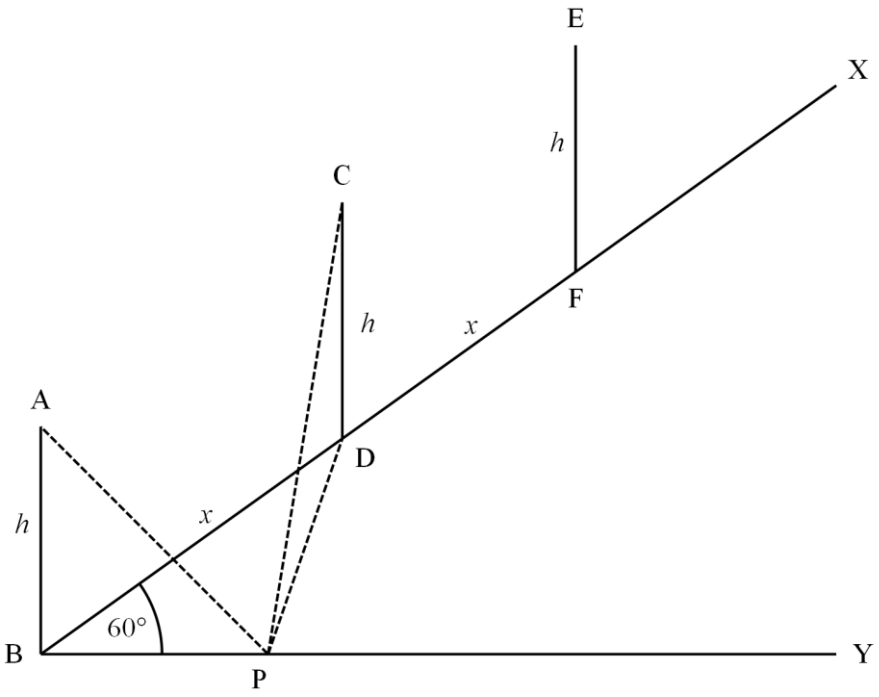
i) Copy or trace the diagram into your answer booklet.

ii) Give a reason why $\angle MBC = \angle BAC$. **1**

iii) Show that *MNCB* is a cyclic quadrilateral. **2**

Question 12 (15 marks) Begin a NEW answer booklet or answer sheet

a) In the diagram below, *BX* and *BY* represent two roads intersecting at an angle of 60°. On the road *BX* are situated three telegraph poles *AB*, *CD* and *EF*, all of equal height, the same distance, *x* metres, apart (i.e. *BD* = *DF* = *x*). P is a point on the road *BY* and the angles of elevation to *A* and *C* are 45° and 30° respectively.



i) Show that $DP = h\sqrt{3}$. **1**

ii) Show that $\angle BDP = 30^\circ$ and hence that ΔBPD is right angled at *P*. **2**

iii) Prove that $x = 2h$. **1**

iv) Show that $PF = h\sqrt{13}$ and hence show that the angle of elevation of *E* from *P* is approximately $15{\cdot}5^\circ$. **3**

b) State the domain and range of $y = \sin^{-1}(2x + 1)$ and then sketch the curve. **2**

c) The position x cm of a particle moving along an x-axis is given by $x = 3t + e^{-2t}$ where t is the time in seconds.

i) What is the position of the particle when $t = \frac{1}{2}$ second? Give your answer in exact form. **1**

ii) What is the initial velocity of the particle? **2**

iii) Show the initial acceleration of the particle is 4 cm/s^2. **2**

iv) Explain why the particle will never come to rest. **1**

Question 13 (15 marks) Begin a NEW answer booklet or answer sheet

a) Find the slope of the tangent to $y = \tan^{-1}\left(\sqrt{1-x}\right)$ at the point $y = \frac{\pi}{4}$. **2**

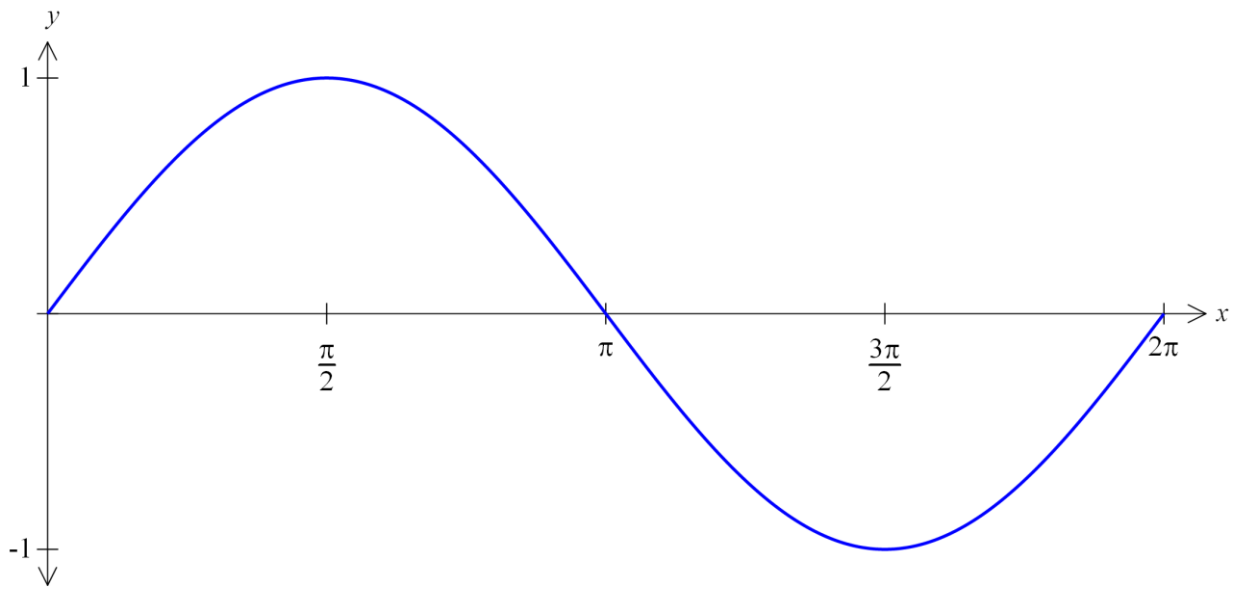
b) Solve $\frac{2}{2-x} \geq 3$. **2**

c) Evaluate $\int_0^{\frac{\pi}{6}} \frac{2\cos x}{1 + 4\sin^2 x}\, dx$ using the substitution $u = 2\sin x$. **4**

d) The region bounded by the curve $y = \sin x$, the x-axis and the lines $x = \frac{\pi}{12}$ and $x = \frac{\pi}{4}$ is rotated through one complete revolution about the x-axis. Find the volume of the solid formed. **3**

e) The curves $y = \ln x$ and $xy = e$ intersect at the point P (e, 1). Show that at the point P $\tan\theta = \frac{2e}{e^2 - 1}$. **2**

f) The diagram shows the graph of $y = \sin x$ for $0 \leq x \leq 2\pi$. Copy or trace the diagram onto your answer sheet.



i) On the same set of axes, sketch the graph of $y = \cos 2x$ for $0 \leq x \leq 2\pi$. **1**

ii) Hence, state the number of solutions in $0 \leq x \leq 2\pi$ to the equation $\sin x = \cos 2x$. **1**

Question 14 (15 marks) Begin a NEW answer booklet or answer sheet

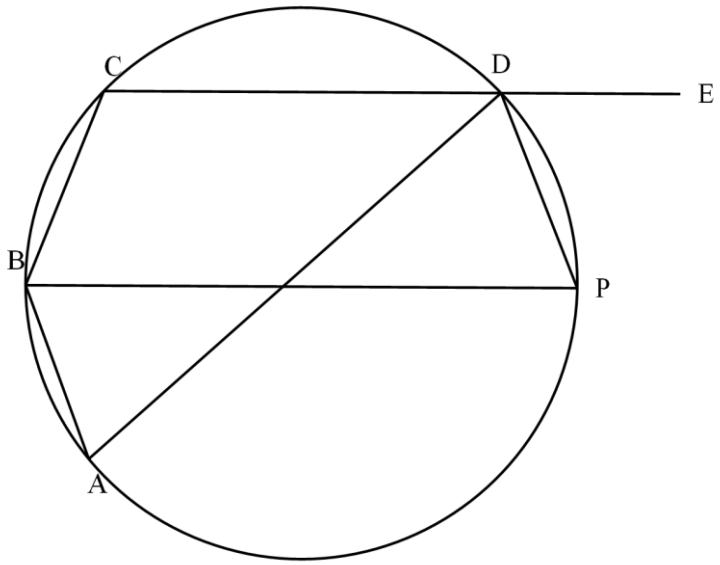
a) An isotope of carbon, C_{14} decays at a rate proportional to the mass present. The rate of change is given by $\frac{dM}{dt} = -kM$ where k is a positive constant and M is the mass present.

i) Show $M = M_0 e^{-kt}$ is a solution to this equation. **1**

ii) The half-life of this isotope is 5600 years. This means it takes 5600 years for 100 grams of C_{14} to decay to 50 grams. Find the value of k correct to 3 significant figures. **1**

iii) Archaeologists use radiocarbon dating to establish the age of discoveries. Calculate the age of an item in which only one-eighth of the original carbon remains. **1**

b) In the diagram below $ABCD$ is a cyclic quadrilateral. CD is produced to E. P is a point on the circle through A, B, C, D such that $\angle ABP = \angle PBC$.



Copy or trace the diagram into your answer booklet marking all the information given.

i) Explain why $\angle ABP = \angle ADP$. **1**

ii) Show that PD bisects $\angle ADE$. **2**

iii) If, in addition, $\angle BAP = 90^\circ$ and $\angle APD = 90^\circ$, explain where the centre of the circle is located. **1**

c) A is the point (-2, 1) and B is the point (x, y). The point P (13, -9) divides AB externally in the ratio 5:3. Find the values of x and y. **2**

d) Find the area under the curve $y = \cos^{-1} x$ between $x = 0$, $x = \frac{1}{2}$ and the x – axis. **2**

e) For the function $f(x) = \frac{1}{1 - x^2}$

i) Find the inverse function. **2**

ii) Using Simpsons Rule, find the area under the inverse function between $x = 1$ and $x = 3$ using five function values. **2**

End of Section II

End of Examination

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x \, , \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

Note $\ln x = \log_e x, \quad x > 0$

TRINITY GRAMMAR SCHOOL
2012, Year 12 Mathematics Extension 1
Half Yearly Examination
SECTION I
ANSWER SHEET

Student Number: …… Class Teacher: ……..

Be sure to write your answers for Section I on this answer sheet. After you have selected an answer, **CIRCLE** the correct answer. To change an answer, erase your previous mark completely, and then record your new answer. Mark only one answer for each question.

Q1.	A	B	C	D
Q2.	A	B	C	D
Q3.	A	B	C	D
Q4.	A	B	C	D
Q5.	A	B	C	D
Q6.	A	B	C	D
Q7.	A	B	C	D
Q8.	A	B	C	D
Q9.	A	B	C	D
Q10.	A	B	C	D