BAULKHAM HILLS HIGH SCHOOL

2015

YEAR 12 HALF-YEARLY

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Attempt all questions
- Start a new page for each question

Total marks – 70
Exam consists of 8 pages.

This paper consists of TWO sections.

Section 1 – Pages 2-4

Multiple Choice

Question 1-10 (10 marks)

Section 2 – Pages 5-8

Extended Response

Question 11- 14 (60 marks)

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet in your booklet for Questions 1-10

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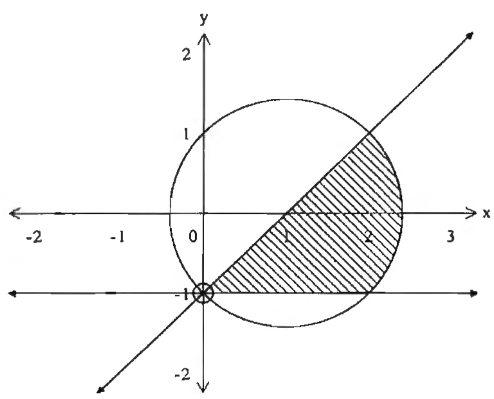
1. What is the remainder when $x^3 + x^2 + 5x + 6$ is divided by $x + i$?

(A) $5 - 4i$
(B) $5 + 6i$
(C) $7 - 6i$
(D) $7 - 4i$

2. Consider the Argand diagram below.



Which pair of inequalities define the shaded area?

(A) $|z - 1| \leq \sqrt{2}$ and $0 \leq \arg(z - i) \leq \frac{\pi}{4}$
(B) $|z - 1| \leq \sqrt{2}$ and $0 \leq \arg(z + i) \leq \frac{\pi}{4}$
(C) $|z - 1| \leq 1$ and $0 \leq \arg(z - i) \leq \frac{\pi}{4}$
(D) $|z - 1| \leq 1$ and $0 \leq \arg(z + i) \leq \frac{\pi}{4}$

3. Given $3x + 2iy - ix + 5y = 7 + 5i$, where x and y are real numbers, then:

(A) $x = -1, y = 2$
(B) $x = \frac{39}{11}, y = -\frac{8}{11}$
(C) $x = -\frac{3}{5}, y = \frac{22}{5}$
(D) $x = -11, y = 8$

4. If z represents a variable point on the Argand diagram, which description best represents the locus of $|z - 4 + i| = |z + 4 - i|$?

(A) Circle
(B) Ellipse
(C) Hyperbola
(D) Line

5. If ω is a non-real cube root of unity the value of $\frac{1}{1+\omega} + \frac{1}{1+\omega^2}$ is equal to

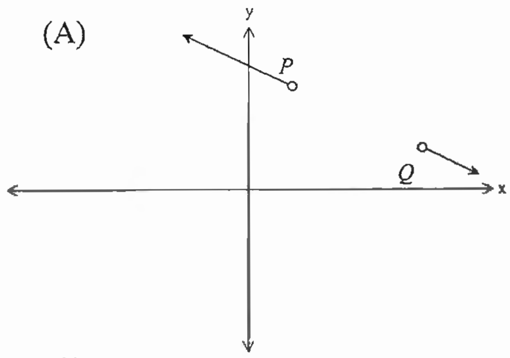
(A) -1
(B) 0
(C) 1
(D) ω

6. The polynomial equation $P(z) = 0$ has real coefficients, and has roots including a double root of $-3 + 2i$ and a single root of 3.
What is the minimum possible degree of $P(z)$?
(E) 3
(F) 4
(G) 5
(H) 6

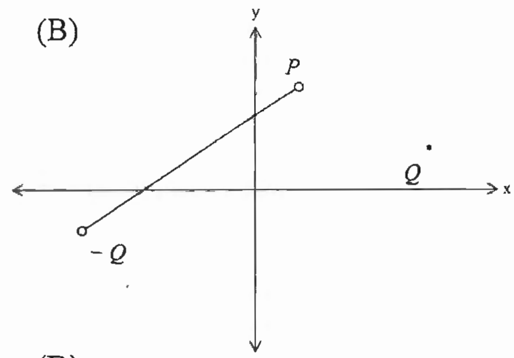
7. The points P, Q and R are represented by the complex numbers p, q and r respectively.
The points P, Q and R are joined to form a triangle.
If $r + p\,i = q + r\,i$ what is the value of the ratio of RQ:PQ?
(A) $\frac{1}{\sqrt{2}}$
(B) 1
(C) $\sqrt{2}$
(D) 2

8. P and Q are two points representing the complex numbers p and q respectively. Which of the following shows the locus of z defined by $\arg(z+q) - \arg(z-p) = \pi$?

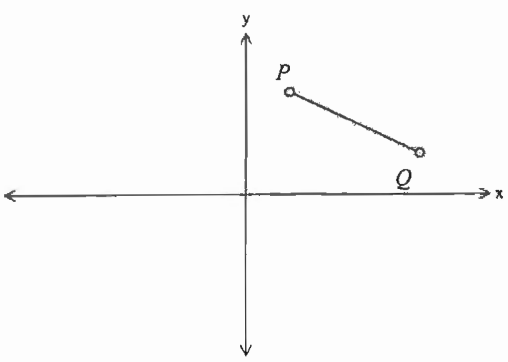
(A)



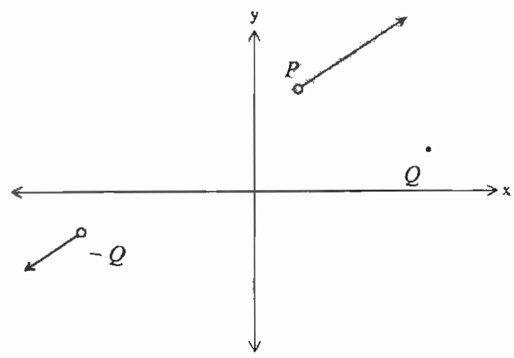
(B)



(C)



(D)



9. If $\log_b a = c$ and $\log_x b = c$, then $\log_a x$ equals

(A) a

(B) c^{-2}

(C) b

(D) b^2

10. For what values of k will $\dfrac{x^2}{|2k|-3} + \dfrac{y^2}{k-2} = 1$ be a hyperbola?

(A) $-\dfrac{3}{2} < k < \dfrac{3}{2}$ or $k < 2$

(B) $\dfrac{3}{2} < k < 2$

(C) $k < 2$

(D) $k < -\dfrac{3}{2}$ or $\dfrac{3}{2} < k < 2$

End of Section I

Section II

60 marks
Attempt Questions 11-14
Allow about 1hour and 45minutes for this section

Answer each question on the relevant page of your writing booklet.

In Questions 11-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Answer on the relevant page of your writing booklet.

(a) Consider the complex number $z = \sqrt{3} + i$

(i) Express z in modulus-argument form. **1**

(ii) Write down $\dfrac{1}{z^2}$ in modulus-argument form. **1**

(iii) Find the least positive value of n for which z^n is a positive real number. **1**

(b) Sketch the locus of z if $\arg\left(\dfrac{z-3}{z+5}\right) = \dfrac{2\pi}{3}$ **2**

(c) Find integers m and n such that $(x+1)^2$ is a factor of $x^5 + 2x^2 + mx + n$ **3**

(d) Consider the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$.

(i) Find the eccentricity **1**

(ii) Find the equation of the directrices **1**

(iii) Find the coordinates of the foci. **1**

(iv) Sketch the graph of the ellipse, clearly showing all intercepts with the axes and the foci and directrices. **1**

(e) The product of two of the roots of the cubic $6x^3 - 23x^2 + kx - 12 = 0$ is 2.

(i) Find the value of k. **1**

(ii) Find all the roots of the cubic equation. **2**

End of Question 11

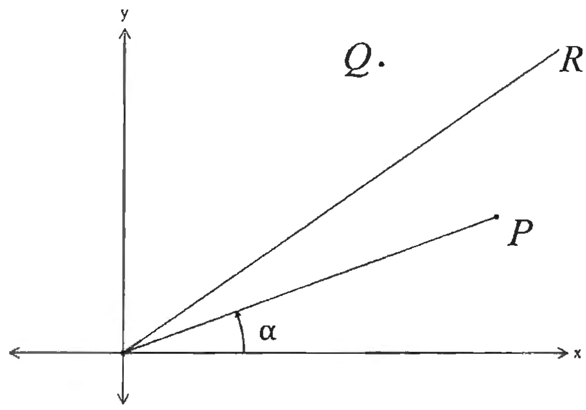
Question 12 (15 marks) Answer on the relevant page of your writing booklet.

(a) Let $P(z) = z^2 + 8 + 6i$

(i) Solve $P(z) = 0$ **2**

(ii) Hence, or otherwise solve $z^2 - (5 - i)z + 8 - i = 0$ **2**

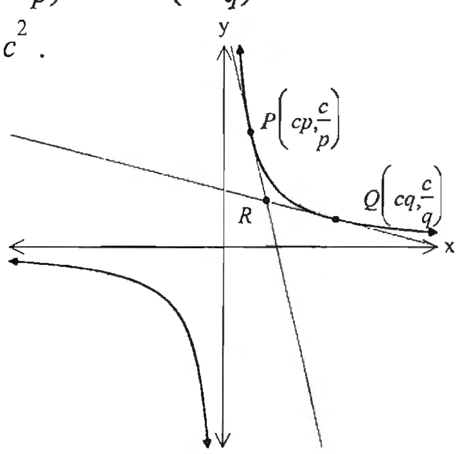
(b) The point P represents the complex number z. The reflection of P in the line OR, making an angle of α with the positive direction of the real axis, is the point Q, representing the complex number w.



(i) Explain why $|z| = |w|$ **1**

(ii) Hence prove that $zw = |z|^2 (\cos 2\alpha + i\sin 2\alpha)$ **2**

(c) The points $P\left(cp, \frac{c}{p}\right)$ and $Q\left(cq, \frac{c}{q}\right)$ are two distinct points on the rectangular hyperbola $xy = c^2$.



(i) Prove that the equation of the tangent at P is $x + p^2 y = 2cp$. **2**

(ii) The tangents at P and Q meet at R. Find the coordinates of R. **2**

(iii) The midpoint of PQ is M. Find the coordinates of M. **2**

(iv) If O is the origin, prove that O, R and M are collinear. **2**

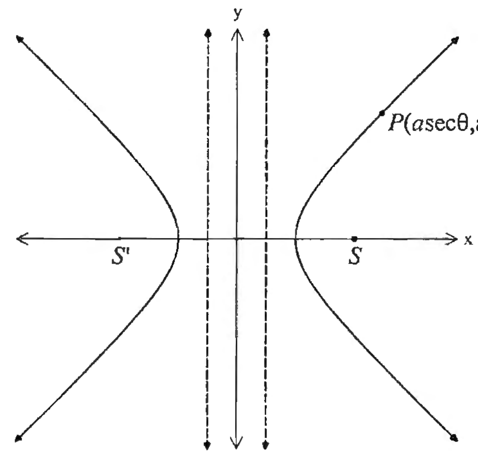
End of question 12

Question 13 (15 marks) Answer on the relevant page of your writing booklet.

(a) Solve $\dfrac{x^2 + x - 6}{x^2 - 4x} \leq 1$ **3**

(b) The roots of the cubic equation $x^3 - 6x^2 - 11 = 0$ are α, β and γ. Form the equation which has roots $\alpha + \beta$, $\beta + \gamma$, and $\gamma + \alpha$. **3**

(c) The point $P(a\sec\theta, a\tan\theta)$ is a point on the rectangular hyperbola $x^2 - y^2 = a^2$.



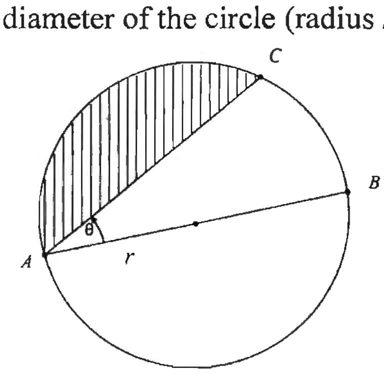
The foci are S and S'.

(i) Show that the eccentricity is $\sqrt{2}$. **1**

(ii) Show that $SP = a\sqrt{2}\,(\sec\theta - \dfrac{1}{\sqrt{2}})$. **2**

(iii) Hence show that $SP.S'P = OP^2$, where O is the origin. **2**

(d) In the diagram, AB is the diameter of the circle (radius r) and $\angle BAC = \theta$.



(i) If the area of the shaded region is one third of the area enclosed by the circle, show that $\sin 2\theta = \dfrac{\pi}{3} - 2\theta$. **2**

(ii) By drawing appropriate graph/s, show that the above equation has only one solution. **2**

End of Question 13

Question 14 Start on the appropriate page in your answer booklet

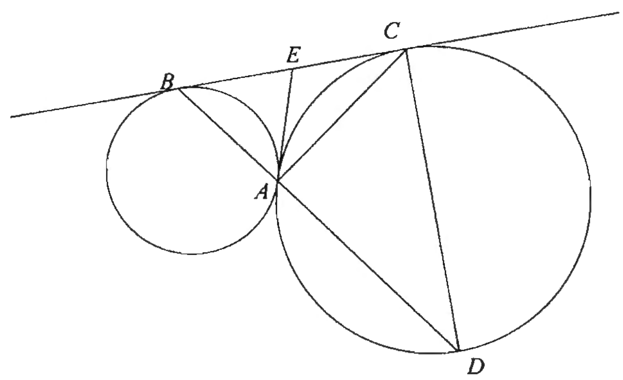
(a) Given $\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$ (DO NOT PROVE THIS RESULT)

(i) Find the roots of $x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$ **2**

(ii) Hence, without the aid of a calculator, prove that **3**

$$\sec^2\frac{\pi}{16} + \sec^2\frac{3\pi}{16} + \sec^2\frac{5\pi}{16} + \sec^2\frac{7\pi}{16} = 32$$

(b) Two circles touch at A. BC and AE are common tangents. BA is produced to meet the second circle at D.



(i) Prove that $AC \perp AB$ **2**

(ii) Prove that the lengths AB, AC and AD are in geometric progression. **2**

(c) (i) Sketch the curve $|z - \alpha| = |\alpha|$ where α is a complex number. **2**

(ii) Hence find the maximum value of $|z - 2\text{Re}(\alpha)|$ **1**

(d) (i) In how many ways is it possible to separate 5 distinct objects into two different boxes so that no box remains empty? **1**

(ii) In how many ways is it possible to factor the number 30 030 into three positive integer factors greater than 1 (note: $30 \times 77 \times 13$ is the same factoring as $13 \times 30 \times 77$). **2**

End of Paper

X2 SOLUTIONS HALF YEARLY 2015

Section I - Multiple choice

1. A $\quad P(-i) = -i^3 + i^2 - 5i + 6$

$= 5 - 4i$

$\therefore A$

2. B $\quad$ circle has centre $(1,0)$ radius $= \sqrt{1+1}$

$= \sqrt{2}$

$\arg(z - (-i)) = \arg(z+i)$

$\therefore B$

3. A $\quad$ Equating real and imaginary parts

$3x + 5y = 7 \quad (1)$

$2y - x = 5 \quad (2)$

sub (2) in (1) $\quad 3(2y-5) + 5y = 7$

$6y - 15 + 5y = 7$

$11y = 22$

$y = 2$

$x = 2 \times 2 - 5$

$x = -1$

$\therefore A$

4. D $\quad$ Locus is of form $|z - z_1| = |z - z_2|$ which results in perpendicular bisector of interval z_1 & z_2.

$\therefore D$

5. C $\quad \dfrac{1}{1+w} + \dfrac{1}{1+w^2} = \dfrac{-1}{w^2} + \dfrac{-1}{w} \qquad$ using $1 + w + w^2 = 0$

$= \dfrac{-1-w}{w^2}$

$= \dfrac{w^2}{w^2}$

$= 1$

$\therefore C.$

6. C $\quad P(z) = (z+3-2i)^2(z-3)(z+3-2i)^2 Q(z)$

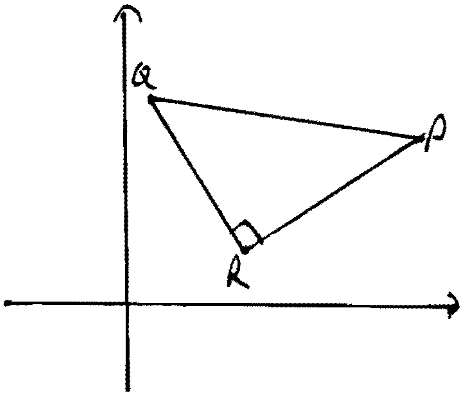
$-3+2i$ is double root $\qquad$ $z=3$ is single root $\qquad$ coefficients are real, roots occur in conjugate pairs

$\therefore$ degree $= 2+2+1+\deg(Q(z))$

$\therefore$ minimum degree $=5$ (if $Q(z)$ is a constant)

$\therefore C$

7. A



$r + pi = q + ri$

$i(p-r) = q-r$

$i\,\overrightarrow{PR} = \overrightarrow{QR}$

$\therefore |PR| = |QR|$ & $|PQ| = \sqrt{2}$

$\therefore \dfrac{RQ}{PQ} = \dfrac{1}{\sqrt{2}}$

$\therefore A$

8. B Section between P and -Q $\quad$ since $\arg(z+q) - \arg(z-p)$
$= \arg(z-(-q)) - \arg(z-p)$

$\therefore B$

9. B $\quad b^c = a$ & $x^c = b$

$(x^c)^c = b^c$

$x^{c^2} = b^c$

$x^{c^2} = a \rightarrow$

$(x^{c^2})^{\frac{1}{c^2}} = a^{\frac{1}{c^2}}$

$\log_a x = c^{-2}$

$\therefore B$

10. Either $|2k|-3>0$ & $k-2<0$ $\quad$ OR $\quad$ $|2k|-3<0$ & $k-2>0$

$|2k|>3$ $\qquad\qquad$ $|2k|<3$ & $k>2$

$\therefore k<-\frac{3}{2}$ or $k>\frac{3}{2}$ & $k<2$ $\qquad$ $-\frac{3}{2}<k<\frac{3}{2}$ & $k>2$

$\therefore$ No sol'n

$\therefore$ Solution is $k<-\frac{3}{2}$ or $\frac{3}{2}<k<2$

$\therefore D$

SECTION II

Q11 a)(i) $z = 2\,\text{cis}\left(\frac{\pi}{6}\right)$ ✓

(ii) $z^{-2} = 2^{-2}\,\text{cis}\left(\frac{\pi}{6}\right)^{-2}$ ✓

$\frac{1}{z^2} = \frac{1}{4}\,\text{cis}\left(\frac{-\pi}{3}\right)$

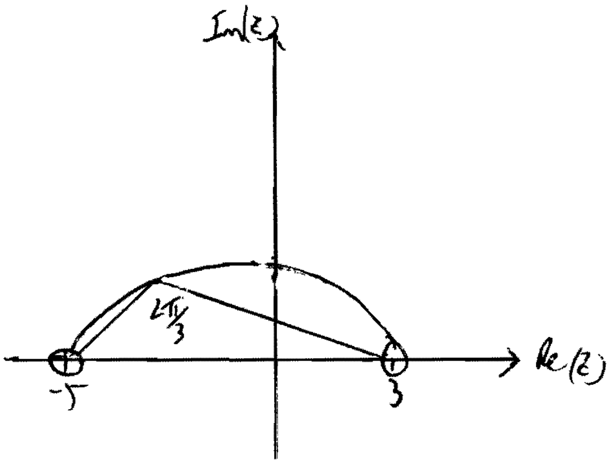
(iii) $z^n = 2^n\,\text{cis}\left(\frac{n\pi}{6}\right)$

$\therefore$ for z^n to be positive real number $\frac{n\pi}{6}$ must be a multiple of 2π

$\therefore \frac{n\pi}{6} = 2\pi k$ for k, an integer

$\therefore n = 12$

b)



(2) minor arc or $\frac{2\pi}{3}$ from to -5
(1) arc from allow (1) for arc

c) $x = -1$ is a double root.

$P(-1) = 0 = -1 + 2 - m + n$

$P'(-1) = 0 = 5 - 4 + m$

$P(x) = x^5 + 2x^2 + mx + n$

$P'(x) = 5x^4 + 4x + m$

$\therefore m = -1$

$\therefore 2 + n = 0$

$n = -2$

$\therefore m = -1$ & $n = -2$.

(3) correct
(2) correct m
(1) using $P'(x) = 0$

d) i) $b^2 = a^2(1 - e^2)$

$4 = 9(1 - e^2)$

$\frac{4}{9} = 1 - e^2$

$e^2 = \frac{5}{9}$

$e = \frac{\sqrt{5}}{3}$ ✓

$$(3\alpha-4)(2\alpha-3)=0$$

$\therefore$ Roots are $1, \frac{4}{3}, \frac{3}{2}$. ✓

12 a) i) $z^2=-8-6i$

let $z=x+iy$

$x^2-y^2+2xyi=-8-6i$

$x^2-y^2=-8$ ✓

$2xy=6$

$xy=3$ ✓

$x=\mp 1, y=\pm 3$

$\therefore z=\pm(\ \ )$

(ii) $z=\dfrac{5-i\pm\sqrt{(5-i)^2-4(8-i)}}{2}$

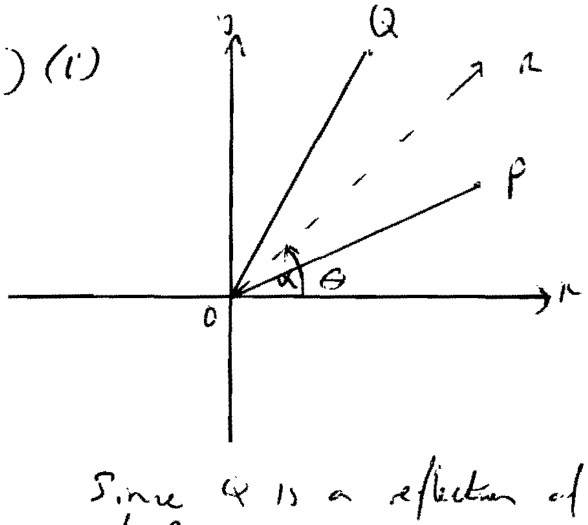
$z=\dfrac{5-i\pm\sqrt{24-10i-32+4i}}{2}$

$=\dfrac{5-i\pm\sqrt{-8-6i}}{2}$

$=\dfrac{5-i\pm(1-3i)}{2}$

$z=3-2i$ or $2+i$

b) (i)



Since Q is a reflection of P in OR $PQ\perp OR$ and midpoint of PQ is on OR.

$\therefore \Delta OPQ$ is isosceles.

$\therefore OP=OQ$.

(i) correct reasoning

b(ii) let $\arg p = \theta$.

(2) correct solution
(1) expression for argument of q ✓

$\angle POR = \alpha - \theta$

Also $\angle POR = \angle ROQ$ (Q is reflection of P in OR)

$\therefore \arg q = \alpha + (\alpha - \theta)$

$= 2\alpha - \theta$ ✓

$zw = |z||w| \text{ cis } (\theta + 2\alpha - \theta)$

$= |z|^2 \text{ cis } 2\alpha$ ✓ since $|w| = |z|$

c) i) $y = \frac{c^2}{x}$

$\frac{dy}{dx} = -c^2 x^{-2}$

$= \frac{-c^2}{x^2}$

(2) correct solution
(1) correct $\frac{dy}{dx}$

At P, $\frac{dy}{dx} = \frac{-c^2}{c^2 p^2}$

$\frac{dy}{dx} = -\frac{1}{p^2}$ ✓

Eqn of tangent $y - \frac{c}{p} = \frac{-1}{p^2}(x - cp)$

$p^2 y - cp = -x + cp$

$x + p^2 y = 2cp$ (1) ✓

ii) Tangent at Q is $x + q^2 y = 2cq$ (2)

for R: (1) − (2) $(p^2 - q^2)y = 2c(p - q)$

$(p+q)(p-q)y = 2c(p-q)$

$y = \frac{2c}{p+q}$ ✓

(2) correct solution
(1) either x or y value correct

$\therefore x + p^2\left(\frac{2c}{p+q}\right) = 2cp$

$x = 2cp - \frac{2p^2 c}{p+q}$

$= \frac{2cp(p+q) - 2p^2 c}{p+q}$

$x = \frac{2cpq}{p+q}$

$\therefore R$ is $\left(\dfrac{2cpq}{p+q}, \dfrac{2c}{p+q}\right)$ ✓

(iii) $M = \left(\dfrac{cp+cq}{2}, \dfrac{\frac{c}{p}+\frac{c}{q}}{2}\right)$

(2) correct solution
(1) either x or y value correct

$M = \left(\dfrac{c(p+q)}{2}, \dfrac{c(p+q)}{2pq}\right)$ ✓

✓ (1) (1)

(iv) For R $\left(\dfrac{2cpq}{p+q}, \dfrac{2c}{p+q}\right) \Rightarrow x = \dfrac{2cpq}{p+q} \quad y = \dfrac{2c}{p+q}$

$\therefore \quad x = pqy$

ie $y = \dfrac{x}{pq}$ $\quad \therefore R$ lies on line gradient $\dfrac{1}{pq}$ through $(0,0)$ ✓

For M, $\quad x = \dfrac{c(p+q)}{2} \quad y = \dfrac{c(p+q)}{2pq}$

$\therefore y = \dfrac{x}{pq}$

which also has same gradient $\dfrac{1}{pq}$ as OR and passes through $(0,0)$ ✓

$\therefore$ O, R M are collinear

OR allow (1) for gradient of OR or OM
(2) for correct solution

3 (a) bottom of next page

(a) bottom of next page

3 b) $x^3 - 6x^2 - 11 = 0$ has roots α, β, γ

$\alpha + \beta + \gamma = 6$

New roots: $\alpha + \beta = 6 - \gamma$, $\beta + \gamma = 6 - \alpha$, $\gamma + \alpha = 6 - \beta$ $\therefore$ we want roots $y = 6 - x$ where $x = \alpha, \beta, \gamma$

$\therefore (6-x)^3 - 6(6-x)^2 - 11 = 0$

$216 - 3\times 6^2 \times x + 3\times 6x^2 - x^3 - 6(36 - 12x + x^2) - 11 = 0$

$-x^3 + 12x^2 - 36x - 11 = 0$

$\therefore x^3 - 12x^2 + 36x + 11$

c) (i) $\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$

$b^2 = a^2(e^2 - 1)$

$1 = e^2 - 1$

$e^2 = 2$

$e = \sqrt{2}$

ii) Directrix $x = \frac{\pm a}{e}$

$x = \frac{\pm a}{\sqrt{2}}$

$e = \frac{PS}{PM}$

$PS = \sqrt{2}\, PM$

$= \sqrt{2}\left(a\sec\theta - \frac{a}{\sqrt{2}}\right)$

$= a\sqrt{2}\left(\sec\theta - \frac{1}{\sqrt{2}}\right)$

(iii) Similarly, replacing a with $\frac{a}{\sqrt{2}}$ yields $PS' = a\sqrt{2}\left(\sec\theta + \frac{1}{\sqrt{2}}\right)$

$SP.S'P = a\sqrt{2}\left(\sec\theta - \frac{1}{\sqrt{2}}\right)(a\sqrt{2})\left(\sec\theta + \frac{1}{\sqrt{2}}\right)$

$= 2a^2\left(\sec^2\theta - \frac{1}{2}\right)$

$OP^2 = (a\sec\theta)^2 + (a\tan\theta)^2$

$= a^2(\sec^2\theta + \tan^2\theta)$

$= a(\sec^2\theta + \sec^2\theta - 1)$

$= a(2\sec^2\theta - 1)$

$= 2a^2\left(\sec^2\theta - \frac{1}{2}\right)$

$= SP.SP'$ as reqd.

13 d) (i) $\triangle ACO$ is isosceles (equal radii)

$\therefore \angle ACO = \theta$

$\therefore \angle AOC = \pi - 2\theta$ ($\angle$ sum of $\triangle AOC$)

Area segment = $\frac{\text{Area of circle}}{3}$

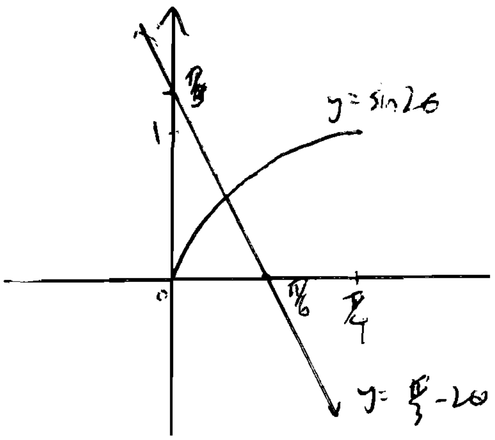
$\frac{1}{2}r^2\left[(\pi-2\theta)-\sin(\pi-2\theta)\right] = \frac{\pi r^2}{3}$ ✓ (1) expression for area of segment with $\frac{1}{3}$ circle

$\pi - 2\theta - \sin 2\theta = \frac{2\pi}{3}$

$\sin 2\theta = \pi - \frac{2\pi}{3} - 2\theta$ ✓

$\sin 2\theta = \frac{\pi}{3} - 2\theta$

ii)



(2) correct solution
(1) one correct graph drawn with points indicated

$\therefore$ one point of intersection $\therefore$ one root

13 (e) $\frac{x^2+x-6}{x^2-4x} \le 1$

$\frac{(x+3)(x-2)}{x(x-4)} \le 1$ $x \ne 0, 4$

Critical points:

$\frac{(x+3)(x-2)}{x(x-4)} = 1$

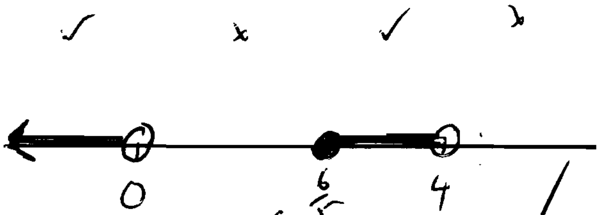
$x^2+x-6 = x^2-4x$

$5x = 6$

$x = 6/5$

(3) correct solution
(2) 1 correct region
(1) critical values/multiply by $(x^2-4x)^2$

OR



$\therefore x < 0$ or $\frac{6}{5} \le x < 4$ ✓

14 a) i) let $\tan 4\theta = 1$ & let $x = \tan\theta$

alternative solutions at end

$$1 = \frac{4x - 4x^3}{1 - 6x^2 + x^4}$$

$x^4 - 6x^2 + 1 = 4x - 4x^3$

$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$ has roots given by $x = \tan\theta$ where $\tan 4\theta = 1$ ✓

$\therefore 4\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$

$\therefore \theta = \frac{\pi}{16}, \frac{5\pi}{16}, \frac{9\pi}{16}, \frac{13\pi}{16}$ ✓

$\therefore$ Roots are $x = \tan\frac{\pi}{16}, \tan\frac{5\pi}{16}, \tan\frac{9\pi}{16}, \tan\frac{13\pi}{16}$

(or $x = \tan\frac{\pi}{16}, \tan\frac{5\pi}{16}, -\tan\frac{7\pi}{16}, -\tan\frac{3\pi}{16}$

(ii)

Eqn of roots $\tan^2\frac{\pi}{16}, \tan^2\frac{5\pi}{16}, \tan^2\frac{7\pi}{16}, \tan^2\frac{3\pi}{16}$

will be found by letting $y = x^2$

$\therefore x = \sqrt{y}$

$y^2 + 4y\sqrt{y} - 6y - 4\sqrt{y} + 1 = 0$ ✓

$(y^2 - 6y + 1)(y^2 - 6y + 1) = \left[4\sqrt{y}(1 - y)\right]^2$

$y^4 - 6y^3 + y^2 - 6y^3 + 36y^2 - 6y + y^2 - 6y + 1 = 16y(1 - 2y + y^2)$

$y^4 - 28y^3 + 70y^2 - 28y + 1 = 0$ ✓

$\therefore \tan^2\frac{\pi}{16} + \tan^2\frac{5\pi}{16} + \tan^2\frac{7\pi}{16} + \tan^2\frac{3\pi}{16} = 28$

But $\tan^2\theta = \sec^2\theta - 1$

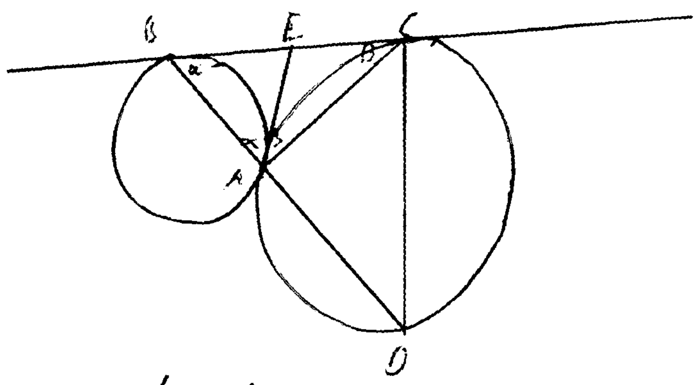
$\therefore \sec^2\frac{\pi}{16} + \sec^2\frac{3\pi}{16} + \sec^2\frac{5\pi}{16} + \sec^2\frac{7\pi}{16} - 4 = 28$

$= 32$ as reqd.

✓ 1 mark

See end for alternative method & marking scheme.

b)



(i) tangent at A meet BC at E

let $\angle EBA = \alpha$

$\angle EAB = \alpha$ (tangents from external point are equal, ΔEAB is isosceles)

similarly let $\angle ECA = \beta$

$\angle EAC = \beta$

Now In ΔBAC

$\alpha + \alpha + \beta + \beta = 180°$ ($\angle$ sum of ΔBAC)

$2(\alpha + \beta) = 180°$

$\alpha + \beta = 90°$

$\therefore \angle CAE = 90°$

$\therefore AC \perp AB$

✓

ii) In ΔBAC & ACD

$\angle BCA = \angle CDA = \beta$ (alternate segment theorem)

$\angle CAB = \angle CAD = 90°$ ($AC \perp AB$ in (i))

$\therefore \Delta ABC ||| \Delta ACD$ (AA)

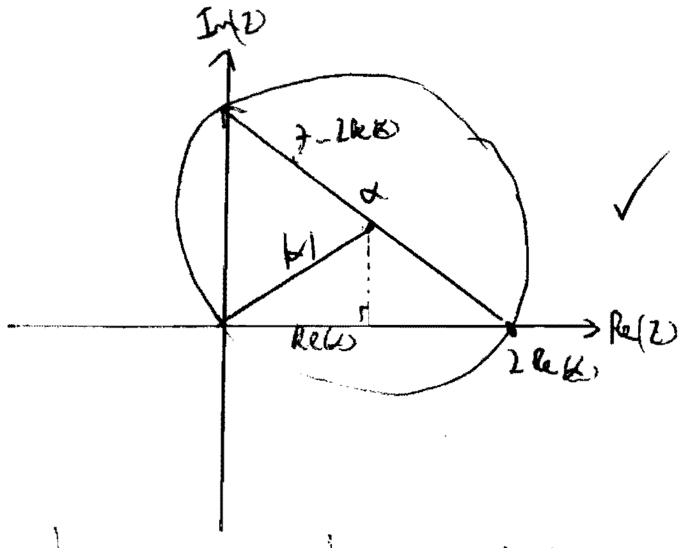
✓

$\therefore \frac{CA}{AD} = \frac{AB}{AC}$ (matching sides in same ratio, of similar Δ's ABC & ACD)

$\therefore \frac{CA}{AB} = \frac{AD}{AC}$

$\therefore AB, AC, AD$ are in geometric progression ✓

c) (i)



✓

(2) correct
(1) Circle centre α (not on real axis)

ii)
For max value $|z - 2\,\mathrm{Re}(\alpha)|$, z must lie on diameter on circle radius α.

$$\therefore |z - 2\,\mathrm{Re}(\alpha)| = 2|\alpha|$$ ✓

d(i) $2^5 - 2$
$= 30$

For any method:
(2) correct
(1) significant progress

ii) Factors: 2, 3, 5, 7, 11, 13

$\underset{\text{factor 1}}{\underline{\qquad}} \times \underset{\text{factor 2}}{\underline{\qquad}} \times \underset{\text{factor 3}}{\underline{\qquad}}$

$$= \frac{3^6 - 3\times 2^6 + 3}{3!}$$
$$= \frac{540}{6}$$
$$= 90$$

(each of 6 factors have 3 choices but we need to subtract no of ways of having one empty box 3×2^6 AND add back in all three empty)

Alternatively using (i)

Possible combinations − 1 box empty − 2 boxes empty

$$= \frac{3^6 - 3(2^6 - 2) - 3\times 1^6}{3!}$$
$$= \frac{540}{6}$$
$$= 90$$