BAULKHAM HILLS HIGH SCHOOL

2016

YEAR 12 HALF-YEARLY

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes
- Working time – 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work
- Attempt all questions
- Start a new page for each question

Total marks – 70
Exam consists of 8 pages.

This paper consists of TWO sections.

Section 1 – Pages 2-4
Multiple Choice
Question 1-10 (10 marks)

Section 2 – Pages 5-8
Extended Response
Question 11- 14 (60 marks)

Reference sheet is provided

Section I - 10 marks
Allow about 15 minutes for this section

1. $x = 1$ is a root of the equation $3x^5 - 5x^4 + 5x - 3 = 0$. What is its multiplicity?

(A) 1

(B) 2

(C) 3

(D) 4

2. If $z = 1 + i$, evaluate z^{12}

(A) 64

(B) -64

(C) $64i$

(D) $-64i$

3. The equation of a curve is $x^2 + 2y^2 - 2xy + x = 8$. What is the gradient of the curve at the point (3,2) ?

(A) $\frac{1}{4}$

(B) $-\frac{1}{4}$

(C) $\frac{3}{2}$

(D) $-\frac{3}{2}$

4. Which of the following is a focus of the hyperbola $\frac{y^2}{4} - \frac{x^2}{10} = 1$

(A) $(\sqrt{14}, 0)$

(B) $(0, \sqrt{14})$

(C) $\left(\sqrt{\frac{28}{5}}, 0\right)$

(D) $\left(0, -\sqrt{\frac{28}{5}}\right)$

5. The equation $|z-4|+|z+4|=10$ defines an ellipse. What is the length of its semi-minor axis?

(A) 2.4 units

(B) 3 units

(C) 4 units

(D) 5 units

6. The roots of the polynomial $P(x)=4x^3+4x-5$ are α, β and γ. Find the value of $(\alpha+\beta-3\gamma)(\beta+\gamma-3\alpha)(\alpha+\gamma-3\beta)$

(A) -80

(B) -16

(C) 16

(D) 80

7. The polynomial equation $P(x)=0$ has real coefficients and has roots which include $x=-2+i$ and $x=2$. What is the minimum possible degree of $P(x)$?

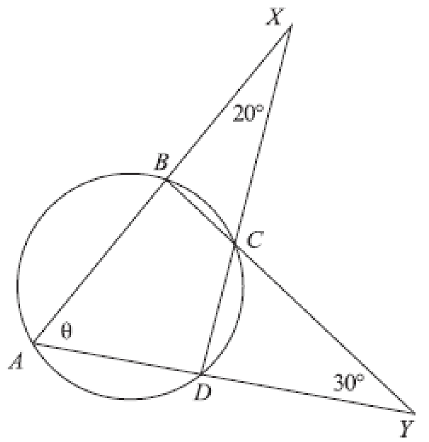
(A) 1

(B) 2

(C) 3

(D) 4

8.



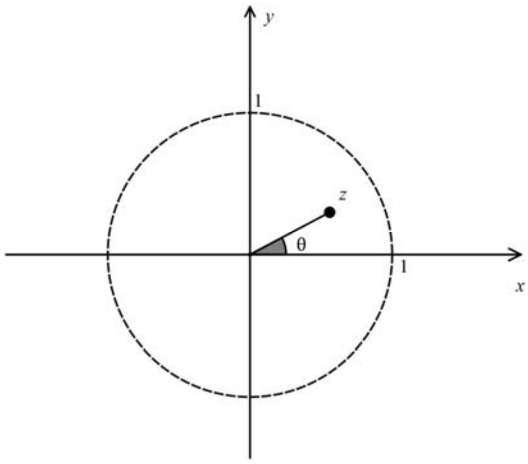
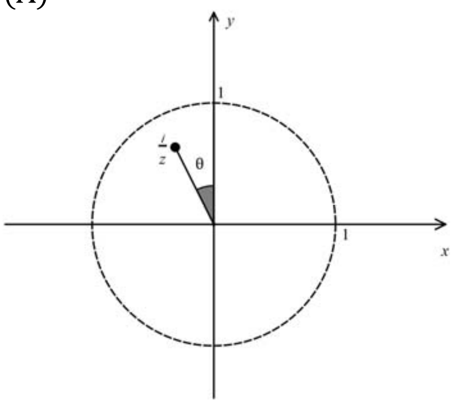
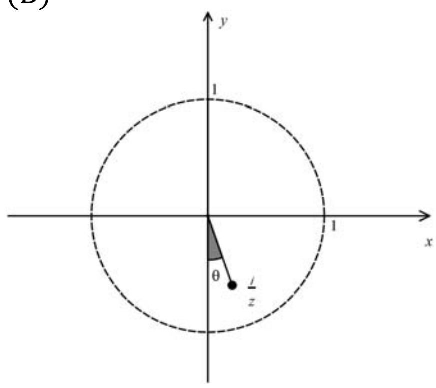
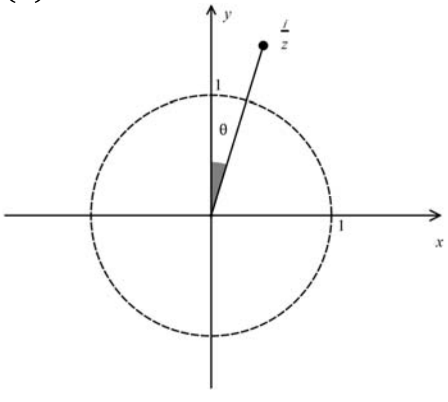
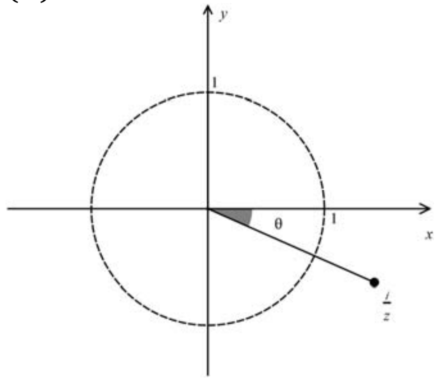
The size of θ in the diagram is:

(A) 50°

(B) 55°

(C) 60°

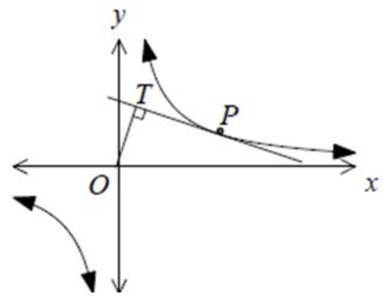
(D) 65°

9.	The cubic equation $P(x) = 0$ has roots α, β and γ. Which of the following cubic equations has roots $2\alpha + 1, 2\beta + 1$ and $2\gamma + 1$? (A) $P(2x) + 1 = 0$ (B) $P(2x + 1) = 0$ (C) $P\left(\frac{x}{2} + 1\right) = 0$ (D) $P\left(\frac{x-1}{2}\right) = 0$
10.	 The Argand diagram above shows the complex number z. Which diagram below best represents the complex number $\frac{i}{z}$? (A)  (B)  (C)  (D) 
	End of Section I

Section II – Extended Response

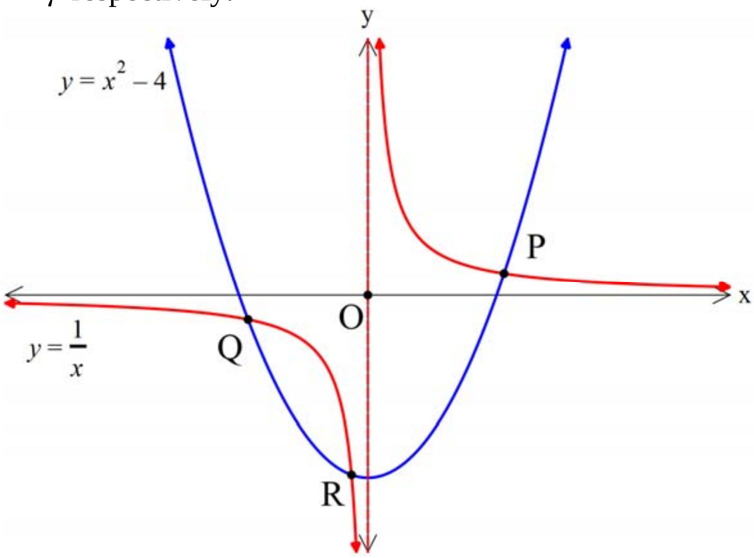
Attempt questions 11-14. Show all necessary working.
Answer each question on a SEPARATE PAGE Clearly indicate question number.
Each piece of paper must show your BOS number.
All necessary working should be shown in every question.

Question 11 (15 marks)		**Marks**
(a)	Let $z = \sqrt{3} - i$ and $w = 1 - i$, find:	
	(i) $i\bar{z} + w$	**2**
	(ii) $\frac{z}{w}$ in modulus argument form	**2**
(b)	(i) Sketch the region on the Argand diagram where both of the following inequalities apply: $$\lvert z - 2 - 2i\rvert \leq 2$$ $$0 \leq \arg(z - 2 - 2i) \leq \frac{\pi}{4}$$	**3**
	(ii) State the range of values for $\lvert z\rvert$ in this region.	**2**
(c)	(i) Solve the equation $z^5 = -1$. (You may leave your answer in modulus-argument form).	**2**
	(ii) Hence factorise $z^5 + 1$ over the real field.	**1**
	(iii) Hence or otherwise, prove that $\cos\frac{\pi}{5}\cos\frac{2\pi}{5} = \frac{1}{4}$	**3**
	End of Question 11	

Question 12 (15 marks)		**Marks**
a)	The polynomial $P(x) = 4x^3 - 3x - 1$ has a double zero which is real. Find the value of this zero.	**2**
b)	Find a polynomial $P(x)$with real coefficients having $2i$ and $1 - 3i$ as zeroes. Express your answer as the product of two real quadratic factors.	**2**
(c)	ω is a non-real root of $z^3 = 1$.	
	(i) Show that $1 + \omega + \omega^2 = 0$.	**1**
	(ii) Evaluate $(1 + \omega)^3(3 + 3\omega^2)$, expressing your answer in simplest form.	**2**
(d)	The point $P\left(ct, \frac{c}{t}\right)$ lies on the hyperbola $xy = c^2$. The point T lies at the foot of the perpendicular drawn from the origin to the tangent at P.	
		
	(i) Show that the tangent at P has equation $x + t^2y = 2ct$	**1**
	(ii) Show that the locus of T is given by $(x^2 + y^2)^2 = 4c^2xy$	**3**
e)	The point $P(acos\theta, bsin\theta)$ lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. The tangent at P cuts the y-axis at B , and M is the foot of the perpendicular from P to the y-axis.	
	(i) Show that the equation of the tangent to the ellipse at point P is given by $\frac{xcos\theta}{a} + \frac{ysin\theta}{b} = 1$	**2**
	(ii) Show that $OM \times OB = b^2$	**2**
	End of Question 12	

Question 13 (12 marks)		**Marks**
a)	For the hyperbola $16x^2 - 9y^2 = 144$	
	(i) Find the eccentricity	**1**
	(ii) Neatly sketch the hyperbola clearly showing the foci, directrices, asymptotes and intercepts with the coordinate axes.	**3**

b) The curves $y = x^2 - 4$ and $y = \frac{1}{x}$ intersect at the points P, Q and R where $x = \alpha$, $x = \beta$ and $x = \gamma$ respectively.



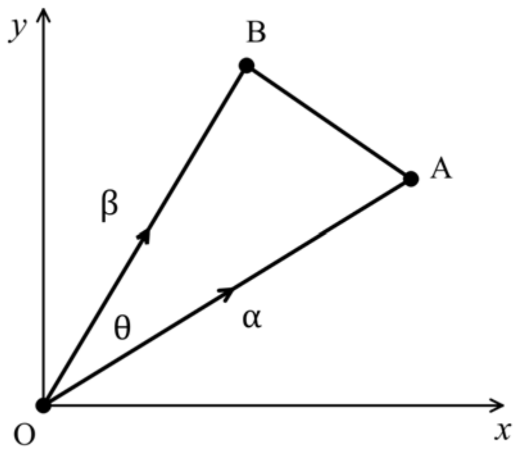
		Marks
(i)	Explain why α, β and γ are the roots of $x^3 - 4x - 1 = 0$.	**1**
(ii)	Find a polynomial equation with integer coefficients whose roots are α^2, $\beta^2$and γ^2	**2**
(iii)	Find a polynomial equation with integer coefficients whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}$ and $\frac{1}{\gamma^2}$.	**2**
(iv)	Hence find the numerical value of $OP^2 + OQ^2 + OR^2$	**2**

c) From a pack of eight cards numbered 1,2,3,4,5,6,7 and 8, three cards are drawn at random and laid on a table from left to right to form a three-digit number.

How many of the numbers formed in this way will:

		Marks
(i)	contain two even digits and one odd digit?	**2**
(ii)	have their digits in ascending order?	**2**

End of Question 13

Question 14 (15 marks)		**Marks**
a)	x and y are real numbers such that $\frac{x}{i} - \frac{y}{1+i} = -1 - 3i$. Find the values of x and y.	**2**
b)	(i) Given $z + \bar{z} = 2Re(z)$ and $\|z\|^2 = z\bar{z}$ (you do not need to prove these results) , prove that: $$\|\alpha\|^2 + \|\beta\|^2 - \|\alpha - \beta\|^2 = 2Re(\alpha\bar{\beta})$$ where α and β are complex numbers.	**3**
	(ii) The diagram below shows the angle θ between the complex numbers α and β on the Argand diagram.  Prove that $\|\alpha\|\|\beta\|cos\theta = Re(\alpha\bar{\beta})$	**2**
c)	(i) Prove that $$\frac{\cos x - \cos(x+2y)}{2\sin y} = \sin(x+y)$$	**3**
	(ii) Prove by induction that $$\sin A + \sin 3A + \sin 5A + \cdots + \sin(2n-1)A = \frac{1-\cos 2nA}{2\sin A}$$ for all positive integers n.	**3**
	(iii) Simplify $\sin 5A + \sin 7A + \sin 9A + \cdots + \sin 19A$, expressing your answer as a single fraction.	**2**
	End of Question 14	

End of Paper