Fort Street H.S.
2002 - Half-yearly Assessment
Mathematics Extension II (90 min)

QUESTION 1 Start a NEW page. **Marks**

a) Show that 1-2i is a zero of the polynomial $P(x) = x^3-5x^2+11x-15$. Hence resolve P(x) into irreducible factors over the field of i) complex numbers, ii) real numbers. **6**

b) Find the roots of $x^4-6x^3+12x^2-10x+3$ given that it has a root of multiplicity 3. **4**

QUESTION 2 Start a NEW page.

a) Find the equation of the locus of z if

i) $\left|\dfrac{z-i}{z+2}\right| = 1$ **3**

ii) $\arg\left(\dfrac{z-i}{z+2}\right) = \dfrac{\pi}{2}$ **4**

b) If $\dfrac{z^2}{z-1}$ is always real, show that the locus of the point representing z consists of the real axis and a circle. **3**

QUESTION 3 Start a NEW page.

The ellipse E has cartesian equation

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

(a) Write down its eccentricity, the co-ordinates of its foci S and S^1, and the equation of each directrix. **3**

(b) Sketch the curve and indicate on your diagram the foci and directrices. **1**

(c) P is an arbitrary point on E.

(i) Prove that the sum of the distances SP, S^1P is independent of P. **2**

(ii) Derive the equation of the normal line to E at P. **4**

QUESTION 4 Start a NEW page. **Marks**

With respect to the x and y axes, the line x=1 is a directrix, and the point (2,0) a focus of a conic of eccentricity $\sqrt{2}$.

(a) Find the equation of the conic, show that it is a rectangular hyperbola, and sketch the curve, indicating its asymptotes, foci and directrices. **4**

(b) Find the equation of the normal to the curve at any point P on it. **3**

(c) The normal to the curve at P meets the x and y axes at (X,0) and (0,Y) respectively, and T is the point (X,Y). Show that as P varies on the curve, T always lies on the hyperbola $x^2 - y^2 = 8$. **3**

QUESTION 5 Start a NEW page.

(a) P ($a\sec\theta$, $b\tan\theta$) lies in the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
The tangent at P meets the tangents at the ends of the major axis at Q and R. Show that QR subtends a right angle at either focus. **4**

(b) Deduce that if P is the point (5, $\frac{4}{3}$) on the hyperbola $\frac{x^2}{9} - y^2 = 1$ with foci S and S^1, then Q,R,S, S^1 are concyclic, and find the equation of the circle through theses points. **6**

QUESTION 6 Start a NEW page

(a) α, β, γ are non-zero and the roots of the cubic equation $x^3 + px + q = 0$. Find the equation whose roots are $\frac{\alpha}{\beta\gamma}, \frac{\beta}{\alpha\gamma}, \frac{\gamma}{\alpha\beta}$. **5**

(b) (i) Show that if α is a multiple root of the polynomial $f(x) = 0$ then $f(\alpha) = f'(a) = 0$. **1**

(ii) The polynomial $\alpha x^{n+1} + \beta x^n + 1$ is divisible by $(x-1)^2$. Show that $a = n$ and $\beta = -(1+n)$. **2**

(iii) Prove that $1 + x + \frac{x^2}{2!} + ... + \frac{x^n}{n!}$ has no multiple roots for any $n \geq 1$. **2**