Name______________________________ Teacher________________________

GOSFORD HIGH SCHOOL

2012

HIGHER SCHOOL CERTIFICATE

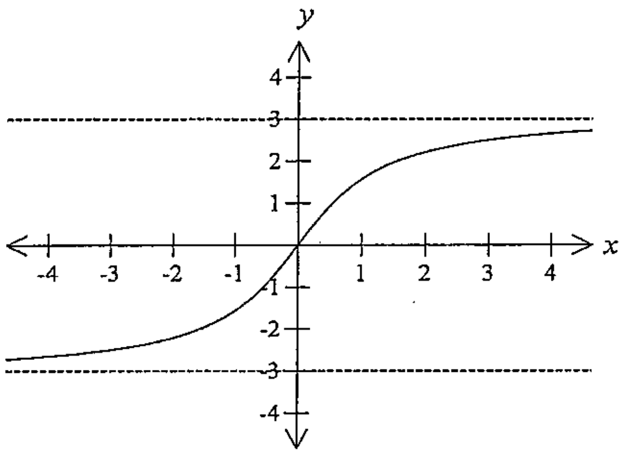
ASSESSMENT TASK 2

MATHEMATICS – EXTENSION 2

Duration- 90 minutes plus 5 minutes reading time

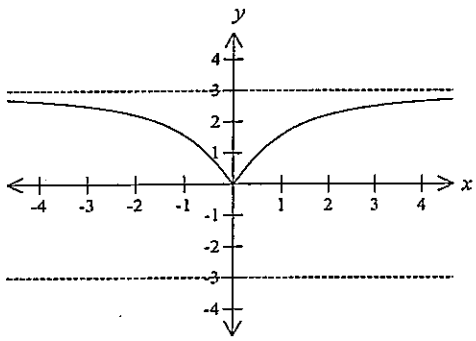
Multiple choice	**8** questions worth **1 mark each.** (Answer this section on the **test paper**)	/8
Graphs	**2** questions worth **13 marks each.** (Answer this section on your **own paper**. Start a new page for each question.)	/26
Polynomials	**2** questions worth **13 marks each.** (Answer this section on your **own paper**. Start a new page for each question.)	/26
TOTAL		**/60**

1 The diagram shows the graph of the function $y = f(x)$.

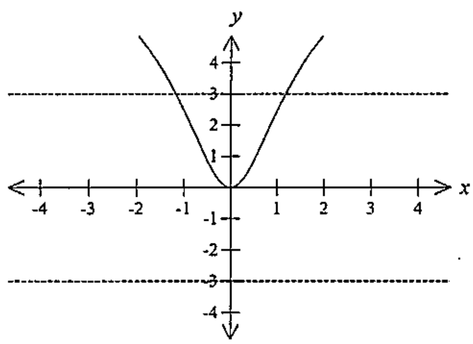


Which of the following is the graph of $y = \sqrt{f(x)}$?

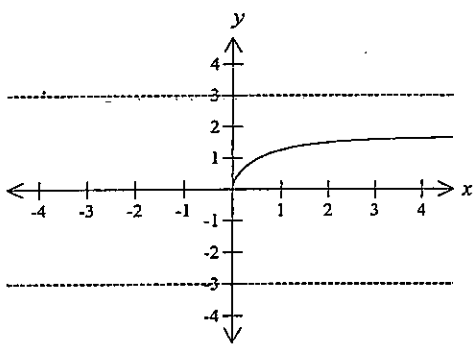
(A)



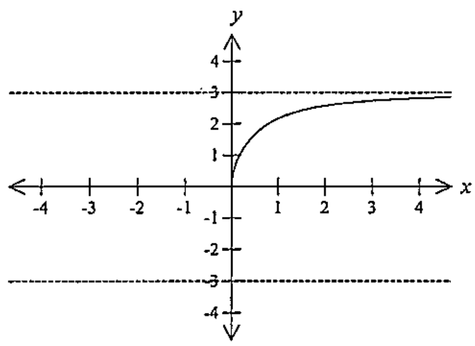
(B)



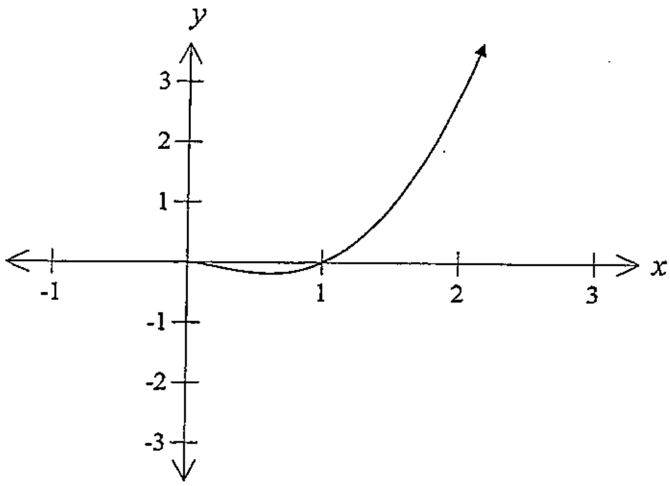
(C)



(D)

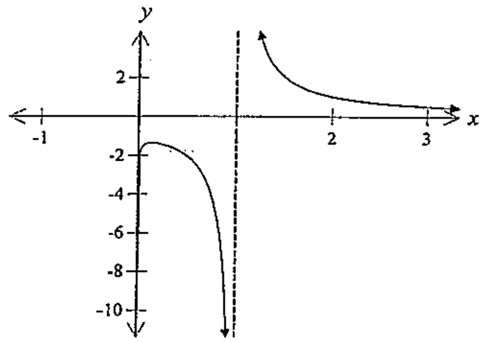


2 The diagram shows the graph of the function $y = f(x)$.

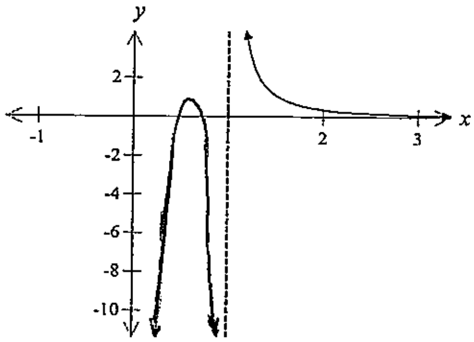


Which of the following is the graph of $y = \frac{1}{f(x)}$?

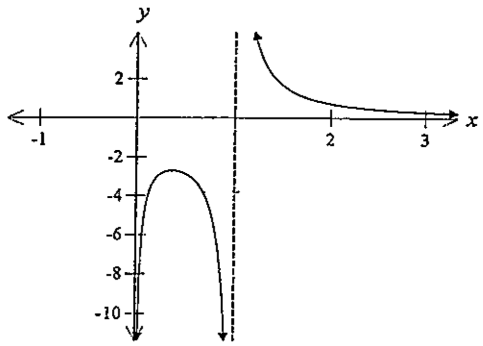
(A)



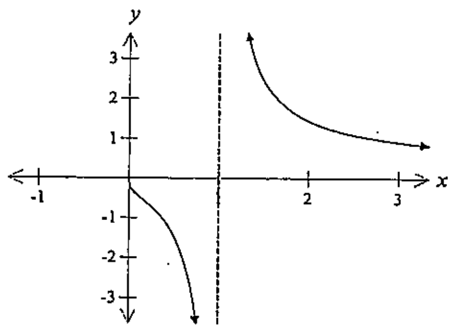
(B)



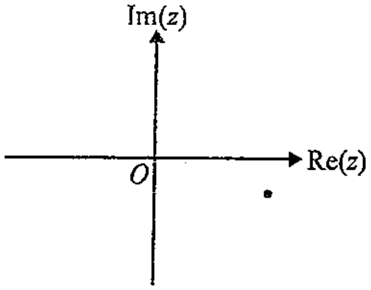
(C)



(D)



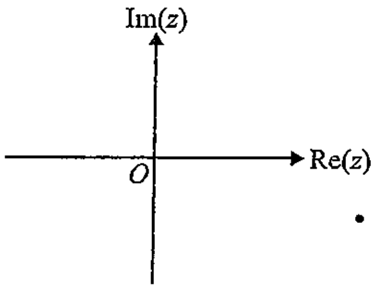
3 A certain complex number z, where $|z|>1$, is represented by the point on the following argand diagram.



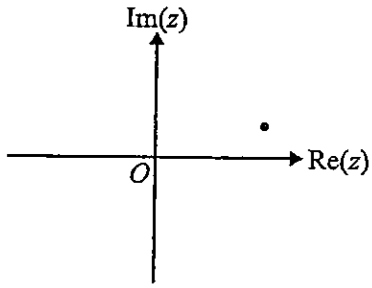
All axes below have the **same scale** as those in the diagram above.

The complex number $\frac{1}{\bar{z}}$ is best represented by

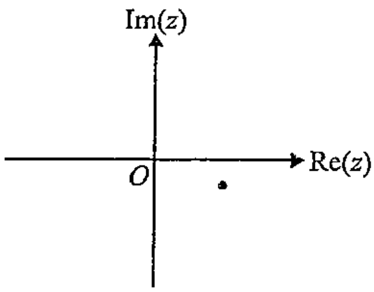
A.



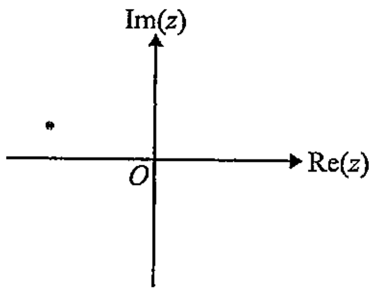
B.



C.



D.



4 Let $z = cis[150°]$

The imaginary part of $z - i$ is

(A) $\frac{-i}{2}$

(B) $\frac{-1}{2}$

(C) $\frac{-\sqrt{3}}{2}$

(D) $\frac{-3i}{2}$

5 The polynomial $P(z)$ has real coefficients. Four of roots of the equation $P(z) = 0$ are

$z = 0,\ z = 1 - 2i.\ z = 1 + 3i,\ z = 1 - 3i\ and\ z = 3i.$

The **minimum** number of roots that the equation $P(z) = 0$ could have is

(A) 5

(B) 6

(C) 7

(D) 8

6 Given that $z = 4cis[120°]$ it follows that the best answer to $Arg(z^5)$ in degrees is

(A) $(120^5)°$

(B) $24°$

(C) $600°$

(D) $-120°$

7 The distance between the points z and $-\bar{z}$ in the complex plane is given by

(A) $2\,Re\,(z)$

(B) $2\,Im\,(z)$

(C) $2\,|z|$

(D) $2\,Re\,(z) + 2\,Im\,(z)$

8 The algebraic fraction $\frac{7}{(x-3)(x^2+4)}$ can be reduced to its partial fractions by equating it to

(A) $\frac{a}{(x-3)} + \frac{b}{(x^2+4)}$

(B) $\frac{a}{(x-3)} + \frac{b}{(x+2)(x-2)}$

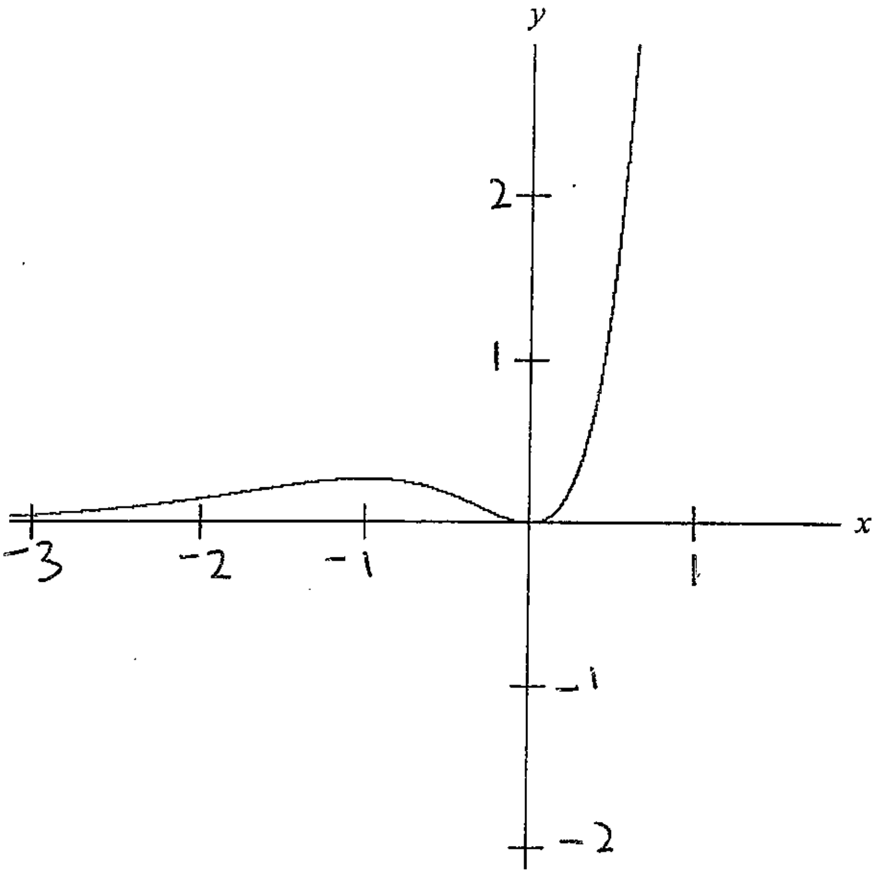
(C) $\frac{ax+b}{(x-3)} + \frac{c}{(x^2+4)}$

(D) $\frac{a}{(x-3)} + \frac{bx+c}{(x^2+4)}$

End of Multiple choice section (Total 8 marks)

Graphs (13 marks) Answer the rest of the exam on your own paper

Q1 The diagram below shows the graph of $y = f(x)$



Draw separate neat sketches of the following

i.	$y = -f(x)$	1
ii.	$y = f(-x)$	2
iii.	$y = f\lvert x\rvert$	2
iv.	$y = \sqrt{f(x)}$	2
v.	$y = \frac{1}{f(x)}$	2
vi.	$y = \lvert f(x)\rvert$	2
vii.	$y^2 = f(x)$	2

Graphs (13 marks)

Q2 (i) Show that $\frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}$ 4

Hence by adding ordinates or otherwise

sketch the graph of $y = \frac{x^2}{x+1}$

showing all important features.

(ii) Consider the function $f(x) = \frac{3x}{(x-1)(4-x)}$

(a) Express $f(x)$ as partial fractions 2

(b) Find the x **coordinates** of any turning points 3

(iii) On the same axes, neatly sketch the graphs of $y = (x+1)^2$

and $y = \frac{2}{x}$ (draw as dotted graphs for reference) By

multiplying ordinates or otherwise and using the 4

same axes, draw the graph of $y = \frac{2(x+1)^2}{x}$

(Make sure this graph is clearly distinguished from the

reference graphs and **not** dotted)

Polynomials (13 marks)

Q1 (a) The polynomial $P(x) = x^3 - 3x^2 + 4$ has a root of multiplicity 2. Find this root and fully factorise $P(x)$ 3

(b) Factorise $x^3 - 4x^2 + 6x - 4$ into 3 linear factors 4

(c) Form a new equation whose roots are reciprocals of the roots of $3x^3 + 4x^2 - 5x + 3 = 0$ 2

(d) The equation $x^3 - x^2 - 3x + 2 = 0$ *has roots* α, β, γ

(i) Use the value of $\alpha + \beta + \gamma$ to find the monic polynomial equation with roots $2\alpha + \beta + \gamma,\ \alpha + 2\beta + \gamma,\ \alpha + \beta + 2\gamma$ 2

(e) α, β, γ are the roots of $x^3 + 2x^2 - 2x + 3 = 0$.
Form the equation whose roots are $\alpha^2, \beta^2, \gamma^2$ 2

Polynomials (13marks)

Q2 (a) It is given that $3 - i$ is a root of $P(z) = z^3 + rz + 60$, where r is a real number

(i) State why $3 + i$ is also a root of $P(z)$ 1

(ii) Factorise $P(z)$ over the <u>real</u> numbers 2

(b) By applying de Moivre's theorem and by expanding $(cos\theta + isin\theta)^3$ obtain expressions for $cos3\theta$ in terms of $cos\theta$. 4

(c) (i) If $P(x)$ and $Q(x)$ have a common factor $(x - a)$, show that $R(x) = P(x) - Q(x)$ will have the same common factor 2

(ii) If $P(x) = 6x^3 + 7x^2 - x - 2$ and $Q(x) = 6x^3 - 5x^2 - 3x + 2$, find the two zeros that $P(x)$ and $Q(x)$ have in common. 2

(d) When a polynomial $P(x)$ is divided by $(x - 3)$ and $(x - 4)$ the remainders are 5 and 12 respectively. Determine what the remainder must be when $P(x)$ is divided by $(x - 3)(x - 4)$ 2

End of examination

Please ensure

1. Your **name or candidate number** is on **each section** including the **Question Sheet.**

2. Each of the **3 sections** are stapled **separately** and collected for marking.

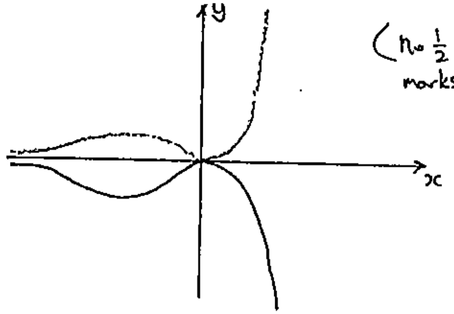
Assess #2 MATHS EXT 2 GRAPHS SOLUTIONS.

Q1) (−1 overall no ruler, untidy)
−½ no x, y labels

(i) $y = -f(x)$

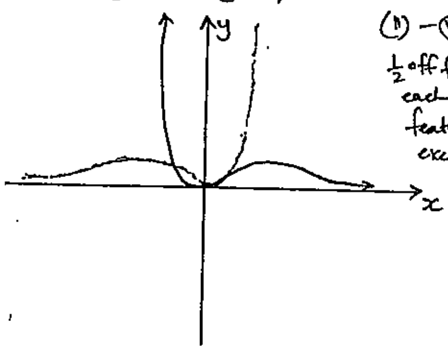
(a reflection in the x axis)

(no ½ marks)

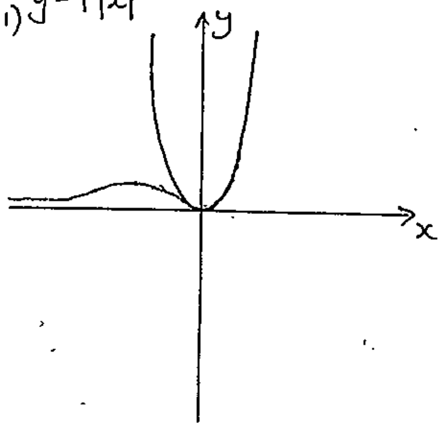


$y = f(-x)$

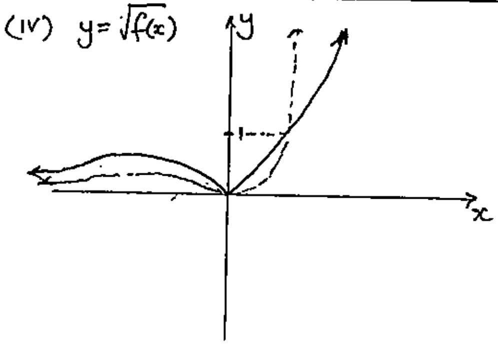
(i) − (vii) ½ off for each feature excluded.



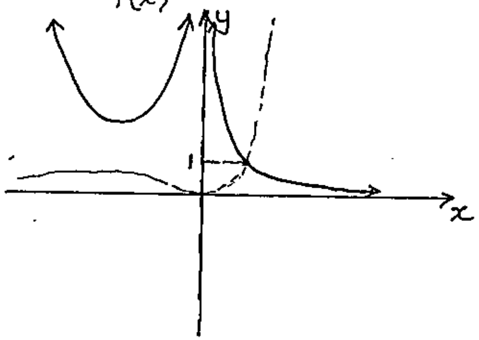
$y = f|x|$



(iv) $y = \sqrt{f(x)}$

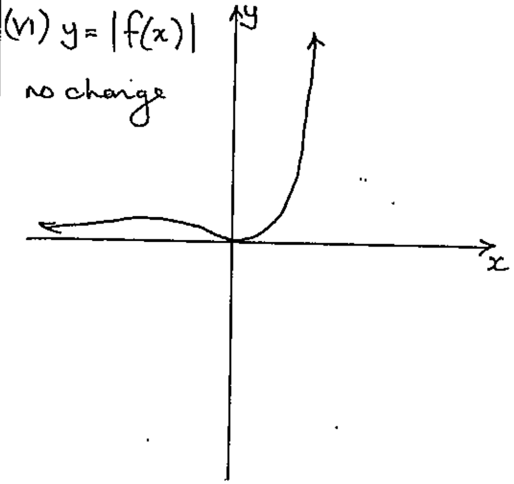


(v) $y = \frac{1}{f(x)}$



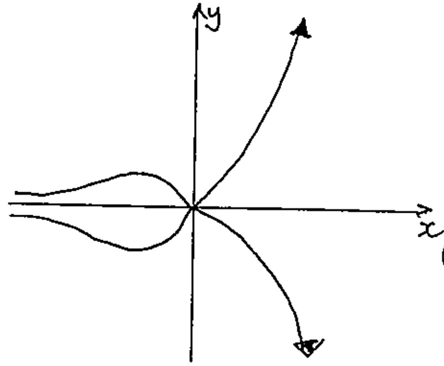
(vi) $y = |f(x)|$

no change



(vii) $y^2 = f(x)$

$y = \sqrt{f(x)}$ + reflection in x axis



Q2 (i) RHS $= \frac{(x-1)(x+1)+1}{x+1}$

$= \frac{x^2-1+1}{x+1}$ ①

$= \frac{x^2}{x+1}$ as req.

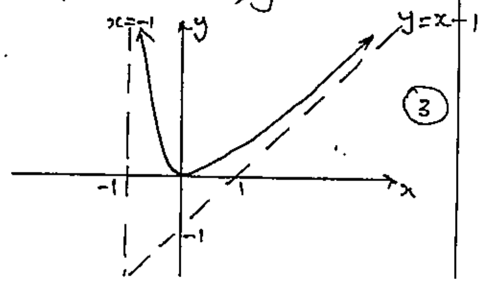
or by P.F's.

otherwise:

(ii) V.A ⇒ $x = -1$

H.A ⇒ $y = x - 1$ is an oblique asymptote

Intercepts ⇒ $x = 0, y = 0$



③

(ii) let $y = \frac{3x}{(x-1)(4-x)}$

a) $\frac{3x}{(x-1)(4-x)} = \frac{a}{x-1} + \frac{b}{4-x}$

$3x = a(4-x) + b(x-1)$

(let $x = 4$)

$12 = 3b$

$4 = b$

(let $x = 1$) ②

$3 = 3a$

$1 = a$

∴ RHS $= \frac{1}{x-1} + \frac{4}{4-x}$

b) stat pts occur when $y' = 0$

$y' = -1(x-1)^{-2} + -4(4-x)^{-2} \times -1$

$= \frac{-1}{(x-1)^2} + \frac{4}{(4-x)^2}$

($y' = 0$) $\frac{4}{(4-x)^2} = \frac{1}{(x-1)^2}$ ③

$4(x-1)^2 = (4-x)^2$

$4x^2 - 8x + 4 = 16 - 8x + x^2$

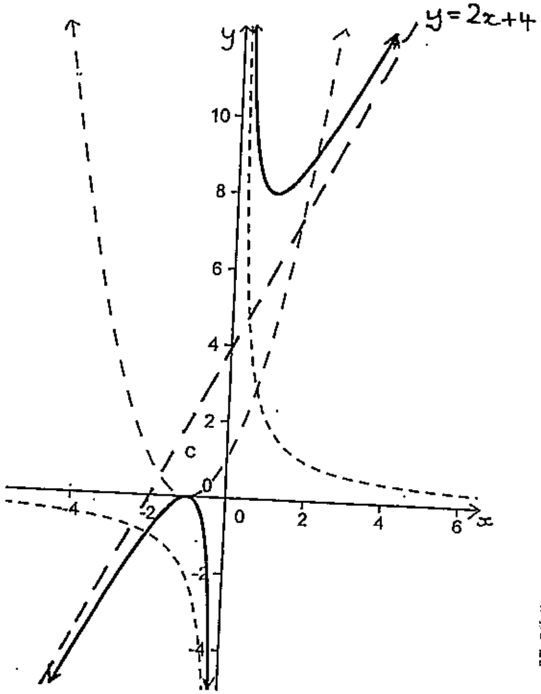
$3x^2 - 12 = 0$

$3(x-2)(x+2) = 0$

∴ $x = \pm 2$

1 - each dotted
(−½ no intercepts)
1 - each arm
(−½ no oblique)

Q2/
(iii)



V.A ⇒ $x = 0$

Intercepts ⇒ $y = \phi$, $x = -1$

HA ⇒ $\frac{2x^2+4x+2}{x} = 2x + 4 + \frac{2}{x}$

∴ Oblique Asymptote ⇒ $y = 2x+4$

Multiple Choice Answers

1/ C

2/ C

3/ C

4/ B

5/ C

6/ D

7/ A

8/ D

Polynomials

Q1/(a) $P(x) = x^3 - 3x^2 + 4$

$P'(x) = 3x^2 - 6x$

$= 3x(x-2)$

$x = 0$ not a root OR $x = 2$

$P(2) = 0$ ∴ 2 is a root of multiplicity 2

∴ $P(x) = (x-2)^2 Q(x)$

$= (x-2)^2 (x+1)$ (by inspection)

(b) $x^3 - 4x^2 + 6x - 4$

check ±1, ±2, ±4 $P(2) = 0$

$$\begin{array}{r} x^2 - 2x + 2 \\ x-2 \enclose{longdiv}{x^3 - 4x^2 + 6x - 4} \\ x^3 - 2x^2 \\ \hline -2x^2 + 6x \\ -2x^2 + 4x \\ \hline 2x - 4 \\ 2x - 4 \\ \hline 0 \end{array}$$

$x^3 - 4x^2 + 6x - 4 = (x-2)(x^2 - 2x + 2)$

$= (x-2)[(x-1)^2 + 1]$

$= (x-2)[(x-1)^2 - i^2]$

$= (x-2)(x-1-i)(x-1+i)$

Q1 Polynomials

(c) $3x^3 + 4x^2 - 5x + 3 = 0$

$\frac{3}{x^3} + \frac{4}{x^2} - \frac{5}{x} + 3 = 0$

multiply by x^3

$3 + 4x - 5x^2 + 3x^3 = 0$

∴ $3x^3 - 5x^2 + 4x + 3 = 0$ has reciprocal roots

(d) $x^3 - x^2 - 3x + 2 = 0$

$\alpha + \beta + \gamma = -\frac{b}{a}$

$\alpha + \beta + \gamma = 1$

∴ $2\alpha + \beta + \gamma = \alpha + 1$

$\alpha + 2\beta + \gamma = \beta + 1$

$\alpha + \beta + 2\gamma = \gamma + 1$

∴ replace x with $(x-1)$

$(x-1)^3 - (x-1)^2 - 3(x-1) + 2 = 0$ ✓

$x^3 - 3x^2 + 3x - 1 - x^2 + 2x - 1 - 3x + 3 + 2 = 0$ } This step not necessary

$x^3 - 4x^2 + 2x + 3 = 0$

(e) $x^3 - 2x^2 + x + 3 = 0$

let $x = \alpha^2$ ∴ $\alpha = \sqrt{x}$ or $-\sqrt{x}$ → you only need one or the other not both.

∴ $\alpha^2, \beta^2, \gamma^2$ satisfies $(\sqrt{x})^3 - 2(\sqrt{x})^2 + (\sqrt{x}) + 3 = 0$

$x\sqrt{x} - 2x + \sqrt{x} + 3 = 0$

$x\sqrt{x} + \sqrt{x} = 2x - 3$

squaring both sides $x^3 + 2x^2 + x = 4x^2 - 12x + 9$

$x^3 - 2x^2 + 13x - 9 = 0$

Polynomials

Q2) (a) (i) $P(z)$ has real co-efficients so if $3-i$ is a root the conjugate $3+i$ is also a root

(ii) Let $Q(z) = z^2 - (3+i+3-i)z + (9+1)$
$= z^2 - 6z + 10$

Now $P(z) = z^3 + rz + 60$

$\therefore z^3 + rz + 60 = (z^2 - 6z + 10)(z+6)$

(b) $\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3$ (De Moivres')
$= \cos^3\theta + 3\cos^2\theta\, i\sin\theta + 3i^2\sin^2\theta\cos\theta + i^3\sin^3\theta$
$= \cos^3\theta - 3\sin^2\theta\cos\theta + i(3\cos^2\theta\sin\theta - \sin^3\theta)$

Equating real ...

$\cos 3\theta = \cos^3\theta - 3\sin^2\theta\cos\theta$
$= \cos^3\theta - 3(1-\cos^2\theta)\cos\theta$
$= \cos^3\theta - 3\cos\theta + 3\cos^3\theta$
$= 4\cos^3\theta - 3\cos\theta$

2 (c) (i) Let $P(x) = (x-a)A(x)$ and
$Q(x) = (x-a)B(x)$

$P(x) - Q(x) = R(x)$

$R(x) = (x-a)A(x) - (x-a)B(x)$
$= (x-a)(A(x) - B(x))$

$\therefore R(x)$ has the same common factor as $P(x)$ and $Q(x)$

(ii) $6x^3 + 7x^2 - x - 2 - 6x^3 + 5x^2 + 3x - 2 = P(x) - Q(x)$
$12x^2 + 2x - 4 = R(x)$

$R(x) = 2(6x^2 + x - 2)$

$3x \quad 2$
$2x \quad -1$

$= 2(3x+2)(2x-1)$

$\therefore$ common zeros are $-\frac{2}{3}, \frac{1}{2}$

2 (d) $P(x) = (x-3)(x-4) \cdot Q(x) + R(x)$

$= (x-3)(x-4) \cdot Q(x) + (ax+b)$

$P(3) = 5$

$\therefore 5 = 3a+b$

$P(4) = 12$

$\therefore 12 = 4a+b$

$4a+b = 12$

$3a+b = 5$

subtract

$a = 7$

$21+b = 5$

$b = -16$

$\therefore$ remainder is $7x-16$