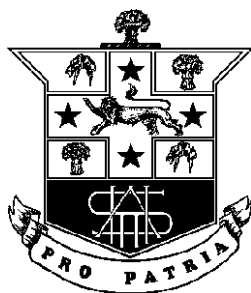# HURLSTONE AGRICULTURAL HIGH SCHOOL

MATHEMATICS

2005

YEAR 12

ASSESSMENT TASK 2

HALF YEARLY EXAMINATION

MATHEMATICS

EXAMINERS ~ S. GEE, P. BIZCO, H. CAVANAGH, S. FAULDS, R. YEN

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working Time – 2 hours.
- Attempt **all** questions.
- Questions are of equal value.
- **All** necessary working should be shown in every question.
- This paper contains ten (10) questions.

- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators and MathAids may be used.
- **Each question is to be started in a new answer booklet**.
- This examination paper must **NOT** be removed from the examination room

STUDENT NAME: _____

TEACHER: _____

Start each question on a separate writing booklet.

**QUESTION ONE   8 marks**   Start a SEPARATE booklet   **Marks**

(a)   Find the value of $\sqrt{\dfrac{19}{4\pi}}$ correct to 2 decimal places   **1**

(b)   Simplify $\dfrac{m+3}{2} - \dfrac{m+2}{3}$ **2**

(c)   Factorise $3k^2 - 8k - 3$ **2**

(d)   A 2.5% increase in the annual Council rates increased the charge by $28.   **1**
      What was the *original* charge?

(e)   Factorise $a^3 - 27$ **2**

QUESTION TWO.   8 marks   Start a NEW booklet.

(a)   Solve the following equation for $0° \le \theta \le 360°$:

$$\sqrt{3}\tan\theta = -1$$ **3**

(b)   Prove the identity:
$$5 - 5\sin^2\theta \equiv 5\cos^2\theta$$ **2**

(c)   A ship is travelling due west at 20 knots.
      From a point A, a lighthouse is sighted on a bearing of 300°.
      Two hours later, at point B, the lighthouse can be seen on a bearing of 345°.

      (i)   Draw a neat diagram which illustrates the information given above.   **1**

      (ii)   How far is the point B from the lighthouse?   **2**
            Give your answer to the nearest nautical mile.

QUESTION THREE.   **8 marks**   Start a NEW booklet.   **Marks**
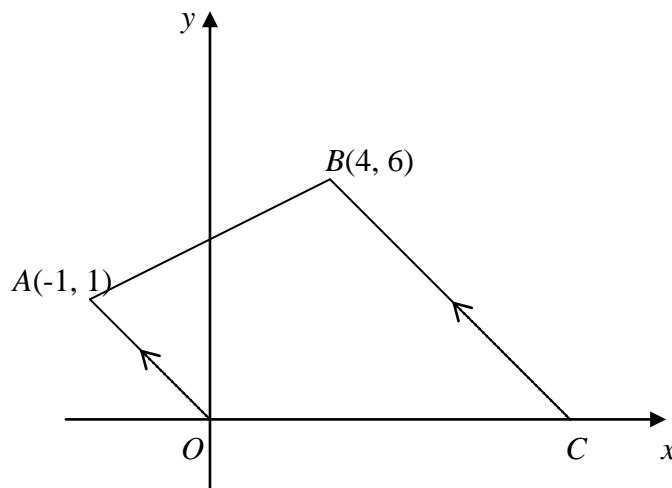
In the diagram, *OABC* is a trapezium with *OA* || *CB*. The coordinates of *O, A* and *B* are (0, 0), (-1, 1) and (4, 6) respectively.

(a)   Calculate the length of *OA*.   **1**

(b)   Write down the gradient of the line *OA*.   **1**

(c)   Find the equation of the line *BC*.   **1**

(d)   Find the coordinates of *C*.   **1**

(e)   Show that the perpendicular distance from *O* to the line *BC* is $5\sqrt{2}$.   **2**

(f)   Hence, or otherwise, calculate the area of the trapezium *OABC*.   **2**

QUESTION FOUR.   **8 marks**   Start a NEW booklet.

Consider the curve given by $y = \dfrac{1}{4}x^4 - x^3$.

(a)   Find any turning points and determine their nature.   **3**

(b)   Find any points of inflexion.   **2**

(c)   Sketch the curve for $-1 \le x \le 4$.   **1**

(d)   For what values of *x* is the curve concave down?   **2**
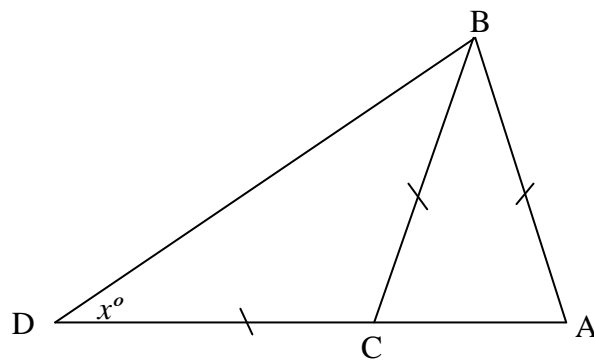
(a)
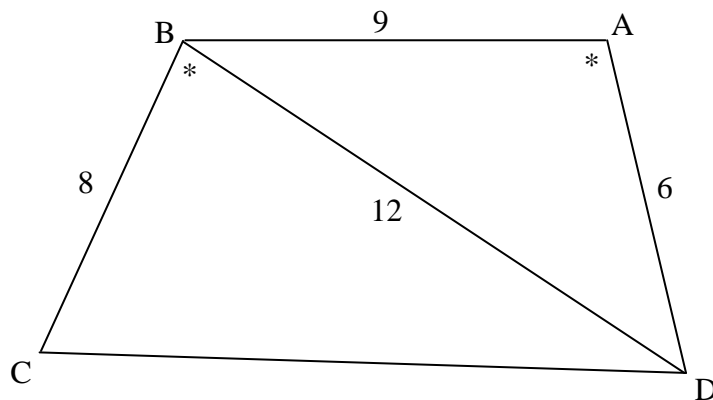
$\angle DAB = \angle CBD$

(NOT TO SCALE)

    (i)   Prove triangles ABD and BDC are similar.   **2**

    (ii)   Find the length of CD.   **1**

    (iii)   Prove that AB and CD are parallel.   **1**

(b)

$$AB = BC = CD; \quad \angle BDC = x^o$$

    (i)   Prove that $\angle CAB = 2x^o$ **2**

    (ii)   If $\angle ABD = 120^o$, find the value of x.   **2**

QUESTION SIX   8 marks   Start a SEPARATE booklet   **Marks**

(a)   Three consecutive terms of a sequence are $2x + 5$, T_2 and $8x + 19$.
       Find T_2 in terms of x if the sequence is to be arithmetic.   **2**

(b)   For the sequence 3, $\dfrac{11}{2}$, 8, … find:

    (i)   the 37th term   **2**

    (ii)   the sum of 37 terms.   **2**

(c)   Does the sequence $\dfrac{3}{4}$, 1, $\dfrac{4}{3}$, ... have a limiting sum?

    Explain your answer, stating S_∞ if it exists.   **2**

QUESTION SEVEN   8 marks   Start a SEPARATE booklet

(a)   Show the equation of a parabola is $x^2 - 2x - 2y - 13 = 0$ is also given by
       $(x-1)^2 = 2(y+7)$.   **1**

    Find:

    (i)   the coordinates of its vertex   **1**

    (ii)   the focal length   **1**

    (iii) the equation of its directrix   **1**

(b) $A(1, 0)$ and $B(4, 0)$ are points on the number plane.
       The point $P(x, y)$ moves such that the length of PB is twice the length of PA.

    (i) Write a formula for the length of PB.   **1**

   (ii) Prove that the locus of P is a circle and determine its centre and radius.   **3**

QUESTION EIGHT   8 marks   Start a SEPARATE booklet

(a)   If $2x^2 - 7x + 4 = a(x+2)^2 + b(x+2) + c$ for all values of x, find a, b and c.   **2**

(b)   Find all values of k for which the quadratic equation $x^2 + (k-3)x + k = 0$ has real roots   **3**

(c)   Find all real numbers x which satisfy the equation $x^4 = 4(x^2 + 8)$ **3**

QUESTION NINE   8 marks   Start a SEPARATE booklet   **Marks**

(a)   For the function $y = x + \dfrac{900}{x}$

    (i)   Find $\dfrac{dy}{dx}$ **1**

    (ii)   Show that y has a relative minimum value of 60.   **2**

(b)   In the triangle ABC, AB = 20m, BC = 15m and angle ABC = $90°$. BPQR is a
rectangle inscribed in ABC, as shown, with PQ = x metres.
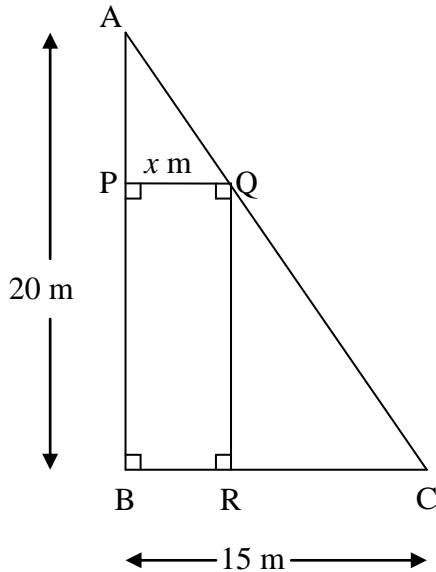
    (i)   Prove that ΔAPQ ||| ΔABC   **1**

    (ii)   Find the length of AP in terms of x and hence show that the area of the
rectangle BPQR is given by $x(20 - \dfrac{4x}{3})$ m^2 **2**

    (iii)   Hence find the maximum possible area of the rectangle BPQR   **2**

QUESTION TEN   8 marks   Start a SEPARATE booklet

(a)   Find the primitive of $x^2 + 2x - 3$.   **1**

(b)   Expand and simplify $(x^2 + 2)^2$.   **2**
Hence find the primitive of $(x^2 + 2)^2$.

(c)   The curve $y = f(x)$ has a gradient function   **3**

$$\frac{dy}{dx} = 3x^2 - 2x + 1.$$

If the curve passes through the point Q(2,3), find its equation.

(d)   Find the domain for which $y = \dfrac{1}{x^2 + 1}$ is a decreasing function.   **2**