HURLSTONE AGRICULTURAL HIGH SCHOOL

PRO PATRIA

<u>MATHEMATICS – EXTENSION 2</u>

2006

YEAR 12

HALF YEARLY EXAMINATION

Examiners ~ Z Pethers, G Rawson

GENERAL INSTRUCTIONS

- Reading Time – 5 minutes.
- Working Time – 2 HOURS.
- Attempt **all** questions.
- **All** necessary working should be shown in every question.
- This paper contains five (5) questions.
- Marks may not be awarded for careless or badly arranged work.
- Board approved calculators may be used.
- **Each question is to be started in a new booklet.**
- This examination paper must **NOT** be removed from the examination room.

STUDENT NAME: ______________________________

TEACHER: ______________________________

Question 1 (15 marks) Start a NEW answer booklet

Marks

(a) Find all pairs of integers *x* and *y* that satisfy $(x+iy)^2 = 33+56i$ **2**

(b) (i) Let $z = \cos\frac{\pi}{8} + i\sin\frac{\pi}{8}$. Find z^8. **2**

(ii) Plot on the Argand diagram, all complex numbers that are solutions of $z^8 = -1$. **2**

(c) Find all the roots of the equation $5z^3 - 31z^2 + 56z - 10 = 0$, given that $(3+i)$ is one of the roots. **3**

(d) (i) If $z_1 = 2+4i$, sketch the locus of (α) $|z - z_1| = 3$ **1**

(β) $\arg(z - z_1) = \frac{\pi}{4}$ **1**

(ii) Find the area specified by $|z - z_1| \leq 3$ *and* $\arg(z - z_1) \leq \frac{\pi}{4}$ **2**

(iii) If $|z - z_1| = 3$, find the greatest value of $|z - 7|$. **2**

Question 2 (15 marks) Start a NEW answer booklet

Marks

(a) (i) Suppose the polynomial $P(x)$ has a double root at $x = \alpha$.
Prove that $P'(x)$ also has a root at $x = \alpha$. **2**

(ii) The polynomial $P(x) = x^4 + ax^3 + bx + 21$ has a double root at $x = 1$.
Find the values of a and b. **3**

(iii) Factorise the polynomial $P(x) = x^4 + ax^3 + bx + 21$ over the real numbers. **2**

(b) If $\dfrac{2x+31}{(x-1)^3(x+2)} = \dfrac{a}{(x-1)} + \dfrac{b}{(x-1)^2} + \dfrac{c}{(x-1)^3} + \dfrac{d}{(x+2)}$, find the values of a, b, c, d. **3**

(c) The equation $x^4 + 3x^3 + 5x^2 - 7x + 2 = 0$ has roots $\alpha, \beta, \gamma, \delta$.
Find the equation with roots $\alpha^2, \beta^2, \gamma^2, \delta^2$. **3**

(d) The equation $x^3 - 4x^2 + 5x + 2 = 0$ has roots α, β, γ.
Find the value of $\alpha^2 + \beta^2 + \gamma^2$. **2**

Question 3 (15 marks) Start a NEW answer booklet

Marks

Consider the hyperbola: $x^2 - 3y^2 = 6$.

(a) Find the eccentricity of the hyperbola. **1**

(b) Find the coordinates of the foci and the equations of the directrices and asymptotes of the hyperbola. **3**

(c) Sketch the graph of the hyperbola showing clearly all of the above features and the intercepts on the coordinate axes. **2**

(d) Use differentiation to derive the equations of the tangent and normal to the hyperbola at the point $P(3, -1)$. **3**

(e) The tangent meets the transverse axis in M and normal meets the conjugate axis in N. Find the distance MN. **1**

(f) Determine the angle between the asymptotes. **2**

(g) Find the equations of the tangents to $x^2 - 3y^2 = 6$ which are parallel to the line $2x - y = 7$. **3**

Question 4 (15 marks) Start a NEW answer booklet

Marks

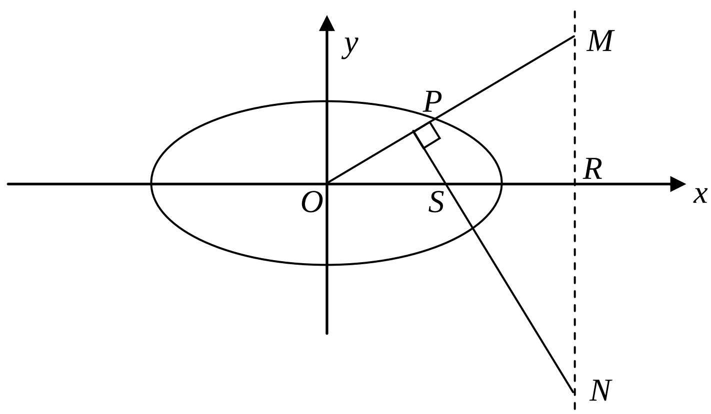
(a) (i) Find the equation of the ellipse with parametric equations $x = 2\cos\theta$ and $y = \sin\theta$. **2**

(ii) Sketch the ellipse, showing its foci and directrices. Show the auxillary circle on your sketch and state its equation. **2**

(b) Using differentiation, find the equation of the normal to the curve whose parametric equations are $x = 2\sec\theta$ and $y = 3\tan\theta$ at the point P where $\theta = \dfrac{\pi}{3}$. **3**

(c) Prove that the condition for the line $y = mx + c$ to be a tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is $c^2 = a^2m^2 + b^2$. **3**

(d) $P(a\cos\theta, b\sin\theta)$ lies on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with centre O.



OP produced meets a directrix in M, and the perpendicular from the corresponding focus onto OM meets the same directrix in N. If R is the foot of the directrix, prove that the product of RM and RN is independent of the position of P. **5**

Question 5 (15 marks) Start a NEW answer booklet

Marks

(a) Consider the polymonial equation $x^4 + ax^3 + bx^2 + cx + d = 0$, where *a*, *b*, *c*, and *d* are all integers. Suppose the equation has a root of the form *ki*, where *k* is real, and $k \neq 0$.

(i) State why the conjugate –*ki* is also a root. **1**

(ii) Show that $c = k^2 a$. **2**

(iii) Show that $c^2 + a^2 d = abc$. **2**

(iv) If 2 is also a root of the equation, and $b = 0$, show that *c* is even. **2**

(b) Let $\rho = \cos\frac{2\pi}{7} + i\sin\frac{2\pi}{7}$.

The complex number $\alpha = \rho + \rho^2 + \rho^4$ is a root of the quadratic equation $x^2 + ax + b = 0$, where *a* and *b* are real.

(i) Prove that $1 + \rho + \rho^2 + \ldots \rho^6 = 0$. **2**

(ii) The second root of the quadratic equation is β. Express β in terms of positive powers of ρ. Justify your answer. **2**

(iii) Find the values of the coefficients of *a* and *b*. **2**

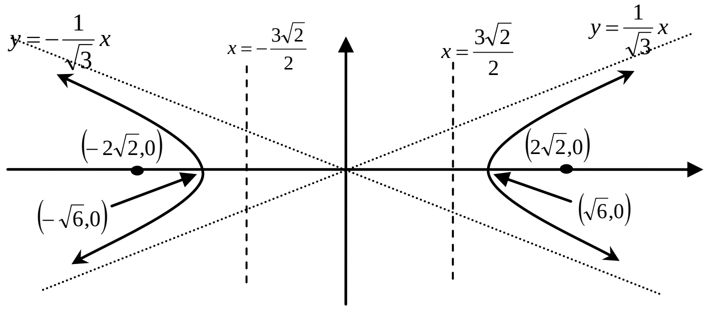
(iv) Deduce that $-\sin\frac{\pi}{7} + \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7} = \frac{\sqrt{7}}{2}$. **2**

Year 12	Mathematics Extension 2	HY Examination 2006
Question No. 3	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

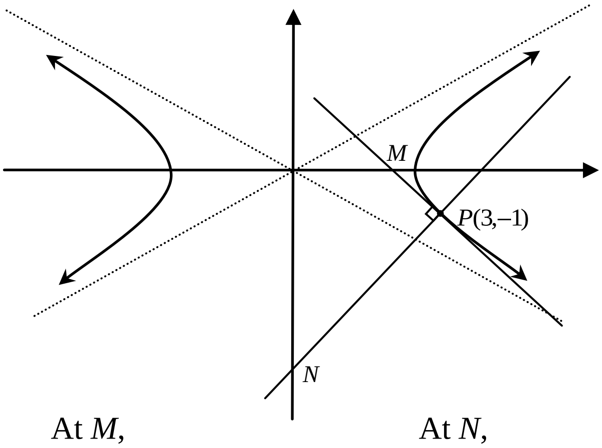
E3 uses the relationship between algebraic and geometric representations of conic sections

E4 uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections

Outcome	Solutions	Marking Guidelines
(a) **E3, E4**	$x^2 - 3y^2 = 6$ $\frac{x^2}{6} - \frac{y^2}{2} = 1 \Rightarrow$ $b^2 = a^2(e^2 - 1)$ $e = \sqrt{\frac{b^2}{a^2} + 1}$ $= \sqrt{\frac{2}{6} + 1}$ $= \frac{2}{\sqrt{3}}$	<u>1 mark</u> : correct answer
(b) **E3, E4**	**Foci** $(\pm ae, 0)$ $= \left(\pm\sqrt{6} \times \frac{2}{\sqrt{3}}, 0\right)$ $= \left(\pm 2\sqrt{2}, 0\right)$ **Directrices** $x = \pm\frac{a}{e}$ $x = \pm\frac{\sqrt{6} \times \sqrt{3}}{2}$ $x = \pm\frac{3\sqrt{2}}{2}$ **Asymptotes** $y = \pm\frac{b}{a}x$ $y = \pm\frac{\sqrt{2}}{\sqrt{6}}x$ $y = \pm\frac{1}{\sqrt{3}}x$	<u>3 marks</u> : all correct *OR* <u>2 marks</u> : 2 correct *OR* <u>1 mark</u> : 1 correct
(c) **E3, E4**		<u>2 marks</u> : correct diagram with *x*-intercepts *OR* <u>1 mark</u> : correct diagram, but partially labelled
(d) **E3, E4**	$x^2 - 3y^2 = 6$ $2x - 6y.\frac{dy}{dx} = 0$ $\frac{dy}{dx} = \frac{x}{3y}$ $= \frac{3}{-3}$ $= -1$ at $P(3, -1)$ Eq'n of tangent $y + 1 = -1(x - 3)$ $x + y = 2$ Eq'n of normal $y + 1 = 1(x - 3)$ $x - y = 4$	<u>3 marks</u> : correct solution *OR* <u>2 marks</u> : substantially correct *OR* <u>1 mark</u> : partially correct

Q.3 pg2

(e)
E3, E4



At M,
$y = 0$ and $x + y = 2$
ie $x = 2$
so M is (2, 0)

At N,
$x = 0$ and $x - y = 4$
ie $y = -4$
so N is (0, –4)

Thus
$$MN = \sqrt{2^2 + 4^2} = 2\sqrt{5} \ \ units$$

1 mark : correct solution

(f)
E3, E4

Asymptote $y = \frac{1}{\sqrt{3}}x$ makes an angle

$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$ with the x-axis

By symmetry, the (acute) angle between the asymptotes is $2 \times \frac{\pi}{6} = \frac{\pi}{3}$

Alternatively, use $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$ with $\begin{cases} m_1 = \frac{1}{\sqrt{3}} \\ m_2 = -\frac{1}{\sqrt{3}} \end{cases}$

2 marks : correct solution
OR
1 mark : substantially correct

(g)
E3, E4

Required equations have $m = 2$, ie they are of form $y = 2x + k$

Solving simultaneously with $x^2 - 3y^2 = 6$
$$x^2 - 3(2x + k)^2 = 6$$
Gives
$$x^2 - 3(4x^2 + 4kx + k^2) - 6 = 0$$
$$11x^2 + 12kx + 3(k^2 + 2) = 0$$

Which must have equal roots for tangent

i.e. $(12k)^2 - 4.11.3(k^2 + 2) = 0$
$$12k^2 - 11(k^2 + 2) = 0$$
$$k^2 = 22$$
$$k = \pm\sqrt{22}$$

So equations of required tangents are $y = 2x \pm \sqrt{22}$

3 marks : correct solution
OR
2 marks : substantially correct
OR
1 mark : partially correct

Q.3 pg3	Alternate solution: $x^2 - 3y^2 = 6 \ldots (1)$ $2x - 6y\dfrac{dy}{dx} = 0$ $\dfrac{dy}{dx} = \dfrac{x}{3y}$ tangent must have gradient = 2 so $\dfrac{dy}{dx} = \dfrac{x}{3y} = 2$ ie $x = 6y$ subbing this into (1) $x^2 - 3y^2 = 6$ $36y^2 - 3y^2 = 6$ $y = \pm\dfrac{\sqrt{2}}{\sqrt{11}}$ and this gives $x = \pm\dfrac{6\sqrt{2}}{\sqrt{11}}$ now, using $y - y_1 = m(x - x_1)$ $y - \dfrac{\sqrt{2}}{\sqrt{11}} = 2\left(x - \dfrac{6\sqrt{2}}{\sqrt{11}}\right)$ $y = 2x - \sqrt{22}$ also, $y = 2x + \sqrt{22}$	

Year 12	Mathematics Extension 2	HY Examination 2006
Question No. 4	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

E3 uses the relationship between algebraic and geometric representations of conic sections

E4 uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections

Outcome	Solutions	Marking Guidelines

(a)(i) E3, E4

$$\left.\begin{aligned} x &= 2\cos\theta \\ y &= \sin\theta \end{aligned}\right\} \Rightarrow a = 2, b = 1$$

so ellipse has equation $\dfrac{x^2}{4} + y^2 = 1$

Marking Guidelines: 2 marks : correct solution
OR
1 mark : substantially correct

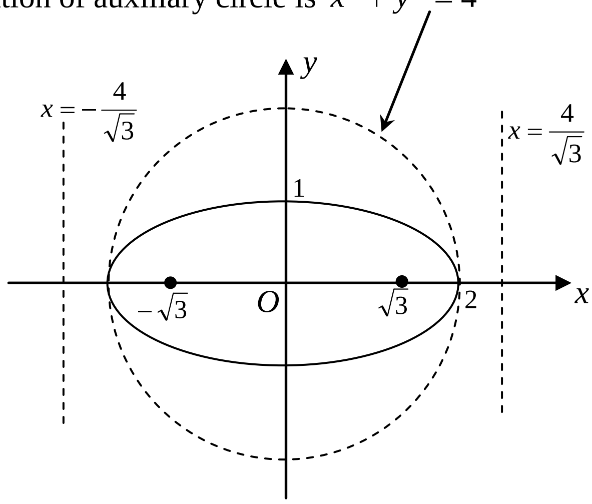
(a)(ii) E3, E4

$$b^2 = a^2\left(1 - e^2\right) \Rightarrow e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \tfrac{1}{4}} = \frac{\sqrt{3}}{2}$$

foci: $(\pm ae, 0) \Rightarrow \left(\pm 2 \times \dfrac{\sqrt{3}}{2}, 0\right)$ *ie* $\left(\pm\sqrt{3}, 0\right)$

directrices: $x = \pm\dfrac{a}{e} = \pm\dfrac{2}{\frac{\sqrt{3}}{2}} = \pm\dfrac{4}{\sqrt{3}}$

equation of auxiliary circle is $x^2 + y^2 = 4$



Marking Guidelines: 2 marks : correct diagram with auxiliary circle
OR
1 mark : correct ellipse, partially labelled

(b) E3, E4

$x = 2\sec\theta$ $\qquad$ $y = 3\tan\theta$

$\dfrac{dx}{d\theta} = 2\sec\theta\tan\theta$ $\qquad$ $\dfrac{dy}{d\theta} = 3\sec^2\theta$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3\sec^2\theta}{2\sec\theta\tan\theta} = \frac{3}{2\sin\theta}$$

Q.4 pg2

Now, when $\theta = \frac{\pi}{3}$

Normal has

$$m = -\frac{2\sin\theta}{3} = -\frac{2.\frac{\sqrt{3}}{2}}{3} = -\frac{\sqrt{3}}{3}$$

$$x = 2\sec\theta = 2\sec\frac{\pi}{3} = 2\times 2 = 4$$

$$y = 3\tan\theta = 3\tan\frac{\pi}{3} = 3\sqrt{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 3\sqrt{3} = -\frac{\sqrt{3}}{3}(x - 4)$$

$$\sqrt{3}x + 3y - 13\sqrt{3} = 0$$

3 marks : correct solution

OR

2 marks : substantially correct

OR

1 mark : partially correct

(c)
E3, E4

Tangent at $P(x_1, y_1)$ is $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \quad \ldots①$

To find condition for $y = mx + c$ to touch ellipse,
we write this as $mx - y = -c$

$$-\frac{mx}{c} + \frac{y}{c} = 1 \quad \ldots②$$

Both ① and ② represent the same tangent,
So, comparing coefficients:

$$\frac{x_1}{a^2} = -\frac{m}{c} \quad \text{and} \quad \frac{y_1}{b^2} = \frac{1}{c}$$

$$x_1 = \frac{-a^2 m}{c} \qquad y_1 = \frac{b^2}{c}$$

Subbing these values into $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, we have:

$$\frac{a^4 m^2}{a^2 c^2} + \frac{b^4}{b^2 c^2} = 1$$

$$\frac{a^2 m^2}{c^2} + \frac{b^2}{c^2} = 1$$

$$c^2 = a^2 m^2 + b^2$$

Alternate solution:

Solving $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ and $y = mx + c$ gives:

$$\frac{x^2}{a^2} + \frac{(mx+c)^2}{b^2} = 1$$

$$b^2x^2 + a^2m^2x^2 + 2a^2mcx + a^2c^2 - a^2b^2 = 0$$

which must have one solution for tangent, ie $\Delta = 0$.

$$\Delta = 4a^4 - 4(a^2m^2 + b^2)(a^2c^2 - a^2b^2) = 0$$

$$\Downarrow$$

$$c^2 = a^2m^2 + b^2$$

3 marks : correct solution

OR

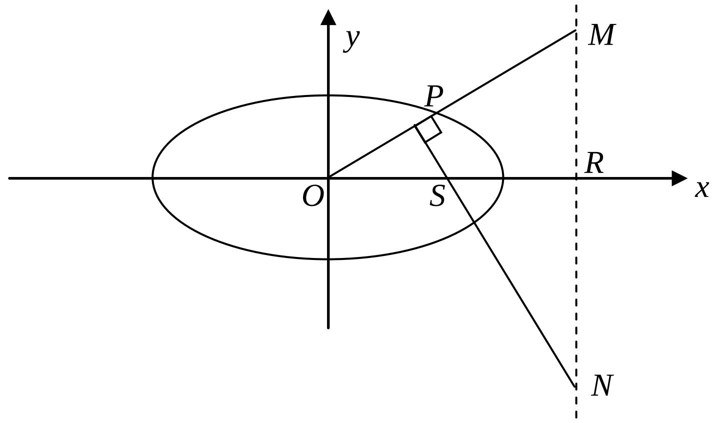
2 marks : substantially correct

OR

1 mark : partially correct

Q.4 pg3

(d)
E3, E4



Equation of OP is $y = \dfrac{b\sin\theta}{a\cos\theta}x$ and OP meets the directrix $x = \dfrac{a}{e}$ in *M*, thus, the coordinates of *M* are

$$x = \frac{a}{e} \quad , \quad y = \frac{b\sin\theta}{a\cos\theta}.\frac{a}{e} = \frac{b\sin\theta}{e\cos\theta}$$

Hence, $RM = \left|\dfrac{b\sin\theta}{e\cos\theta}\right|$

$SN \perp OP$ so *SN* has gradient $-\dfrac{a\cos\theta}{b\sin\theta}$ & passes through the focus $(ae, 0)$

so has equation $y - 0 = -\dfrac{a\cos\theta}{b\sin\theta}(x - ae)$

SN meets the directrix $x = \dfrac{a}{e}$ in *N*, thus, the coordinates of *N* are

$$x = \frac{a}{e} \quad , \quad y = \frac{-a\cos\theta}{b\sin\theta}\left(\frac{a}{e} - ae\right)$$

$$= \frac{-a^2(1-e^2)\cos\theta}{eb\sin\theta}$$

$$= \frac{-b^2\cos\theta}{eb\sin\theta} \quad , \qquad \text{since } b^2 = a^2(1-e^2).$$

Hence, $RN = \left|\dfrac{b\cos\theta}{e\sin\theta}\right|$

$$\therefore RM \times RN = \left|\frac{b\sin\theta}{e\cos\theta}\right|.\left|\frac{b\cos\theta}{e\sin\theta}\right| = \frac{b^2}{e^2}$$

which is independent of θ.

<u>5 marks</u> : correct solution

OR

<u>4 marks</u> : substantially correct

OR

<u>3 marks</u> : correctly finding the length of either *RM* or *RN* and partially correct solution thereafter.

OR

<u>2 marks</u> : correctly finding the length of either *RM* or *RN*

OR

<u>1 mark</u> : some evidence of an appropriate method of solution

Year 12	Mathematics Extension 2	HY Examination 2006
Question No. 5	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

E2 chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings

E4 uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving polynomials

E9 communicates abstract ideas and relationships using appropriate notation and logical argument

Outcome	Solutions	Marking Guidelines
(a)(i) E4	For polynomials with real coefficients, complex roots occur in conjugate pairs. So, if *ki* is a root, –*ki* must also be a root.	1 mark : correct statement – must include reference to real coefficients
(a)(ii) E4	$P(x) = x^4 + ax^3 + bx^2 + cx + d$ $P(ki) = k^4 - ak^3 i - bk^2 + cki + d = 0$ Equating imaginary parts: $-ak^3 + ck = 0$ $c = ak^2$ ***OR…*** $(x - ki)(x + ki) = x^2 + k^2$ is a factor so, $P(x) = x^4 + ax^3 + bx^2 + cx + d = \left(x^2 + k^2\right)\left(x^2 + Ax + B\right)$ Equating coefficients of x^3 : $a = A$ Equating coefficients of x : $c = Ak^2$ $\therefore c = ak^2$ ***OR…*** Let the roots of $P(x)$ be ki , $-ki$, α , β $\sum \alpha = \alpha + \beta = -a$ $\sum \alpha\beta\gamma = ki\alpha\beta - ki\alpha\beta + k^2\alpha + k^2\beta$ $= k^2(\alpha + \beta) = -c$ $\therefore k^2(-a) = -c$ $c = ak^2$	2 marks : correct solution *OR* 1 mark : substantially correct
(a)(iii) E4	From $P(ki) = 0$ in (ii), equate real parts ie $k^4 - bk^2 + d = 0$ & noting $k^2 = \dfrac{c}{a}$ $\dfrac{c^2}{a^2} - b\dfrac{c}{a} + d = 0$ $c^2 - abc + ad = 0$ $c^2 + ad = abc$	2 marks : correct solution *OR* 1 mark : substantially correct

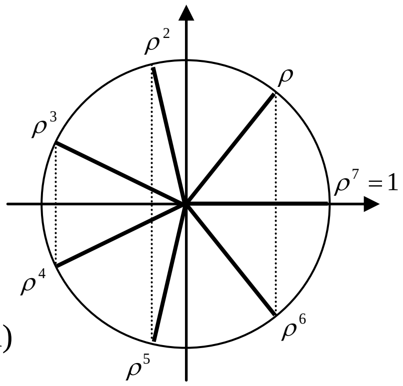
Q.5 pg2 (a)(iv) E2, E4	$b=0 \Rightarrow P(x)=x^4+ax^3+cx+d=0$ roots are ki, $-ki$, 2, α $\sum\alpha = 2+\alpha = -a$ $\sum\alpha\beta = k^2+2\alpha = 0$ $\alpha = -(a+2)$ $k^2 = 2\alpha$ Noting $k^2 = \frac{c}{a}$ $\Rightarrow$ $\frac{c}{a} = -2\alpha$ $c = -2\alpha$ $c = 2a(a+2)$ a is an integer, $\therefore c$ is even ***OR…*** $P(2)=0 \Rightarrow 16+8a+4b+2c+d=0$ since b = 0: $16+8a+2c+d=0$ Now, $c^2+a^2d=abc$ $b=0 \Rightarrow a^2d=-c^2$ $d=\frac{-c^2}{a^2}$ $\therefore 16+8a+2c-\frac{c^2}{a^2}=0$ $c^2-2a^2c-16a^2-8a^3=0$ Quadratic Formula gives $c=2a(a+2)$ or $c=-4a$ which are both even	<u>2 marks</u> : correct solution *OR* <u>1 mark</u> : substantially correct
(b)(i) E2	$\rho = \cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}$ $\rho^7 = \left(\cos\frac{2\pi}{7}+i\sin\frac{2\pi}{7}\right)^7$ $=\cos 2\pi + i\sin 2\pi$ $=1$ $\therefore \rho$ is a root of $x^7-1=0$ i.e. ρ is a root of $(x-1)(x^6+x^5+x^4+x^3+x^2+x+1)=0$ but $x\neq 0$, so ρ satisfies $x^6+x^5+x^4+x^3+x^2+x+1=0$ i.e. $1+\rho+\rho^2+\rho^3+\rho^4+\rho^5+\rho^6=0$ ***OR…*** $1+\rho+\rho^2+\ldots+\rho^6$ is a GP with $a=1$ $r=\rho$ $n=7$ $so,\ 1+\rho+\rho^2+\ldots+\rho^6 = \frac{1(\rho^7-1)}{\rho-1}$ $=\frac{(cis\frac{2\pi}{7})^7}{\rho-1}$ $=\frac{\cos 2\pi + i\sin 2\pi - 1}{\rho-1}=0$	<u>2 marks</u> : correct solution *OR* <u>1 mark</u> : substantially correct *Note: this must be proven/shown. Marks not awarded for using calculator*

Q.5 pg3

(b)(ii)
E2

Since the coefficients of $x^2 + ax + b = 0$ are real, and $\alpha = \rho + \rho^2 + \rho^4$ is a complex root, then $\overline{\alpha}$ must also be a root.

$$\therefore \beta = \overline{\alpha} = \overline{\rho + \rho^2 + \rho^4}$$
$$= \overline{\rho} + \overline{\rho^2} + \overline{\rho^4}$$
$$= \rho^6 + \rho^5 + \rho^3$$



Since $\overline{\rho} = \rho^6$, $\overline{\rho^2} = \rho^5$ and $\overline{\rho^4} = \rho^3$ (see diagram)

<u>2 marks</u> : correct solution

OR

<u>1 mark</u> : substantially correct

(b)(iii)
E2, E4

sum of roots:

$$\alpha + \beta = -a$$
$$\text{ie } a = -(\alpha + \beta)$$
$$= -\left(\rho + \rho^2 + \rho^3 + \rho^4 + \rho^5 + \rho^6\right) \text{ from (ii)}$$
$$= 1 \quad \text{from (i)}$$

product of roots:

$$b = \alpha\beta$$
$$= \left(\rho + \rho^2 + \rho^4\right)\left(\rho^3 + \rho^5 + \rho^6\right)$$
$$= \rho\left(1 + \rho + \rho^3\right)\rho^3\left(1 + \rho^2 + \rho^3\right)$$
$$= \rho^4\left(\underbrace{1 + \rho + \rho^2 + \rho^3 + \rho^4 + \rho^5 + \rho^6}_{\Downarrow} + 2\rho^3\right)$$
$$= \rho^4\left(\qquad 0 \qquad + 2\rho^3\right)$$
$$= 2\rho^7$$
$$= 2 \qquad \left(\rho^7 = 1\right)$$

i.e. $a = 1$ & $b = 2$

<u>2 marks</u> : correct solution
(both *a* **and** b correct)

OR

<u>1 mark</u> : partially correct
(either *a* **or** b correct)

(b)(iv)
E2

from (iii), the quadratic equation is:

$$x^2 + x + 2 = 0$$
$$\text{so... } x = \frac{-1 \pm \sqrt{1-8}}{2}$$
$$= \frac{-1 \pm i\sqrt{7}}{2}$$

$\therefore \text{Im}(\alpha) = \dfrac{\pm\sqrt{7}}{2}$, but $\alpha = \rho + \rho^2 + \rho^4$
and $\text{Im}(\alpha) > 0$, from diagram in (ii)

$$\therefore \text{Im}(\alpha) = \frac{\sqrt{7}}{2}$$

Q.5 pg3	Now, $\operatorname{Im}(\alpha) = \operatorname{Im}\left(\rho + \rho^2 + \rho^4\right)$ $= \sin\frac{2\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{8\pi}{7}$ $= \sin\frac{2\pi}{7} + \sin\left(\pi - \frac{3\pi}{7}\right) + \sin\left(\pi + \frac{\pi}{7}\right)$ $= \sin\frac{2\pi}{7} + \sin\frac{3\pi}{7} - \sin\frac{\pi}{7}$ $= \frac{\sqrt{7}}{2}$, as required	<u>2 marks</u> : correct solution *OR* <u>1 mark</u> : substantially correct *Note: this must be deduced, ie from earlier parts. Marks not awarded for using calculator*