HURLSTONE AGRICULTURAL HIGH SCHOOL

YEAR 12

MATHEMATICS EXTENSION 2

2010

HSC COURSE

HALF YEARLY EXAMINATION
(ASSESSMENT TASK 2)

EXAMINERS ~ G. RAWSON AND S. GEE

GENERAL INSTRUCTIONS

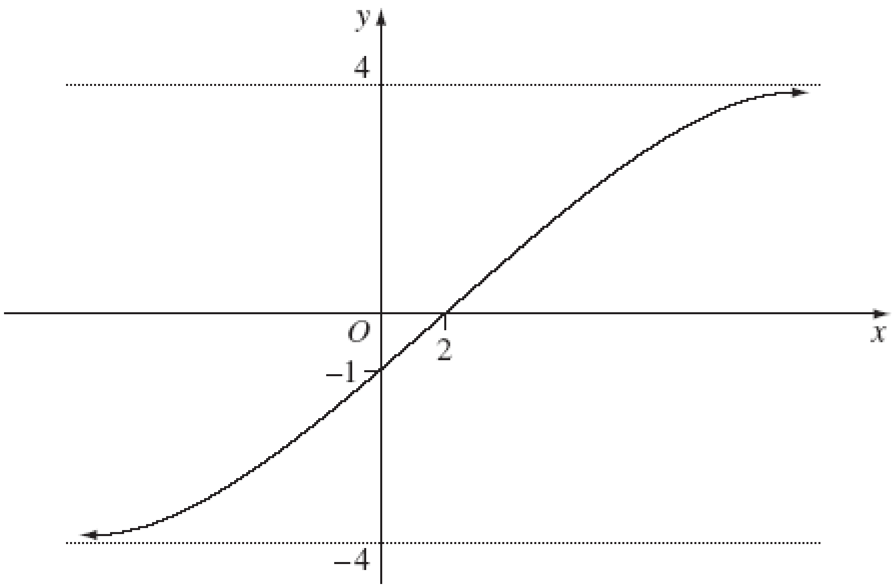
- Reading Time – 5 minutes.
- Working Time – Two Hours.
- Attempt all questions.
- Questions are of equal value.
- All necessary working should be shown in every question.
- **This paper contains Five (5) questions.**
- **Total Marks – 75 marks**
- Marks may not be awarded for careless or badly arranged work.
- Board approved Calculators and Templates may be used.
- Each question is to be started in a new Examination Booklet.
- This assessment task must **NOT** be removed from the Examination Room.

STUDENT NAME / NUMBER: ______________________________

TEACHER: ______________________________

QUESTION ONE **15 marks** *Start a SEPARATE sheet*

(a) The diagram shows the graph of $y = f(x)$.



Draw neat separate sketches of at least one third of a page for

(i) $y = f(x+3)$ **1**

(ii) $y = |f(x)|$ **1**

(iii) $y = \sqrt{f(x)}$ **1**

(iv) $y = f(|x|)$ **1**

(v) $|y| = f(x)$ **1**

(b) Given the function $g(x) = (x-1)^2(x+2)^2$

Draw neat separate sketches of at least one third of a page for

(i) $y = g(x)$ **1**

(ii) $y = \dfrac{1}{g(x)}$ **1**

(iii) $y = -g(x)$ **1**

(iv) $y = g(x) + 2$ **1**

Question 1 continued on next page…

Question 1 continued…

(c) Given the function $f(x)=\dfrac{x^2+1}{x}$

(i) Show $y = f(x)$ is an odd function. **1**

(ii) Find the equations of any asymptotes. **1**

(iii) Show $y = f(x)$ has a relative minimum point at $(1, 2)$. **1**

(iv) Without using calculus explain why $y = f(x)$ has a relative maximum point at $(-1, -2)$. **1**

(v) Sketch the curve $f(x)=\dfrac{x^2+1}{x}$ **1**

(vi) With reference to your sketch in part (v),
explain why the inequation $\left|\dfrac{x^2+1}{x}\right| \le 2$ has only two solutions. **1**

QUESTION TWO **15 marks** *Start a SEPARATE sheet*

(a) If $z = 3 - i$ and $w = 1 + 3i$, find (in the form $x + iy$) the simplest answer for

(i) $z - w$ **1**

(ii) zw **1**

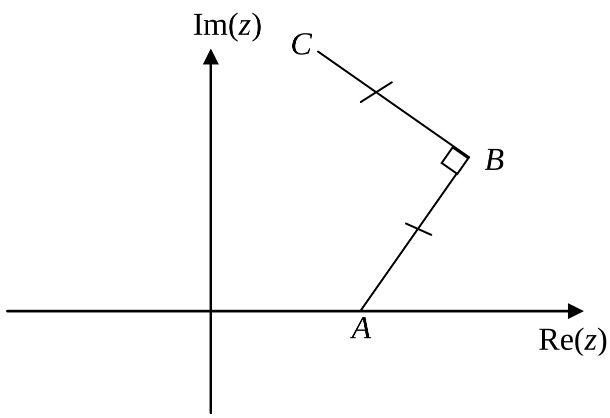
(iii) $w\overline{w}$ **2**

(iv) $\dfrac{z}{w}$ **2**

(b) (i) Find the modulus and argument (in radians) of $5 + 5\sqrt{3}\, i$ **2**

(ii) Hence, or otherwise, find the two square roots of $5 + 5\sqrt{3}\, i$ **3**

(c)



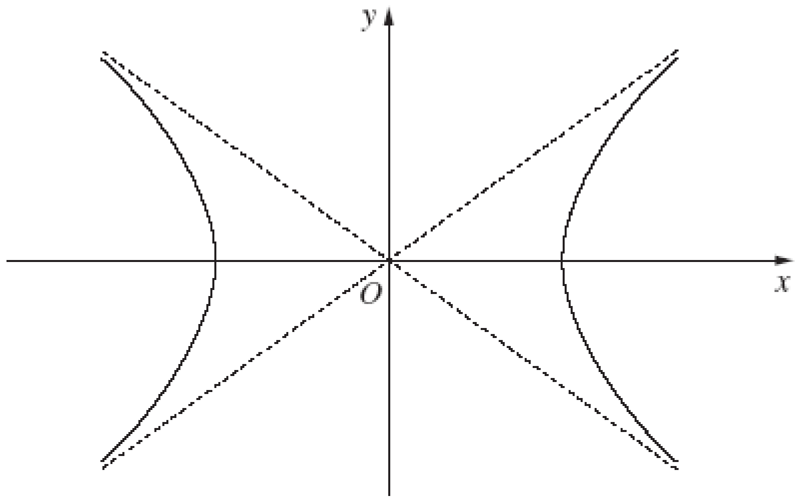
The diagram above shows the fixed points *A*, *B* and *C* in the Argand plane, where $AB = BC$, $\angle ABC = \dfrac{\pi}{2}$, and *A*, *B* and *C* are in anticlockwise order. The point *A* represents the complex number $z_1 = 2$ and the point *B* represents the complex number $z_2 = 3 + \sqrt{5}\, i$.

(i) Find the complex number z_3 represented by the point *C*. **2**

(ii) *D* is the point on the Argand plane such that *ABCD* is a square.
Find the complex number z_4 represented by *D*. **2**

QUESTION THREE **15 marks** *Start a SEPARATE sheet*

(a) The sketch shows a hyperbola in the form $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$



Given $a^2=144$ and $b^2=25$

(i) Find the coordinates of the points where the hyperbola meets the x- axis. **1**

(ii) Find the coordinates of the foci of the hyperbola. **2**

(iii) Find the equations of the asymptotes and directrices of the hyperbola. **2**

(iv) If a remains constant, what value must b become for the hyperbola to become rectangular? **1**

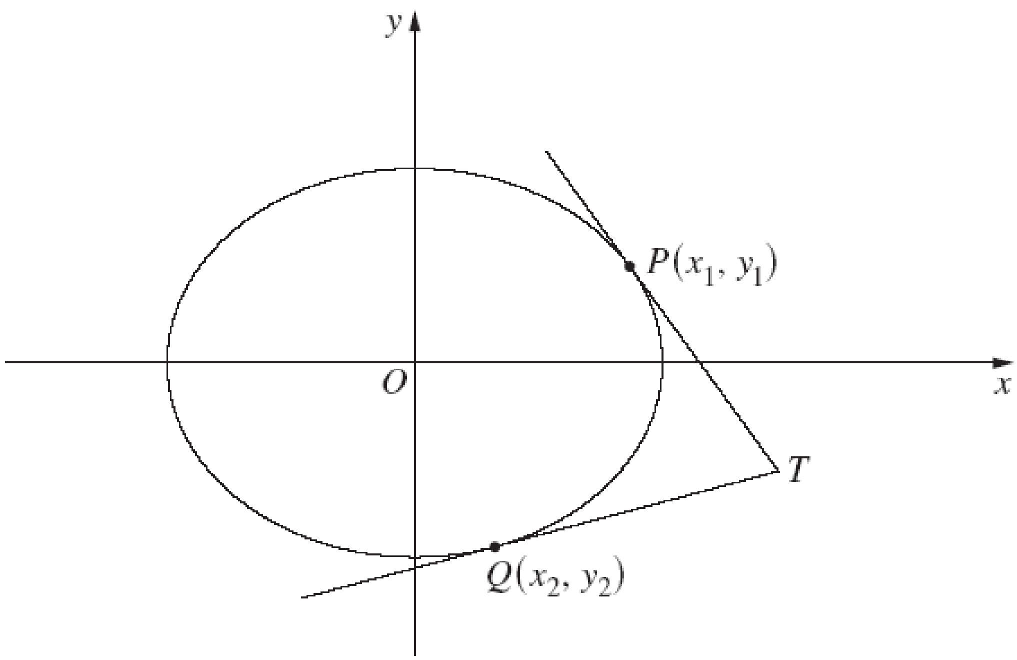
(v) Find the equation of the tangent to the curve at the point $P\left(13, \dfrac{1}{12}\right)$. **1**

(vi) Find the equation of the focal chord SP where S is the closest focus. **1**

Question 3 continued on next page…

(b) The points $P(x_1, y_1)$ and $Q(x_2, y_2)$ lie on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

The tangents at P and Q meet at T.



(i) Show that the equation of the tangent at P is $\dfrac{x_1}{a^2}x + \dfrac{y_1}{b^2}y = 1$. **2**

(ii) Show that T lies on the line $\dfrac{(x_1 - x_2)}{a^2}x + \dfrac{(y_1 - y_2)}{b^2}y = 0$. **2**

(iii) Let M be the midpoint of PQ.

Show that O, M and T are collinear. **3**

QUESTION FOUR **15 marks** *Start a SEPARATE sheet*

(a) If $\left|z_1 - z_2\right| = \left|z_1 + z_2\right|$ show that $\arg z_1 - \arg z_2 = \dfrac{\pi}{2}$. **2**

(b) Plot on the Complex Plane, the values of z for which $z^3 - 8i = 0$. **3**

(c) What is the maximum value of $|z|$ for $|z - 1 - i| \le 2$? **2**

(c) Sketch the locus of z satisfying:

(i) $\arg(z-4) = \dfrac{3\pi}{4}$ **3**

(ii) $\operatorname{Im} z = |z|$ **2**

(d) Given $z = x + iy$, sketch the locus of z if $\arg\left(\dfrac{z-1}{z+1}\right) = \dfrac{\pi}{4}$, showing all important features.

Write the Cartesian equation of this locus. **3**

QUESTION FIVE **15 marks** *Start a SEPARATE sheet*

(a) Factorise $2^{n+1}+2^n$ and hence write $\dfrac{2^{1001}+2^{1000}}{3}$ as a power of 2. **2**

(b) Consider the circle $x^2+y^2-2x-14y+25=0$

(i) Show that if the line $y = mx$ intersects the circle in two distinct points, then

$(1+7m)^2-25\left(1+m^2\right)>0$. **3**

(ii) For what values of m is the line $y = mx$ tangent to the circle? **2**

(c) Find the equation of $P(x)$ given it is monic of degree 5,
has a double root at $x = 2$ and is odd. Sketch $y = P(x)$. **2**

(d) (i) Prove De Moivre's Theorem using mathematical induction.
ie. $(\cos\theta+i\sin\theta)^n=\cos(n\theta)+i\sin(n\theta)$, for integral values of $n\geq 0$. **3**

(ii) Show De Moivre's Theorem also holds for integral negative values. **2**

(iii) If $z=1+i\sqrt{3}$ find the principal argument of z^5 in radians. **1**

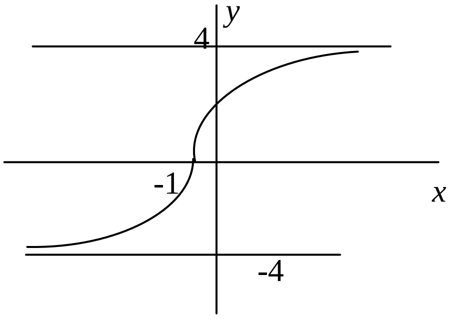
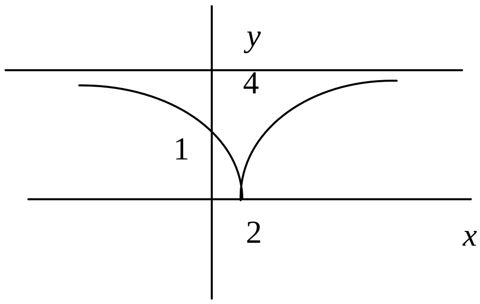
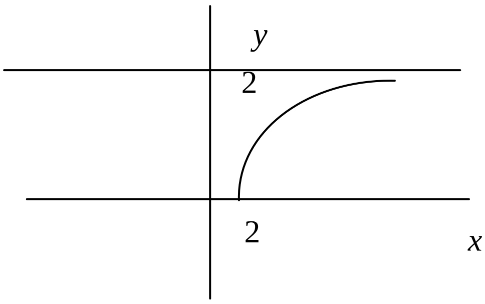
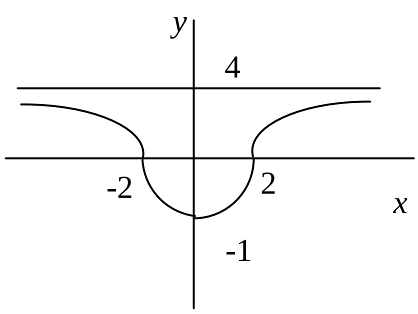
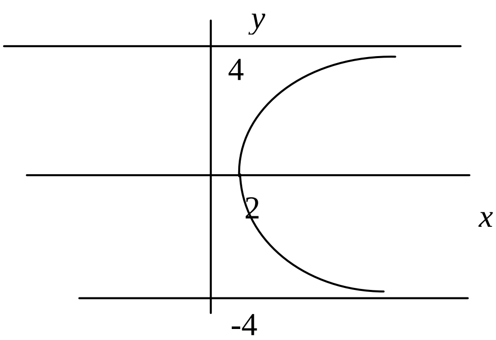
END OF TEST

Year 12 Extension 2 Mathematics HSC Task 2 Half Yearly 2010

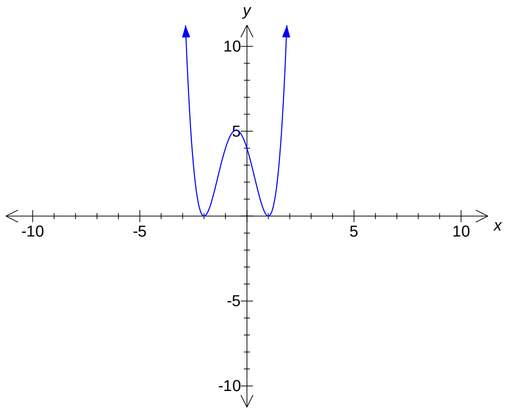
Question No.1 Solutions and Marking Guidelines

Outcomes Addressed in this Question

E6 combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions

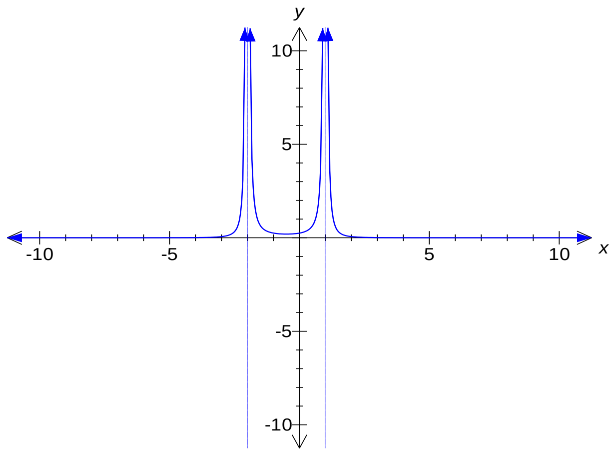
Outcome	**Solutions**	**Marking Guidelines**
	(a)(i) $y = f(x+3)$ 	1 mark correct graph
	(ii) $y = \lvert f(x) \rvert$ 	1 mark correct graph
	(iii) $y = \sqrt{f(x)}$ 	1 mark correct graph
	(iv) $y = f(\lvert x \rvert)$ 	1 mark correct graph
	(v) $\lvert y \rvert = f(x)$ 	1 mark correct graph

b(i) $g(x)=(x-1)^2(x+2)^2$



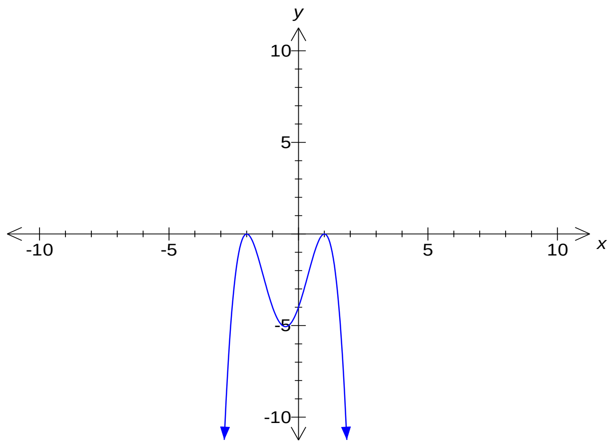
1 mark correct graph

(ii) $y=\dfrac{1}{g(x)}$



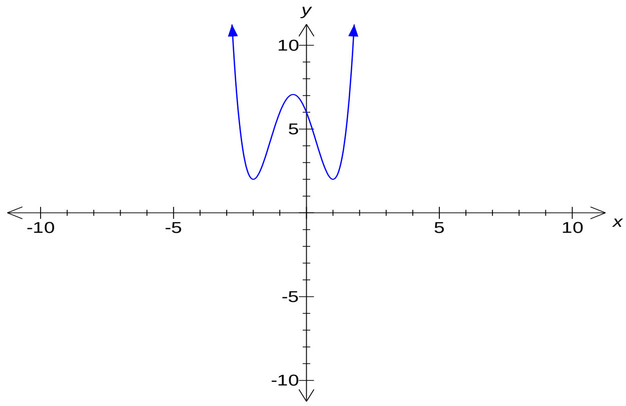
1 mark correct graph

(iii) $y=-g(x)$



1 mark correct graph

(iv) $y=g(x)+2$



1 mark correct graph

c

(i)

Odd Function if $f(-x) = -f(x)$

$$f(x) = \frac{x^2+1}{x}$$

$$f(-x) = \frac{(-x)^2+1}{-x}$$

$$= -\frac{x^2+1}{x}$$

$$= -f(x) \therefore \text{odd function}$$

1 mark correct answer

(ii)

Asymptotes $x = 0,\ y = x$

1 mark correct answer

(iii)

$$f(x) = x + \frac{1}{x}$$

$$f(x) = x + x^{-1}$$

$$f'(x) = 1 - x^{-2}$$

Stationary points at $f'(x) = 0$

$$1 - \frac{1}{x^2} = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

When $x = 1$ the point required is $(1, 2)$

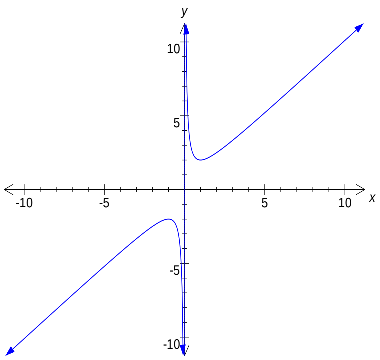
1 mark correct answer

(iv)

Since is an odd function it has point symmetry through the origin. Therfore a relative minima will have a relatve maxima as an image point and $(-1, -2)$ will be this point.

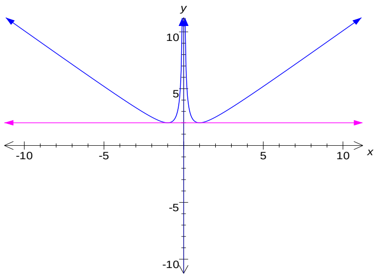
1 mark correct reasoning

(v)



1 mark correct graph

(vi)



Since the line y=2 cuts the curve above exactly twice and y<2 has no intersecting points then there is exactly 2 solutions to the inequation

$$\left|\frac{x^2+1}{x}\right| \le 2$$

1 mark correct reasoning

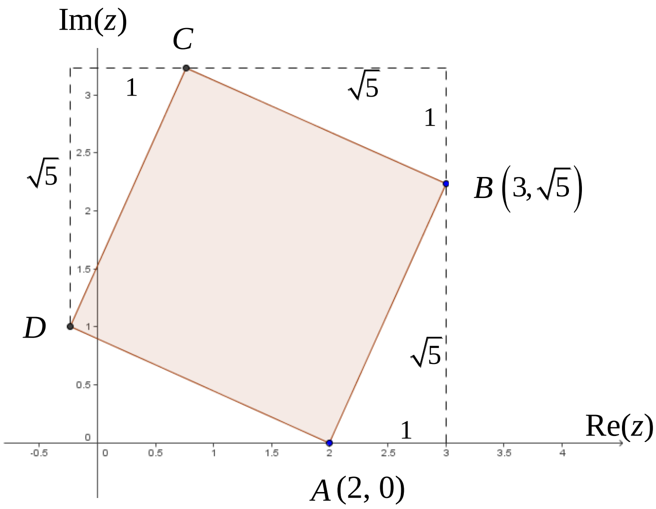
Year 12	Mathematics Extension 2	Half Yearly 2010 HSC
Question No. 2	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

E3 uses the relationship between algebraic and geometric representations of complex numbers

Outcome	Solutions	Marking Guidelines
	(a) $z=2-i$, $w=1+2i$	
	(i) $z-w=3-i-(1+3i)$ $=2-4i$	<u>1 mark</u> : correct answer
	(ii) $zw=(3-i)(1+3i)$ $=3+9i-i-3i^2$ $=6+8i$	<u>1 mark</u> : correct answer
	(iii) $w\bar{w}=(1+3i)(\overline{1+3i})$ $=(1+3i)(1-3i)$ $=1-9i^2$ $=10$	<u>2 marks</u> : correct answer <u>1 mark</u> : partially correct
	(iv) $\dfrac{z}{w}=\dfrac{3-i}{1+3i}\times\dfrac{1-3i}{1-3i}$ $=\dfrac{3-9i-i+3i^2}{10}$ $=\dfrac{-10i}{10}=-i$	<u>2 marks</u> : correct answer <u>1 mark</u> : partially correct
	(b) (i) $\lvert z\rvert=\sqrt{5^2+\left(5\sqrt{3}\right)^2}$ $=10$ $\arg(z)=\tan^{-1}\dfrac{5\sqrt{3}}{5}$ $=\dfrac{\pi}{3}$	<u>2 marks</u> : correct solution <u>1 mark</u> : partially correct solution
	(b) (ii) $(r\text{cis}\theta)^2=10\text{cis}\dfrac{\pi}{3}$ $r^2\text{cis}2\theta=10\text{cis}\dfrac{\pi}{3}$ $r^2=10$ $2\theta=\dfrac{\pi}{3}\pm2k\pi$ $r=\sqrt{10}$ $\theta=\dfrac{\pi}{6}\pm k\pi$ so the two square roots are $\sqrt{10}\text{cis}\dfrac{\pi}{6}$ and $\sqrt{10}\text{cis}\dfrac{-5\pi}{6}$ $\left\{\text{or } \pm\dfrac{\sqrt{10}}{2}\left(\sqrt{3}+i\right)\right\}$ $\left\{\begin{array}{l}(x+iy)^2=5+5\sqrt{3}i \\ x^2-y^2+2xyi=5+5\sqrt{3}i \\ \text{ie} \quad x^2-y^2=5 \\ \text{and} \quad 2xy=5\sqrt{3} \quad \Rightarrow \begin{cases} x=\frac{\sqrt{30}}{2},-\frac{\sqrt{30}}{2} \\ y=\frac{\sqrt{10}}{2},-\frac{\sqrt{10}}{2}\end{cases}\end{array}\right\}$	<u>3 marks</u> : correct solution <u>2 marks</u> : substantially correct solution <u>1 mark</u> : partially correct solution

(c)



(i) $A \to B$ is across (right) 1, up $\sqrt{5}$

$B \to C$ is up 1, across (left) $\sqrt{5}$

So B is the point $\left(3-\sqrt{5}, 1+\sqrt{5}\right)$

i.e. the number $\left(3-\sqrt{5}\right)+\left(1+\sqrt{5}\right)i$

<u>2 marks</u> : correct solution

<u>1 mark</u> : partially correct

(ii) $C \to D$ is across (left) 1, down $\sqrt{5}$

so D is the point $\left(2-\sqrt{5}, 1\right)$

i.e. the number $2-\sqrt{5}+i$

<u>2 marks</u> : correct solution

<u>1 mark</u> : partially correct

Alternatively…

(i)

$$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC}$$
$$= 2 + \left(1+\sqrt{5}i\right) + i\left(1+\sqrt{5}i\right)$$
$$\therefore z_3 = 3-\sqrt{5} + \left(1+\sqrt{5}\right) i$$

(ii)

$$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$$
$$= \overrightarrow{OA} + \overrightarrow{BC}$$
$$= 2 + i\left(1+\sqrt{5}i\right)$$
$$\therefore z_4 = 2-\sqrt{5}+i$$

Outcomes Addressed in this Question

E4 uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections.

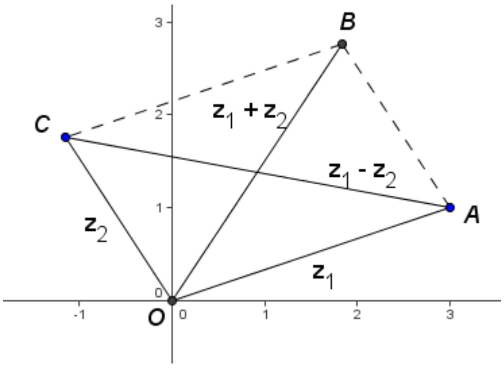
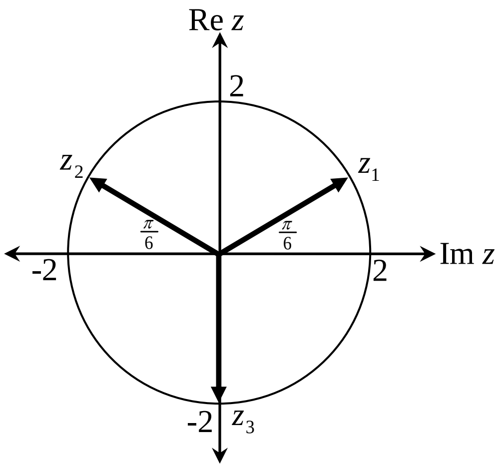
Outcome	Solutions	Marking Guidelines
	a $a^2 = 144, b^2 = 25$ $\frac{x^2}{144} - \frac{y^2}{25} = 1$ (i) $y = 0 : \frac{x^2}{144} = 1$ $x^2 = 144$ $x = \pm 12$	1 mark correct answer
	(ii) $b^2 = a^2\left(e^2 - 1\right)$, focii $\left(\pm ae, 0\right)$ $\frac{25}{144} = e^2 - 1$ $e^2 = \frac{169}{144}$ $e = \frac{13}{12}, e > 0$ $focii\ at \left(\frac{13}{12} \times 12, 0\right) and \left(-\frac{13}{12} \times 12, 0\right)$ $\left(13, 0\right), \left(-13, 0\right)$	2 marks correct method leading to correct conclusion 1 marks substantial progress in proof with correct method leading to an appropriate conclusion
	(iii) asymptotes : $y = \pm \frac{b}{a} x \quad y = \pm \frac{5}{12} x$	1 mark correct answer asymptote
	$directrices : x = \pm \frac{a}{e} \quad x = \pm \frac{12}{\frac{13}{12}} \quad \therefore x = \pm \frac{144}{13}$	1 mark correct answer directrices
	(iv) $b = \pm 12, a = b$ for rectangular hyperbola	1 mark correct answer
	(v) tangent of form $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$ $\frac{13x}{144} - \frac{\frac{1}{12} y}{25} = 1$ $\frac{13x}{144} - \frac{y}{300} = 1$	1 mark correct answer
	(vi) Chord SP $\left(13,0\right), \left(13, \frac{1}{12}\right)$ $x = 13$	1 mark correct answer

	b (i) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$ $\frac{dy}{dx}=-\frac{b^2x}{a^2y}$ gradient of tangent at (x_1,y_1): $m=-\frac{b^2x_1}{a^2y_1}$ equation of tangent at $(x_1,y_1): y-y_1=-\frac{b^2x_1}{a^2y_1}(x-x_1)$ $a^2y_1y-a^2y_1{}^2=-b^2x_1x+b^2x_1{}^2$ $a^2y_1y+b^2x_1x=b^2x_1{}^2+a^2y_1{}^2$ $\frac{y_1y}{b^2}+\frac{x_1x}{a^2}=\frac{x_1{}^2}{a^2}+\frac{y_1{}^2}{b^2}$ $\frac{y_1y}{b^2}+\frac{x_1x}{a^2}=1$ as $\frac{x_1{}^2}{a^2}+\frac{y_1{}^2}{b^2}=1$	2 marks correct method leading to correct equation 1 marks substantial progress in proof with correct method leading to an appropriate conclusion
	(ii) Similarly Q lies on $\frac{y_2y}{b^2}+\frac{x_2x}{a^2}=1$ T is the point of intersection $\therefore$ solve simultaneously $\therefore\frac{y_1y}{b^2}+\frac{x_1x}{a^2}=\frac{y_2y}{b^2}+\frac{x_2x}{a^2}$ $\frac{x_1x}{a^2}-\frac{x_2x}{a^2}+\frac{y_1y}{b^2}-\frac{y_2y}{b^2}=0$ $\frac{x(x_1-x_2)}{a^2}+\frac{y(y_1-y_2)}{b^2}=0$	2 marks correct method with reasoning leading to correct conclusion 1 marks substantial progress in proof with correct method leading to an appropriate conclusion
	(iii) If M lies on line in Part (ii) then it should satisfy the equation $M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ Consider $\frac{x(x_1-x_2)}{a^2}+\frac{y(y_1-y_2)}{b^2}$ $\frac{x_1+x_2}{2}\times\frac{(x_1-x_2)}{a^2}+\frac{(y_1-y_2)}{b^2}\times\frac{y_1+y_2}{2}$ $\frac{1}{2}\left[\frac{x_1{}^2}{a^2}+\frac{y_1{}^2}{b^2}-\left(\frac{x_2{}^2}{a^2}+\frac{y_2{}^2}{b^2}\right)\right]$ $\frac{1}{2}[1-1]=0$ as both $\frac{x_2{}^2}{a^2}+\frac{y_2{}^2}{b^2}=1$ and $\frac{x_1{}^2}{a^2}+\frac{y_1{}^2}{b^2}=1$ $\therefore$ M lie on the same line as T from part (ii) Substitute $(0,0)$ into $\frac{x(x_1-x_2)}{a^2}+\frac{y(y_1-y_2)}{b^2}=0$ $\frac{0\times(x_1-x_2)}{a^2}+\frac{0\times(y_1-y_2)}{b^2}=0$ which is true $\therefore O, M$ and T are collinear.	3 marks correct method leading to correct conclusion 2 marks substantial progress in proof with correct method leading to an appropriate conclusion 1 mark elementary progress towards correct solution Note if you used gradients you had to state they had a point in common therefore were not parallel but coincident

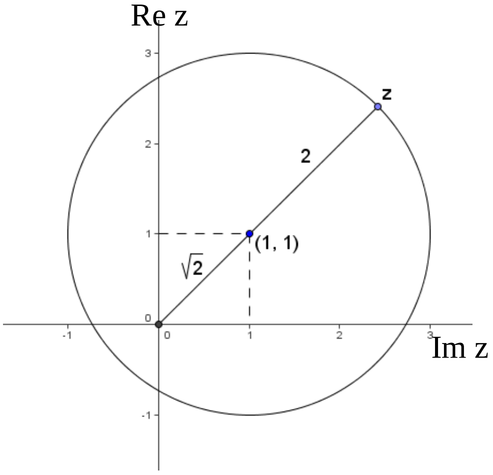
Year 12	Mathematics Extension 2	Half Yearly 2010 HSC
Question No. 4	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

E3 uses the relationship between algebraic and geometric representations of complex numbers

Outcome	Solutions	Marking Guidelines
	(a)  $\overrightarrow{OB}$ represents z_1+z_2 and $\overrightarrow{CA}$ represents z_1-z_2. $\lvert z_1-z_2\rvert=\lvert z_1+z_2\rvert \Rightarrow OB=CA$ $\therefore OABC$ is a rectangle $\therefore z_1 \perp z_2$ so $\arg z_1-\arg z_2=\dfrac{\pi}{2}$	<u>2 marks</u> : correct solution <u>1 mark</u> : partially correct solution
	(b) $z^3=8i$ $r^3 cis 3\theta = 8 cis\dfrac{\pi}{2}$ so $r^3=8$, $r=2$ and $3\theta=\dfrac{\pi}{2}+2k\pi$, $\theta=\dfrac{\pi}{6}+\dfrac{2k\pi}{3}$ $\quad k=0,\pm1$ $k=0 \Rightarrow z_1=2cis\dfrac{\pi}{6}=\sqrt{3}+i$ $k=1 \Rightarrow z_2=2cis\dfrac{5\pi}{6}=-\sqrt{3}+i$ $k=-1 \Rightarrow z_3=2cis\dfrac{-\pi}{2}=-2i$ 	<u>3 marks</u> : correct solution <u>2 marks</u> : substantially correct solution <u>1 mark</u> : partially correct solution

(c) $|z-1-i| \le 2$ represents a circle, centre (1, 1), radius 2.



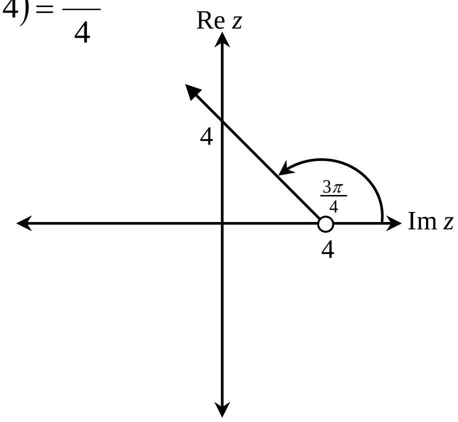
Maximum value of $|z|$ is $2+\sqrt{2}$

2 marks : correct solution

1 mark : progress towards correct solution

(d) (the second (c)… oops)

(i) $\arg(z-4)=\dfrac{3\pi}{4}$



3 marks : correct solution

2 marks : substantially correct

1 mark : partially correct

(ii) $\text{Im}\, z = |z|$ let $z = x + iy$

$$\text{Im}(x+iy) = |x+iy|$$

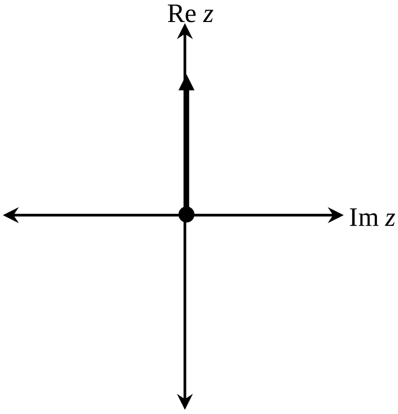
$$y=\sqrt{x^2+y^2}$$

$$y^2 = x^2 + y^2$$

$$x^2 = 0$$

$$x = 0$$

note: as $|z| \ge 0$, $y \ge 0$



2 marks : correct solution

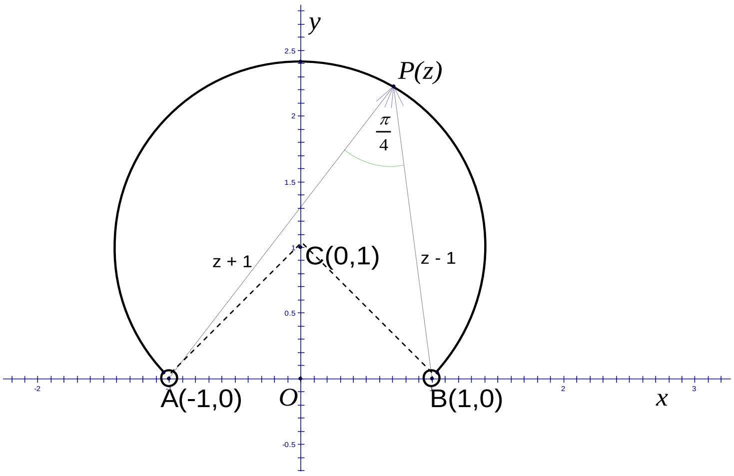
1 mark : progress towards correct solution

(e) ((d) on the paper)

$$\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$$

$$\arg(z-1)-\arg(z+1)=\frac{\pi}{4}$$

$$\arg(z+1)=\arg(z-1)-\frac{\pi}{4}$$



Cartesian equation is

$x^2+(y-1)^2=2$, for $y>0$

OR, algebraically:

$$\frac{z-1}{z+1}=\frac{x+iy-1}{x+iy+1}$$

$$=\frac{\left[(x-1)+iy\right]}{\left[(x+1)+iy\right]}\cdot\frac{\left[(x+1)-iy\right]}{\left[(x+1)-iy\right]}$$

$$=\frac{\left[\left(x^2-1^2\right)+y^2\right]+\left[y(x+1)-y(x-1)\right]i}{(x+1)^2+y^2}$$

$$=\frac{\left[x^2+y^2-1\right]}{(x+1)^2+y^2}-\frac{2y}{(x+1)^2+y^2}i$$

$$\tan\left[\arg\left(\frac{z-i}{z+i}\right)\right]=\frac{-\frac{2y}{(x+1)^2+y^2}}{\frac{x^2+y^2-1}{(x+1)^2+y^2}}$$

$$=-\frac{2y}{x^2+y^2-1}$$

as $\tan\frac{\pi}{4}=1$

$$-\frac{2y}{x^2+y^2-1}=1$$

so the relation is $x^2+y^2+2y-1=0$

Note: this method alone will not show that the locus is above the *x*-axis only.

<u>3 marks</u> : correct solution

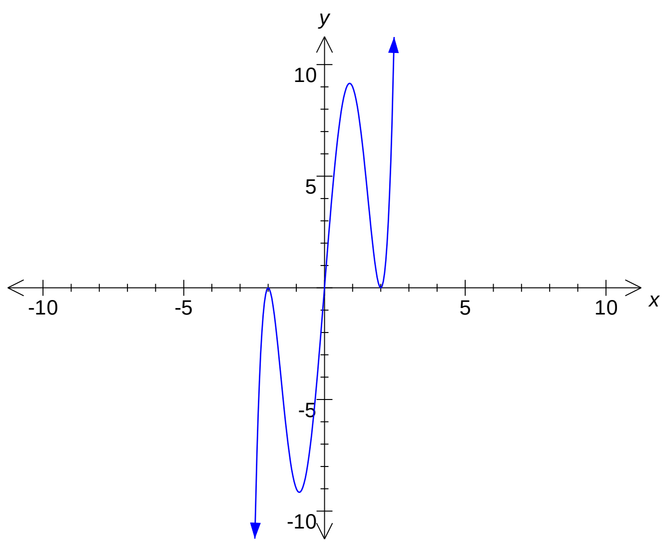
<u>2 marks</u> : substantially correct

<u>1 mark</u> : partially correct

Year 12	Extension 2 Mathematics	HSC Task 2 Half Yearly 2010
Question No.5	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

E2 chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings

Outcome	Solutions	Marking Guidelines
	a $2^{n+1}+2^n=2\times 2^n+2^n$ $=2^n(2+1)$ $=3\times 2^n$	1 mark correct index expression
	$\dfrac{2^{1001}+2^{1000}}{3}=\dfrac{3\left(2^{1000}\right)}{3}$ $=2^{1000}$	1 mark correct answer
	b (i) $x^2+y^2-2x-14y+25=0$ $y=mx$ Solve simultaneously and consider discriminants $x^2+(mx)^2-2x-14(mx)+25=0$ $\left(m^2+1\right)x^2-2(1+7m)x+25=0$ for two solutions $\Delta>0$ $4(1+7m)^2-4\times\left(m^2+1\right)\times 25>0$ $(1+7m)^2-\left(m^2+1\right)\times 25>0$	3 marks correct method leading to correct conclusion 2 marks substantial progress in proof with correct method leading to an appropriate conclusion 1 mark elementary progress towards correct solution
	(ii) for tangent $\Delta=0$ $(1+7m)^2-\left(m^2+1\right)\times 25=0$ $49m^2+14m+1-25m^2-25=0$ $24m^2+14m-24=0$ $12m^2+7m-12=0$ $(3m+4)(4m-3)=0$ $m=\dfrac{-4}{3},\dfrac{3}{4}$	2 marks correct method leading to correct conclusion 1 marks substantial progress in proof with correct method leading to an appropriate conclusion
	c $P(x)=x(x-2)^2(x+2)^2$	1 mark correct answer
		1 mark correct graph

	d (i) Use Induction to prove $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$ Prove True for $n = 0,1$ $LHS: (\cos\theta + i\sin\theta)^0 = 1$ $RHS: \cos 0\times\theta + i\sin 0\times\theta = 1$ $LHS = RHS$ also $n = 1: \cos\theta + i\sin\theta = \cos\theta + i\sin\theta$ $\therefore$ true for $n = 0,1$ Assume true for $n = k$ $i.e. (\cos\theta + i\sin\theta)^k = \cos k\theta + i\sin k\theta$ Prove true for $n = k+1$ $i.e. (\cos\theta + i\sin\theta)^{k+1} = \cos(k+1)\theta + i\sin(k+1)\theta$ $LHS: (\cos\theta + i\sin\theta)^k(\cos\theta + i\sin\theta)$ $(\cos k\theta + i\sin k\theta)(\cos\theta + i\sin\theta)$ equating the real and the imaginary parts $\cos k\theta\cos\theta - \sin k\theta\sin\theta + i(\sin k\theta\cos\theta + \cos k\theta\sin\theta)$ $\cos(k+1)\theta + i\sin(k+1)\theta$ $= RHS$ $\therefore$ if true for $n = k$ is true for $n = k+1$ Since proved true for $n = 1$ and $n = k+1$ is true for $n = 2,3,..$ and hence by the process of mathematical induction is true for all positve integralvalues of n.	3 marks correct method leading to correct conclusion 2 marks substantial progress in proof with correct method leading to an appropriate conclusion 1 mark elementary progress towards correct solution
	ii $(\cos\theta + i\sin\theta)^{-n} = \dfrac{1}{(\cos\theta + i\sin\theta)^n}$ *by part* (i) $= \dfrac{1}{(\cos n\theta + i\sin n\theta)}$ realising the denominator $= \dfrac{1}{(\cos n\theta + i\sin n\theta)} \times \dfrac{(\cos n\theta - i\sin n\theta)}{(\cos n\theta - i\sin n\theta)}$ $= \dfrac{(\cos n\theta - i\sin n\theta)}{\cos^2 n\theta + \sin^2 n\theta}$ $= \cos n\theta - i\sin n\theta$ $= \cos(-n\theta) + i\sin(-n\theta)$ $\therefore$ true for negatives	3 marks correct method leading to correct conclusion 2marks substantial progress in proof with correct method leading to an appropriate conclusion 1 mark elementary progress towards correct solution
	iii $\arg(z) = \tan^{-1}\left(\dfrac{\sqrt{3}}{1}\right) = \dfrac{\pi}{3}$ $\arg(z^5) = 5\times\dfrac{\pi}{3} = -\dfrac{\pi}{3}$ (principle argument)	1 mark correct solution