JAMES RUSE AGRICULTURAL HIGH SCHOOL
4 UNIT MATHEMATICS ASSESSMENT
TERM 2 1996

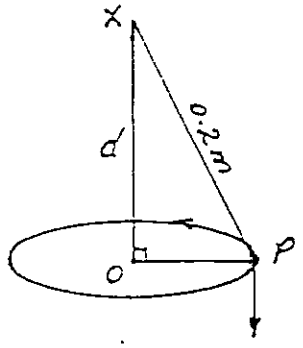
Time: 85 minutes.

Students to attempt all questions .
Approved silent calculators may be used.
Show all necessary working.
Each question to be **handed in separately**.

Question 1 (Start a new page) (5 marks)

The region bounded by the curve $y = x^2(1-x)$ and the x- axis is rotated about the y - axis. By using the method of cylindrical shells, find the volume of revolution.

Question 2 (Start a new page) (6 marks)



A mass P of 0.4 kg is suspended from a fixed point X by a light inextensible string length 0.2m. The mass P is observed to be in uniform circular motion of 150 revolutions per minute in a horizontal plane below point X.

Find the depth, d, to the nearest millimetre, of the mass P below the suspension point X . (take $g = 9.8\ m/s^2$)

Question 3 (Start a new page) (10 marks)

The equation of motion of a particle which moves with velocity v m/s along a straight line x metres from O after a time $t \geq 0$ is given by :

$$\frac{d^2x}{dt^2} + 5 \sin x = 0$$

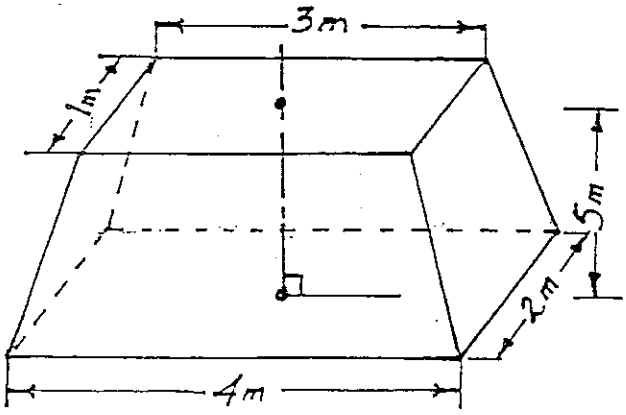
(a) If $v = 2$ when $x = \frac{\pi}{3}$, express v^2 as a function of x.

(b) Find the values of x when the particle is at rest.

(c) Find its maximum velocity, and its position at this instant.

(d) Briefly describe the motion.

Question 4 (Start a new page) (6 marks)



A bridge pier 5m high is to be made from concrete. If the cross sectional area parallel to the base of the pier is rectangular, find the volume of concrete (to 1 decimal place) if the base is 4 m by 2m and the top 3m by 1m.

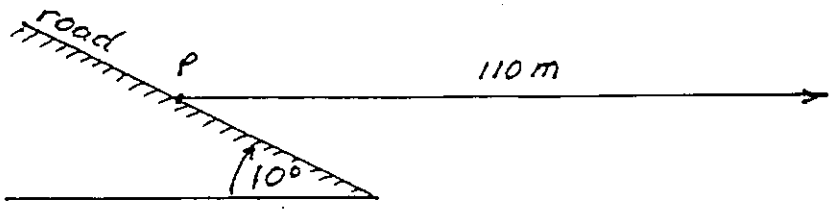
Question 5 (Start a new page) (8 marks)

(a) Shade the region bounded by the curve $y = 1 - \sin x$, the co-ordinate axes and the positive x,y plane.

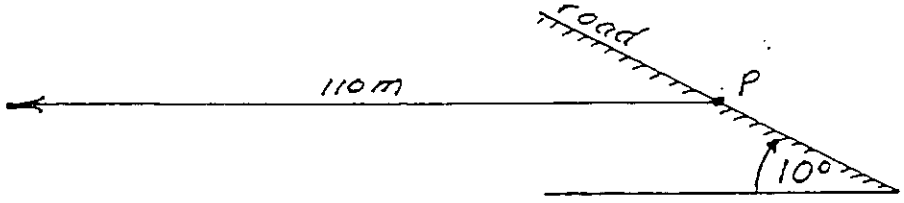
(b) The above region is rotated about the line $y = -1$. By taking slices perpendicular to the axis of rotation, find the volume of the solid of revolution.

Question 6 (Start a new page) (10 marks)

(a) A car P of mass 1500kg travels with constant speed v around a road of 110 metres radius and banked at 10° as shown.Find the speed of the car (in km/hr) so that the car has no tendency to slip.(take $g = 9.8 m/s^2$)



(b) The same car travels around a road of 110 metres radius and is reverse banked at an angle of 10° as shown. If the maximum friction force on the tyres is 0.2N where N is the normal force. find the greatest speed v (in km/hr) to safely travel without slipping.



Question 7 (Start a new page) (15 marks)

(a) Find values A and B which satisfy:

$$\frac{x}{x^2 + 4x + 10} = \frac{A(2x + 4)}{x^2 + 4x + 10} + \frac{B}{x^2 + 4x + 10}$$

(b) A 1 kg mass is projected upwards with an initial velocity V, experiencing a resistive force equal to $4v + v^2$, where v is the velocity. (take $g = 10\ m/s^2$) Show that:

(i) the maximum height H, reached is:

$$H = \frac{1}{2}\ln\left(\frac{V^2 + 4V + 10}{10}\right) - \frac{2}{\sqrt{6}}\tan^{-1}\left(\frac{V\sqrt{6}}{10 + 2V}\right)$$

(ii) the time taken is : $\frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{V\sqrt{6}}{10 + 2V}\right)$

END OF PAPER