# JAMES RUSE AGRICULTURAL HIGH SCHOOL
## 4 Unit Mathematics
## Year 12 Term 2 Assessment 1997

* TIME ALLOWED : 85 Minutes
* Start each question on a new page
* Silent approved calculators may be used
* All questions are of equal value
* Start each question on a new page
* Standard Integrals may be used
* **No equipment may be borrowed during the exam**

**QUESTION 1:(15 marks) Start this question on a new page**

2. A particle starts from rest at a distance b units, right of a fixed point O and moves with acceleration given by :

$$\ddot{x} = -k(x + \frac{b^4}{x^3})$$

where x is the distance from O, $k>0$.

a) show that the particle reaches O in time $\frac{\pi}{4\sqrt{k}}$

b) find the time it takes to reach a point distant $\frac{b}{\sqrt{2}}$.

3. A string is 50cm long, and it is known that it will break if a mass exceeding 40kg is hung from it. A mass of 2kg is attached to one end of the string, and it is revolved in a horizontal circle on a smooth table.

If T is the tension acting on the string, N is the normal reaction and $m$ is the mass,
(i) write down the equations of motion for the above problem
(ii) find the greatest angular velocity which can be imparted without breaking this string.

**QUESTION 2:(15 marks) Start this question on a new page**

1. P and Q are the points ( $cp, \frac{c}{p}$ ) and ( $cq, \frac{c}{q}$ ) on the hyperbola $xy = c^2$.

a) State the equation of the chord PQ

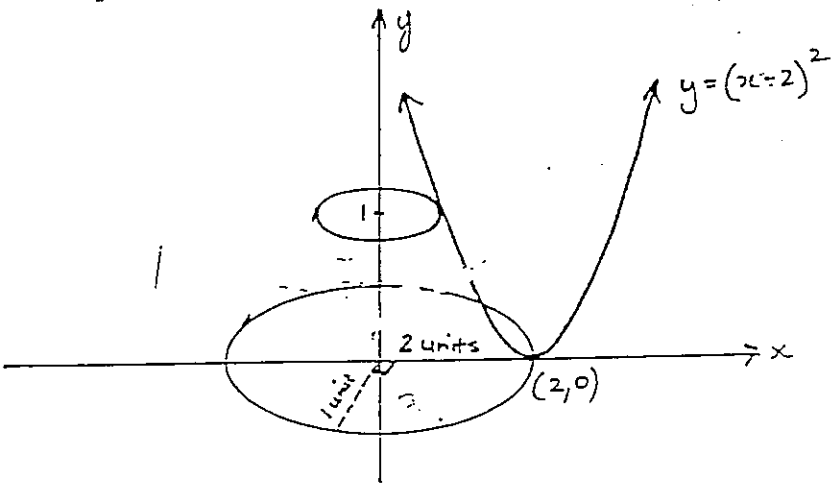
b) Hence, deduce that the equation of the tangent at P is $x + p^2y = 2pc$.

c)TP and TQ are the tangents to the hyperbola and M is the midpoint of the chord PQ, find the coordinates of T and M.

d) If AMBT is a rectangle whose sides are parallel to the coordinate axes, show that A and B are points on the hyperbola.

Show that the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\pi ab$.

b) The diagram shows a solid of height 1 unit. Each cross section is an ellipse with the major axis parallel to the x-axis. The length of the major axis of each ellipse is twice that of the minor axis. With respect to the x-y axes shown, the right hand endpoints of each major axis satisfies the equation $y = (x-2)^2$.




(i) show that the area of an arbitary ellipse at height y is $\pi(2 - 2\sqrt{y} + \frac{1}{2}y)$

(ii) hence, find the volume of the solid.

## QUESTION 3:(15 marks) Start this question on a new page

1. The base of a solid is a circle $x^2 + y^2 = 4$ in the x-y plane. Each plane perpendicular to the x–axis cuts the solid in an equilateral triangle. Find the volume of the solid.

2. The area between the curve $y = 4x - x^2$ and the x-axis is rotated about the line y=6. By taking slices perpendicular to the axis of rotation, find the volume of the solid generated.

3. A particle of unit mass is found to experience a resistive force, in newtons, of $\frac{1}{100}$ of the square of its velocity in metres per second, when it moves through the air. The particle is projected vertically upwards from a point 2 metres above the ground O, with a velocity of $u$ metres per second, $v$ is the velocity after time $t$. The point A, vertically above the point of projection, is the highest point reached by the particle before it starts to fall to the ground again. Assuming the value of g is $10 ms^{-2}$,

a) draw a diagram to show the forces acting on the particle, and show that the equation of motion on the body is $\ddot{x} = -10 - \frac{1}{100}v^2$

b) find the time the particle takes to reach A

c) show that the maximum height it will reach in metres above the ground is

$2 + 50\ln(1 + \frac{u^2}{1000})$