Time: 85 mins

Open book test
Show all necessary working
Each question to be handed in separately

Question 1

(a) The base of a solid is the region bounded by the curve $y=e^x$, $y = 1 - x$ and $x = 4$. Find the volume of the solid if every cross section perpendicular to the x axis is a square with one end point on $y=e^x$ and another endpoint on $y = 1 - x$.

(b) The region in (a) is rotated about the line $x = 4$. Find the volume by using cylindrical shells (i.e slices perpendicular to the x axis)

(c) Given twenty couples :-
   (i)   How many different three-member committees can be formed that do not contain any couple?
   (ii)   What is the probability if a four-member committee is formed that it is comprised of two couples?

Question 2 (Start a new page)

(a) A particle of unit mass moves in a straight line under the action of a constant force P . It encounters a resistive force kv/ unit mass.
   (i)   Write down the equation of motion.
   (ii)   If the initial speed is U find the time taken to increase its speed by 50%
   (iii)   Find the distance covered in the period it takes to increase its speed by 50%

(b) A model plane of mass 10kg, attached to the end of a light inelastic wire of length 60m, the other end of which is fixed, flies in a horizontal circle with angular velocity 0.5rad/sec and angle of elevation α . If the upward (vertical) force of the air on the plane is 200N find (using $g = 10m/s^2$):
   (i)   the value of α
   (ii)   the tension in the wire.

(a) A bomber aeroplane with horizontal velocity of 100m/s and altitude 125m is to hit a train x metres away and moving at constant velocity of 20m/s in the same direction and same vertical plane.
Calculate the angle of depression of the train from the plane at the instant the bomb is released for a direct hit. (Use $g = 10$m/s^2)

(b) A particle of mass m is projected vertically upwards from the ground. The particle experiences a resistance of magnitude mkv^2 where k is a positive constant and the velocity of the particle has magnitude v. During its downward motion, the terminal velocity of the particle is V. Its initial velocity of projection is half this terminal velocity.

(i) By considering the forces on the particle during its downward motion, show that $kV^2 = g$

(ii) Show that during its upward motion, the acceleration of the particle is given by $V^2 \ddot{x} = -g(V^2 + v^2)$, and the distance x travelled by the particle when its velocity is v is given by $x = \dfrac{V^2}{2g}\ln\{\dfrac{5V^2}{4(V^2 + v^2)}\}$

(iii) Find the maximum height h of the particle above its projection point.

Question 4 (Start a new page)

(a) The curve $y = 3x^2 - x^3$ for $x \geq 0$ has a maximum at $P(x_0, y_0)$. The region bounded by the curve and the x axis is rotated about the y axis.

(i) Using the method of slicing perpendicular to the axis of rotation show that the volume V is given by $V = \pi[\int_3^{x_0}(6x^3 - 3x^4)dx - \int_0^{x_0}(6x^3 - 3x^4)dx]$

Hence find V.

(ii) Using the method of cylindrical shells find the volume.

(b) Eight people are seated at a circular table. At the same instant, each person shakes hands with one of the other persons, reaching across the table if necessary, and thus creating four handshakes. If no two pairs of joined hands can cross each other, in how many ways can these handshakes occur?

END OF PAPER