JRAHS TERM 2 4 UNIT ASSESSMENT 2000

QUESTION 1 (20 MARKS) (START A NEW PAGE)

(a) (i) Find the number of ways in which 12 people can be divided into 3 groups of 4.

(ii) If four letters are chosen from the word DIVIDES, find the number of different four letter arrangements that can be formed.

(b) (i) Sketch $y = 6-x$ and $y^2 = x$ clearly showing their points of intersection.

(ii) The area bounded by the line $y = 6-x$ and the parabola $y^2 = x$ is rotated one revolution about the y-axis. Find the volume of this solid.

(c) The retardation acting on a unit mass moving horizontally with velocity v is given by kv^2 where k is a constant. Initially the particle is projected from the origin with velocity u.

(i) Prove that the velocity v after time t is given by $v = \dfrac{u}{ukt + 1}$

(ii) Find an expression for the displacement x in terms of time t.

QUESTION 2 (20 MARKS) (START A NEW PAGE)

(a) A 500 g mass moves in a circular path about a fixed point at the centre of a smooth horizontal table. The mass moves with constant speed 10 m/s and is attached to the centre by a 250 cm string.

(i) Find the tension in the string.

(ii) If the tension in (a) is the maximum tension that the string can withstand before breaking, find the greatest angular velocity that a 2kg mass can revolve with when attached to the end of the above string.

(b) The area bounded by the curve $y = \sqrt{4-x}$ and the lines $x = 4$ and $y = 2$ is rotated one revolution about the line $x = 4$. Find the volume of this solid.

(c) (i) Use the substitution $u^2 = z$ to show that $\displaystyle\int \frac{1}{\sqrt{z}+k}\,dz = 2\sqrt{z} - 2k\ln(\sqrt{z}+k) + c$.

(ii) A toy rocket of mass 1kg is launched at 144m/s from the surface of a small moon. The acceleration due to gravity on the moon is 1m/s^2. The retardation on the motion of the rocket due to the atmosphere equals $\frac{1}{6}$ of the square root of the rocket's velocity (v m/s) at height x metres above the surface.

(α) Show that the time required for the rocket to reach a velocity of v m/s is given by:

$$t = 144 - 12\sqrt{v} + 72\ln\left(\frac{\sqrt{v}+6}{18}\right).$$

(β) Find the time required to reach the maximum height above the surface.

(γ) Find the maximum height reached by the rocket.

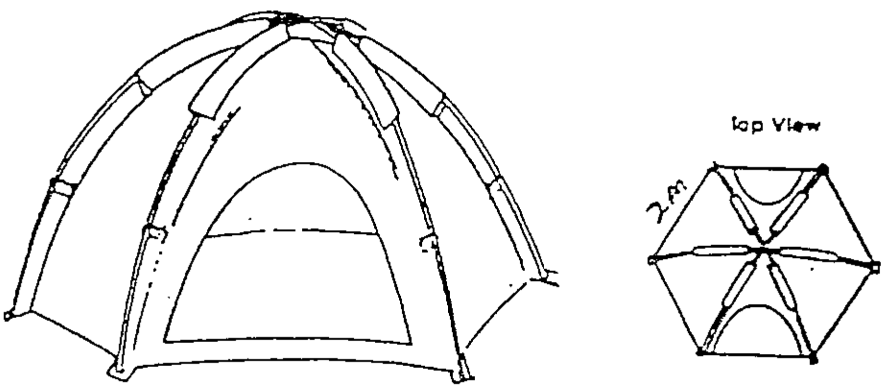
QUESTION 3 (20 MARKS) (START A NEW PAGE)

(a) (i) Show that the volume of a cylindrical shell with inner radius x, outer radius $x+\Delta x$ and height y can be approximated by $V = 2\pi xy\Delta x$ neglecting terms in $(\Delta x)^2$.

(ii) The area bounded by the curve $y = 9x - x^3$ and the positive x-axis is rotated one revolution about the y-axis. Using cylindrical shells, find the volume of this solid.

(b) (i) Show that the area of a regular hexagon of sides s is given by $A = \dfrac{3\sqrt{3}s^2}{2}$.

(ii) The diagram below illustrates a dome tent. When erected, the base is a regular hexagon which measures 2m from one corner to the adjacent corner (internal measurements). Flexible exterior poles extend between opposite corners in semi-circular arcs to support the tent. By taking slices parallel to the base find the volume enclosed by the tent.



(c) A paraboloid is formed by rotating the parabola $x^2 = 4y$ about the y-axis. A particle of mass m is attached by a light string 1.25 units long to a fixed point at the focus of the paraboloid. The particle is then made to describe horizontal circles with constant angular velocity ω while remaining in contact with the smooth inner surface of the paraboloid. (Note: The focus of the parabola and the paraboloid are at the same point)

(i) Assuming that the normal to a parabola at a point P bisects angle SPQ, where S is the focus and PQ (Q above P) is parallel to the axis of the parabola, show that the tension along the string is $m(2\omega^2 - g)$.

(ii) Find the least value of ω for which the particle loses contact with the paraboloid.

THE END