Name:	
Class:	

YEAR 12

ASSESSMENT TEST 2
TERM 1, 2015

MATHEMATICS EXTENSION 2

Time Allowed – 120 Minutes
(Plus 5 minutes Reading Time)

General Instructions:

- *All* questions may be attempted
- *All* questions are of equal value
- Standard Integral Tables will be supplied
- Department of Education approved calculators and templates are permitted
- In every Question, show all relevant mathematical reasoning and/or calculations.
- Marks may not be awarded for careless or badly arranged work
- No grid paper is to be used unless provided with the examination paper

Question 1-5 are to be completed on Multiple choice sheet then Question 6,7,8,9 to be completed on separate sheets of paper and handed in in separate bundles. Each question must show your (in the top right hand corner) Candidate Number.

YEAR 12 – 2015 – EXTENSION 2 TERM 1 -TASK 2
Use the multiple choice answer sheet supplied for question 1 to 5

Q1 The area of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ in units square is

A. 36π B $36\pi^2$ C 6π D $6\pi^2$

Q2 Given any point $P(a\sec\theta, b\tan\theta)$ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

The order of the quadrants on a circle in which $P(a\sec\theta, b\tan\theta)$ from $\theta=0$ to 2π runs is:

A. 1, 2 ,3, 4 B. 1, 4, 2, 3 C. 1, 3, 2 , 4 D. 1, 4, 3, 2

Q3 The equation of the tangent at a point $P(x_0, y_0)$ on the curve: $x^{\frac{2}{3}}+y^{\frac{2}{3}}=a^{\frac{2}{3}}$ in the first quadrant is

A. $y_0^{\frac{1}{3}}x+x_0^{\frac{1}{3}}y=x_0^{\frac{1}{3}}y_0^{\frac{1}{3}}a^{\frac{1}{3}}$.

B. $y_0^{\frac{1}{3}}x+x_0^{\frac{1}{3}}y=x_0^{\frac{1}{3}}y_0^{\frac{1}{3}}a^{\frac{4}{3}}$.

C $y_0^{\frac{1}{3}}x+x_0^{\frac{1}{3}}y=2x_0^{\frac{1}{3}}y_0^{\frac{1}{3}}$.

D. $y_0^{\frac{1}{3}}x+x_0^{\frac{1}{3}}y=x_0^{\frac{1}{3}}y_0^{\frac{1}{3}}a^{\frac{2}{3}}$.

Q4 Using an appropriate substitution

$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec^2 x}{(1+\tan x)^2}$ is equivalent to

A. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{u}du$

C. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{u^2}\,\mathrm{du}$

B. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{u^2}{(1+u)^3}\,\mathrm{du}$

D. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{u^3}\,\mathrm{du}$

Q5. The equation $|z-1-3i|+|z-9-3i|=10$ corresponds to an ellipse in the Argand diagram. Which of the following is the complex number corresponding to the centre of the ellipse?

A. $5+3i$ B. $-5+3i$

C. $-5-3i$ D. $5-3i$

QUESTION 6 (20 Marks)

a) $$\int_{\frac{\pi}{16}}^{\frac{\pi}{12}} \sec 4x \tan 4x \, dx$$ 2

b) Find $\int \frac{1}{\sqrt{x^2 - 6x + 34}} dx$ 2

c) (i) Find real numbers a, b and c such that

$$\frac{3x}{(x+1)(x^2+2x+4)} \equiv \frac{a}{x+1} + \frac{bx+c}{x^2+2x+4}$$ 2

(ii) Find $\int \frac{3x}{(x+1)(x^2+2x+4)} dx$ 2

d) Let $t = \tan\frac{\theta}{2}$

(i) Show that $d\theta = \frac{2}{1+t^2} dt$ 1

(ii) Use the substitution $t = \tan\frac{\theta}{2}$ to find

3

$$\int \frac{1}{3\sin\theta + 4\cos\theta + 5} d\theta$$

e) On separate number planes draw sketches of the following, for $-2\pi \le x \le 2\pi$.

(i) $y^2 = \sin x$ 2

(ii) $|y| = \sin x$ 2

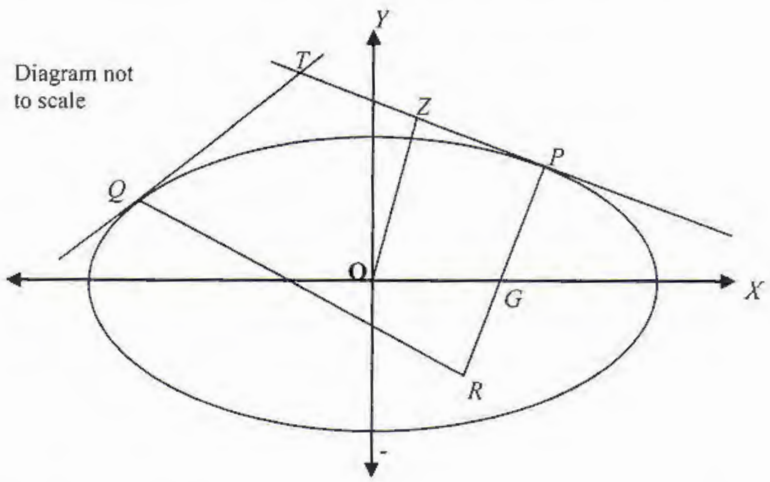
(iii) $y = (\sin^3 x)$ 2

f) If z is any point on the circle $|z - 1| = 1$ prove that arg (z-1) =2argz

2

QUESTION 7 **START A NEW PAGE (20 Marks)**

a) Consider the ellipse $E: x^2 + 4y^2 = 100$. PT and QT are tangents at $P(6, 4)$ and $Q(-8, 3)$ respectively. They meet at the point T.



(i) Show that the equation of PT is given by $3x + 8y = 50$. **2**

(ii) PR and QR are normals at P and Q respectively.
Show the equation of PR is given by $8x-3y=36$. **2**

PR meets the major axis in G, and OZ is the perpendicular from the centre O to the tangent at P.

(iii) Prove that $PG.OZ=25$. **3**

(iv) Find the coordinates of T and R. **4**

(v) Show that the diameter through R is perpendicular to PQ. **2**

b)

(i) Solve $z^3 = \sqrt{2} + \sqrt{2}\,i$, giving answers in the form $R\ cis\,\theta$. **2**

(ii) Hence prove that $\cos\left(\frac{\pi}{12}\right) + \cos\left(\frac{9\pi}{12}\right) + \cos\left(\frac{-7\pi}{12}\right) = 0$ **1**

(c) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 x \sin^5 x dx$ **1**

(d) Find $\int \sqrt{x} \ln x dx$ **3**

QUESTION 8 **START A NEW PAGE (20 Marks)**

a) Given the complex number $z = cos\,\theta + i\,sin\,\theta$

(i) Show that $Z^n + \dfrac{1}{Z^n} = 2\cos n\theta$ 2

(ii) Hence by expanding $(Z + \dfrac{1}{Z})^4$, find an expression for $\cos^4\theta$ in the form

$a\cos 4\theta + b\cos 2\theta + c$ 3

(iii) Hence evaluate $\displaystyle\int_0^{\frac{\pi}{2}} cos^4\theta \, d\theta$ 2

b) (i) Sketch $y = \dfrac{x^2}{x^3+1}$, showing all the essential points 4

(ii) Hence find the number of real roots for $x^3 - 5x^2 + 1 = 0$. 2

c) Let $I_n = \displaystyle\int_0^{\frac{\pi}{4}} \frac{1-\cos 2nx}{\sin 2x}dx$ for n=1, 2, 3...

(i) Evaluate I_1 2

Using the fact that $\cos S - \cos T = -2\sin\left(\dfrac{S+T}{2}\right)\sin\left(\dfrac{S-T}{2}\right)$.

(ii) Show that for $r \geq 1$, $I_{2r+1} - I_{2r-1} = \dfrac{1-(-1)^r}{2r}$ 3

(iii) Hence evaluate I_7 2

QUESTION 9 **START A NEW PAGE (20 MARKS)**

a)

(i) Show that $\int_{-a}^{a} f(x)dx = \int_{-a}^{a} f(-x)dx$ 1

(ii) Hence or otherwise evaluate $\int_{-4}^{4} (e^x - e^{-x})\cos x dx$ 2

(iii) By considering $\int_{0}^{a} \sqrt{a^2 - x^2}\, dx$ find the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ 2

b)

(i) Sketch the hyperbola $\boldsymbol{H}: \frac{x^2}{9} - \frac{y^2}{16} = 1$ showing all the asymptotes, directrices, vertices and foci. 3

(ii) Show the equation of the tangent at any point $P\,(3\sec\theta, 4\tan\theta)$ on the hyperbola $\boldsymbol{H}$ is $4x\sec\theta - 3y\tan\theta = 12$. 2

The tangent at P meets the asymptotes at Q in the first quadrant and R in the fourth quadrant.

(iii) If O is the centre if $\boldsymbol{H}$, prove that P is the mid-point of QR. 4

(iv) Find the area of ΔOQR. 3

(v) If S is the focus of $\boldsymbol{H}$, find $\angle QSR$ to the nearest minute. 3

MATHEMATICS Extension 2: Question 1–5

Suggested Solutions	Marks	Marker's Comments
Q1 Area is πab $\therefore \pi \times 3 \times 2 = 6\pi$ part C	1	
Q2 $\sec\theta$ positive in 1st and 4th $\tan\theta$ positive in 1st and 3rd. $(+,+), (-,-), (-,+)\ (+,-)$ 1st, 3rd, 2nd 4th. part C.	1	
Q3 $y_0^{1/3}x + x_0^{1/3}y = x_0^{1/3}y_0^{1/3}a^{2/3}$ part D see separate sheet.	1	
Q4 let $u = 1+\tan x$ $\frac{du}{dx} = \sec^2 x$ $x = \frac{\pi}{4}$ $u = 2$ $x = -\frac{\pi}{4}$ $u = 0$ $\int_0^2 \frac{1}{u^2}\,du$ most correct answer part C	1	
From $(1,3)$ to $(9,3)$ midpoint is $(5,3)$ so centre is $5+3i$		

Yr 12 Ext 2 2015 T1 Q 6

a) $\left[\frac{1}{4}\sec 4x\right]_{\frac{\pi}{16}}^{\frac{\pi}{12}}$ — 1 — Some forgot $\frac{1}{4}$

$= \frac{1}{4}\left(\sec\frac{\pi}{3} - \sec\frac{\pi}{4}\right)$

$= \frac{1}{4}(2-\sqrt{2})$ # — 1

b) $\int \frac{dx}{\sqrt{(x-3)^2+25}}$ — 1

$= \ln\left(x-3+\sqrt{(x-3)^2+25}\right)+c$ # — 1

c) i) $\frac{3x}{(x+1)(x^2+2x+4)} = \frac{a}{x+1} + \frac{bx+c}{x^2+2x+4}$

$3x = a(x^2+2x+4) + (bx+c)(x+1)$ — $a=-1,\ b=1,\ c=4$

$x=0,\quad 0=4a+c \quad (1)$ — Any 2 correct 1m

$x=-1 \quad -3=3a \quad \therefore a=-1$ — All correct 2m

Sub in (1) $\quad c=4$

$x=1 \quad 3=7a+2b+2c$

$3=-7+2b+8$

$1=b$

ii) $\int\left(\frac{-1}{x+1} + \frac{x+4}{x^2+2x+4}\right)dx$ — 1 m

$= -\ln(x+1) + \frac{1}{2}\int\frac{2x+2}{x^2+2x+4}\,dx + \int\frac{3\,dx}{x^2+2x+4}$ — a lot of students got $\sqrt{3}$ wrong.

$= -\ln(x+1) + \frac{1}{2}\ln(x^2+2x+4) + \sqrt{3}\tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right)+c$ — 1m

#.

d) i) $\frac{dt}{d\theta} = \frac{1}{2}\sec^2\theta/2$

$= \frac{1}{2}(1+\tan^2\theta/2)$

$= \frac{1}{2}(1+t^2)$

$\frac{d\theta}{dt} = \frac{2}{1+t^2}$

1m

ii) $\int \frac{1}{3\times\frac{2t}{1+t^2} + \frac{4(1-t^2)}{1+t^2} + 5} \cdot \frac{2}{1+t^2}\,dt$

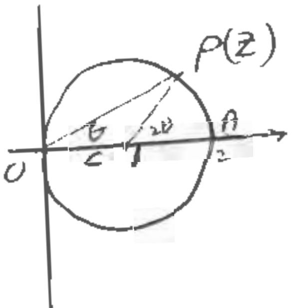
1m for substitution

$= \int \frac{2}{(t+3)^2}\,dt$

1m complete square

$= -\frac{2}{\tan\theta/2+3}$ ct

1m
Some forget to change
Back to θ

f) 

$(x-1)^2+y^2=1$
centre $(1,0)$ at C

1m
must introduce
where is $\arg(z)$
$\arg(z-1)$

Let $\angle POA = \theta = \arg(z)$

$\angle PCA = \arg(z-1)$

$\angle PCA = 2\theta$ (angle at centre twice angle at circumference standing on same arc)

1m

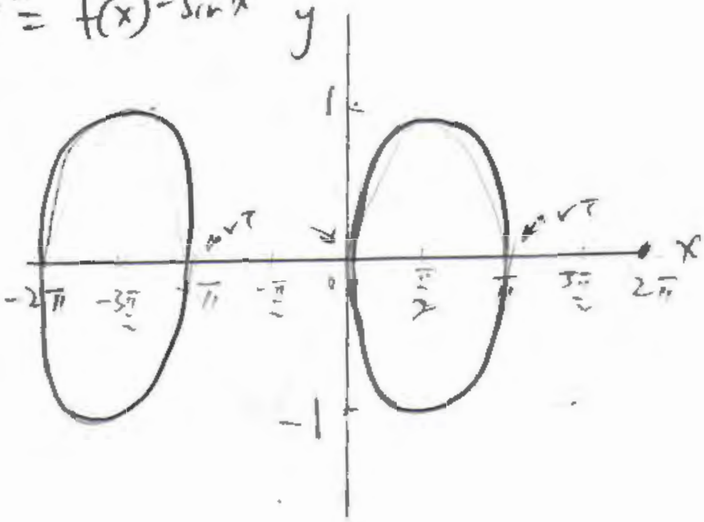
Some forget on same arc did not get this mark

$\therefore \arg(z-1) = 2\arg(z)$

#

6e) $-2\pi \le x \le 2\pi$

i) $y^2 = f(x) = \sin x$

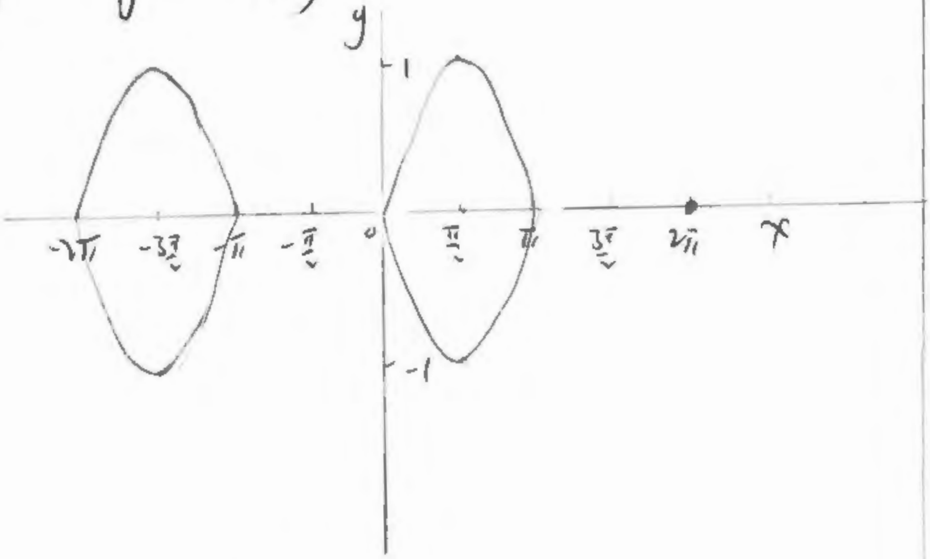


1m for 2 ovals in the Right place

1m for vertical tangents

must locate TP.

ii) $|y| = f(x) = \sin x$



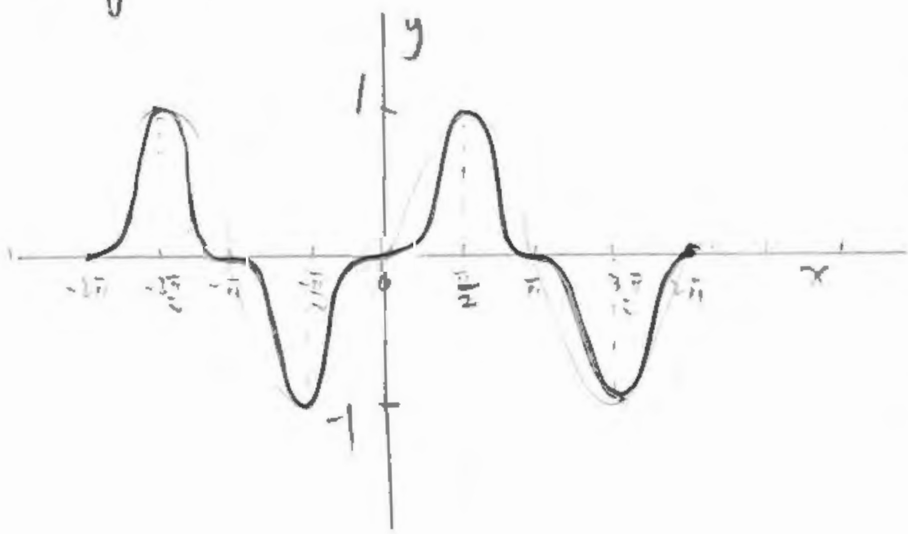
1m for 2 ovals in the right place

1m for correct shape

must locate TP

(i) & (ii) must be different in shape

iii) $y = f^3(x) = \sin^3 x$



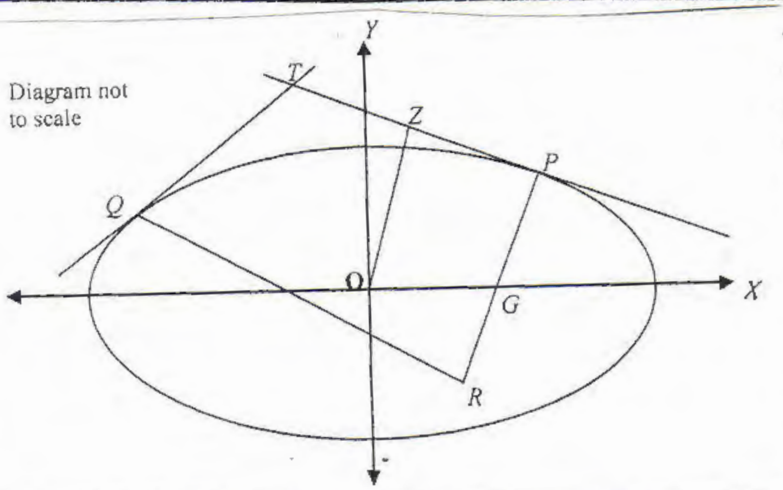
badly done

1 m for correct shape with TP (Concave in)

1m for HPOI

cutting the x-axis

MATHEMATICS Extension 2: Question 7

Suggested Solutions	Marks	Marker's Comments
(a) 		
(i) $x^2+4y^2=100$ $2x+8y\frac{dy}{dx}=0$ $\frac{dy}{dx}=\frac{-x}{4y}$ at $x=6$, $y=4$, $m_T=-3/8$ Equation of tangent at P $y-4=\frac{-3}{8}(x-6)$ $8y-32=-3x+18$ $3x+8y=50$	2	1 gradient of tangent 1 straight line equation
(ii) $m_N=\frac{8}{3}$ Equation of normal at P $y-4=\frac{8}{3}(x-6)$ $3y-12=8x-48$ $\therefore 8x-3y=36$	2	1 gradient 1 straightline equation * can not just quote equations of tangent or normal to ellipse
(iii) when $y=0$ and $8x-3y=36$ $\therefore x=9/2$ $\therefore G=(9/2,0)$ $PG=\frac{\lvert 3(9/2)+8(0)-50\rvert}{\sqrt{3^2+8^2}}=\frac{73}{2\sqrt{73}}$ $OZ=\frac{\lvert 3(0)+8(0)-50\rvert}{\sqrt{3^2+8^2}}=\frac{50}{\sqrt{73}}$ $\therefore PG\times OZ=\frac{73}{2\sqrt{73}}\times\frac{50}{\sqrt{73}}=25$	3	1 $G(9/2,0)$ 1 PG 1 OZ and answer.

MATHEMATICS Extension 2: Question 7

Suggested Solutions	Marks	Marker's Comments
b) (i) $z^3 = \sqrt{2} + \sqrt{2}i = 2\,\text{cis}(\pi/4)$ let $z = R\,\text{cis}\,\theta$ $R^3 \text{cis}\,3\theta = 2\,\text{cis}\,\pi/4$ (By De Moivre) $R = \sqrt[3]{2}$ $\quad 3\theta = \frac{\pi}{4} + 2k\pi \quad k \in Z$ $\theta = \frac{\pi + 8k\pi}{12}$ $k=0 \therefore z_1 = \sqrt[3]{2}\,\text{cis}(\pi/12)$ $k=1 \quad z_2 = \sqrt[3]{2}\,\text{cis}\left(\frac{9\pi}{12}\right) = \sqrt[3]{2}\,\text{cis}\left(\frac{3\pi}{4}\right)$ $k=-1 \quad z_3 = \sqrt[3]{2}\,\text{cis}\left(\frac{-7\pi}{12}\right)$	2	1 $z_1 = \sqrt[3]{2}\,\text{cis}\,\pi/4$ 1 All 3 solutions
$z^3 - 2\,\text{cis}(\pi/4) = 0$ sum of roots $= 0$ coefficient of z^2 term $\sqrt[3]{2}\,\text{cis}\left(\frac{\pi}{12}\right) + \sqrt[3]{2}\,\text{cis}\left(\frac{3\pi}{4}\right) + \sqrt[3]{2}\,\text{cis}\left(\frac{-7\pi}{12}\right) = 0$ Equate Real parts $\therefore \cos\left(\frac{\pi}{12}\right) + \cos\left(\frac{3\pi}{4}\right) + \cos\left(\frac{-7\pi}{12}\right) = 0$	1	1 state sum of roots $=0$ write out sum of roots state equate real part.
(c) $\int_{-\pi/2}^{\pi/2} \cos^4 x \sin^5 x\,dx = 0$ as $\cos^4 x \sin^5 x$ is an odd function	1	Must give reason. (no need to show any integration)
(d) $\int \sqrt{x} \ln x\,dx$ $= \left[\frac{2}{3}x^{3/2}\ln x\right] - \frac{2}{3}\int x^{3/2} \times \frac{1}{x}\,dx$ $= \frac{2}{3}x\sqrt{x}\ln x - \frac{2}{3}\int x^{\frac{1}{2}}\,dx$ $= \frac{2}{3}x\sqrt{x}\ln x - \frac{2}{3} \times \frac{2}{3}x^{3/2} + C$ $= \frac{2}{3}x\sqrt{x}\ln x - \frac{4}{9}x\sqrt{x} + C.$	3	2 integration by parts. 1 Answer.

MATHEMATICS Extension 2: Question 7

Suggested Solutions	Marks	Marker's Comments
(iv) Equation of QT tangent at Q	④	
$m = \frac{-(-8)}{4(3)} = \frac{8}{12} = \frac{2}{3}$		
$y - 3 = \frac{2}{3}(x+8)$		
$3y - 9 = 2x + 16$		
$2x - 3y = -25$ (i)		① Equation of QT
To find T $\quad 3x + 8y = 50$ (ii)		
$6x - 9y = -75$		
$6x + 16y = 100$		
$-25y = -175$		
$y = 7$		① T(−2, 7)
$x = -2$		
$T = (-2, 7)$		
Equation of Normal at Q		
$m = -\frac{3}{2}$		
$y - 3 = -\frac{3}{2}(x+8)$		
$2y - 6 = -3x - 24$		① Equation of QR.
$3x + 2y = -18$ (i)		
$8x - 3y = 36$ (ii)		
$9x + 6y = -54$		
$16x - 6y = 72$		
$25x = 18$		
$x = \frac{18}{25}$		
$y = \frac{-18 - 3(18/25)}{2}$		
$= \frac{-252}{25}$		① $R\left[\frac{18}{25}, \frac{252}{25}\right]$
$R = \left[\frac{18}{25}, \frac{-252}{25}\right]$		
(v) Gradient of PQ $= \frac{4-3}{6-(-8)} = \frac{1}{4}$		① $m_{PQ} = \frac{1}{14}$
Gradient of OR $= -\frac{252}{25} \Big/ \frac{18}{25} = -14$		① m_{OR} and proof of perpendicular
$\therefore m_{PQ} \times m_{OZ} = \frac{1}{4} \times -14$		
$= -1$		
$\therefore$ PQ is perpendicular to OZ		

MATHEMATICS Extension 2: Question 8		
Suggested Solutions	**Marks**	**Marker's Comments**

(a) (i) Given that $z = \cos\theta + i\sin\theta$

$$\text{LHS} = z^n + \frac{1}{z^n}$$

$$= (\cos\theta + i\sin\theta)^n + \frac{1}{(\cos\theta + i\sin\theta)^n}$$

$$= \cos n\theta + i\sin n\theta + \cos(-n\theta) + i\sin(-n\theta)$$ | De Moivres theorem

$$= \cos n\theta + i\sin n\theta + \cos(n\theta) - i\sin n\theta$$ | ($\cos x$ is an even fn, $\sin x$ is an odd fn)

$$= 2\cos n\theta$$

$$= \text{RHS}$$

(ii) $$\left(z + \frac{1}{z}\right)^4 = z^4 + 4z^2 + 6 + 4z^{-2} + z^{-4}$$ (Binomial expansion) |

$$\left(z + \tfrac{1}{z}\right)^4 = \left(z^4 + \tfrac{1}{z^4}\right) + 4\left(z^2 + \tfrac{1}{z^2}\right) + 6$$

$$(2\cos\theta)^4 = 2\cos 4\theta + 4(2\cos 2\theta) + 6$$ $\left(z^n + \frac{1}{z^n}\right) = 2\cos n\theta$ |

$$16\cos^4\theta = 2\cos 4\theta + 8\cos 2\theta + 6$$

$$\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$$ |

(iii) $$\int_0^{\pi/2} \cos^4\theta \, d\theta = \int_0^{\pi/2} \left(\frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}\right) d\theta$$

$$= \left[\frac{\sin 4\theta}{32} + \frac{\sin 2\theta}{4} + \frac{3\theta}{8}\right]_0^{\pi/2}$$ |

$$= \left(\frac{\sin 2\pi}{32} + \frac{\sin \pi}{4} + \frac{3\pi/2}{8}\right) - (0)$$ |

$$= \frac{3\pi}{16}$$

(b) (i) $$y = \frac{x^2}{x^3+1} \qquad x \neq -1$$

$$\frac{dy}{dx} = \frac{2x(x^3+1) - x^2(3x^2)}{(x^3+1)^2}$$

$$= \frac{2x^4 + 2x - 3x^4}{(x^3+1)^2}$$

$$= \frac{x(2 - x^3)}{(x^3+1)^2}$$ |

MATHEMATICS Extension 2: Question 8...

Suggested Solutions	Marks	Marker's Comments

stat pt at $x = \sqrt[3]{2}$ and $x = 0$

$$\frac{d^2y}{dx^2} = \frac{(2-4x^3)(x^3+1)^2 - (2x - x^4)(2 \cdot 3x^2(x^3+1))}{(x^3+1)^4}$$

$$= \frac{(2-4x^3)(x^3+1) - 6x^2(2x - x^4)}{(x^3+1)^3}$$

when $x = \sqrt[3]{2}$, $\frac{d^2y}{dx^2} = \frac{-5(3) - 6\sqrt[3]{2}^2(2\sqrt[3]{2} - \sqrt[3]{2} \cdot 2)}{27}$

$= -5/9$

concave down

$\therefore$ rel. max at $(\sqrt[3]{2}, \approx 0.529)$

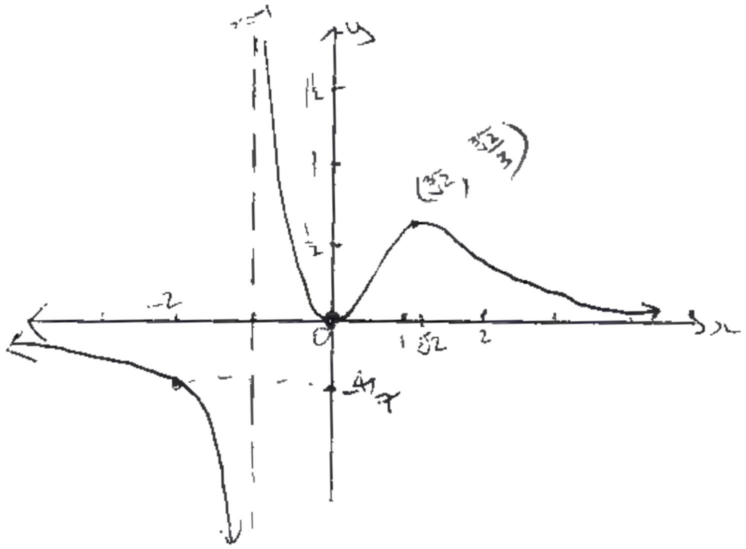
when $x=0$, $\frac{d^2y}{dx^2} = 2$

$\therefore$ rel min at $(2, 4/9)$

1 for data

$(0,0)$ horizontal asymp. at $y=0$

vertical asymp. at $x=-1$



1x for each branch drawn correctly

or If calculus wasn't used at all, 1 for each branch, 1 for vert. asymp and 1 for stat. pts

(b)(ii) $x^3 - 5x^2 + 1 = 0$

intersection between $y = \frac{x^2}{x^3+1}$ and $y = \frac{1}{5}$, from our graph we can see there would be 3 solutions.

1 for stating the 2 graphs

1 for correct ...

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MATHEMATICS Extension 2: Question 8		
Suggested Solutions	**Marks**	**Marker's Comments**
(c) (i) $I_1 = \int_0^{\pi/4} \frac{1-\cos 2x}{\sin 2x}\,dx$		
$= \int_0^{\pi/4} \frac{1-\cos^2 x+\sin^2 x}{2\sin x\cos x}\,dx$		
$= \int_0^{\pi/4} \frac{2\sin^2 x}{2\sin x\cos x}\,dx$		
$= \int_0^{\pi/4} \frac{\sin x}{\cos x}\,dx$	1	
$= \left[-\ln\lvert\cos x\rvert\right]_0^{\pi/4}$		
$= -\ln\frac{1}{\sqrt{2}} + \ln 1$		
$= \ln\sqrt{2} = \frac{1}{2}\ln 2$	1	
(ii) $I_{2r+1} - I_{2r-1}$		
$= \int_0^{\pi/4} \frac{1-\cos 2(2r+1)x}{\sin 2x}\,dx - \int_0^{\pi/4}\frac{1-\cos 2(2r-1)x}{\sin 2x}\,dx$		
$= \int_0^{\pi/4} \frac{1-\cos 2(2r+1)x - 1 + \cos 2(2r-1)x}{\sin 2x}\,dx$		
$= \int_0^{\pi/4} \frac{\cos(4r-2)x - \cos(4r+2)x}{\sin 2x}\,dx$		
$= -\int_0^{\pi/4} \frac{2\sin\left(\frac{4rx-2x+4rx+2x}{2}\right)\sin\left(\frac{4rx-2x-4rx-2x}{2}\right)}{\sin 2x}\,dx$	1	
$= \int_0^{\pi/4} \frac{-2\sin(4rx)\sin(-2x)}{\sin 2x}\,dx$		if you didn't show this or explain why the minus sign goes then you lost a mark
$= \int_0^{\pi/4} \frac{2\sin(4rx)\sin 2x}{\sin 2x}\,dx$ (as $\sin x$ is odd fn)		
$= 2\int_0^{\pi/4} \sin(4rx)\,dx$	1	

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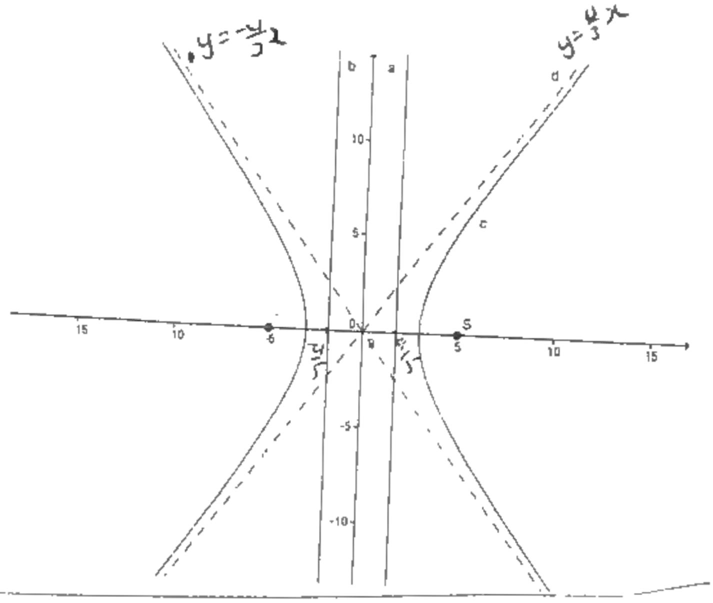
MATHEMATICS Extension 2 Question 8

Suggested Solutions	Marks	Marker's Comments
$= 2\left[-\frac{\cos 4rx}{4r}\right]_0^{\pi/4}$		
$= \frac{-1}{2r}\left[\cos r\pi - \cos 0\right]$	1	
$= \frac{-1}{2r}\left(-1^r - 1\right)$		
$= \frac{1}{2r}\left[1 - (-1)^r\right]$		
$= \frac{1 - (-1)^r}{2r}$		
(iii) so $I_{2r+1} - I_{2r-1} = \frac{1-(-1)^r}{2r}$		
when $r = 3$; $I_7 - I_5 = \frac{1-(-1)^3}{6} = \frac{1}{3}$		
when $r = 2$; $I_5 - I_3 = \frac{1-1}{4} = 0$		1
when $r = 1$; $I_3 - I_1 = \frac{2}{2} = 1$	✓	
now $I_7 - I_5 + I_5 - I_3 + I_3 - I_1 = \frac{1}{3} + 0 + 1$		
$\therefore I_7 - I_1 = \frac{4}{3}$		
$I_7 = \frac{4}{3} + I_1$		
$= \frac{4}{3} + \frac{1}{2}\ln 2$		1

MATHEMATICS Extension 2: Question 9

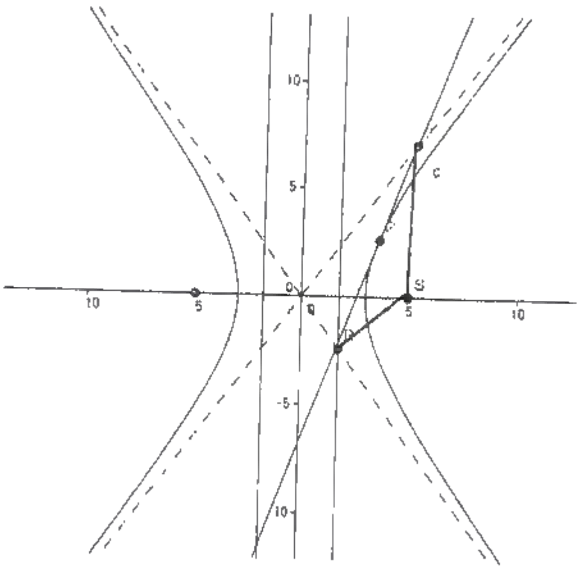
Suggested Solutions	Marks	Marker's Comments
(i) Let $x = -u$ then $x = a \quad u = -a$ $x = -a \quad u = a$ $\frac{dx}{du} = -1$ $\int_a^{-a} f(-u) - du$ $= \int_{-a}^{a} f(-u)\, du$ $= \int_{-a}^{a} f(-x)\, dx$ (use of a dummy variable)	1	students made incorrect assumptions about being odd or even.
(ii) $I = \int_{-4}^{4} (e^x - e^{-x}) \cos x \, dx$ using (i) $= \int_{-4}^{4} e^{-x} - e^{x} \cos(-x)\, dx$ $= -\int_{-4}^{4} e^{x} - e^{-x} \cos x \, dx$ Since $\cos x$ is an even fn $\therefore I = -I$ $2I = 0$ $\underline{I = 0}$	1 1	A number of students obtained $\int_{-4}^{4} 0\, dx$ then stated this was $[x]_{-4}^{4}$

MATHEMATICS Extension 2: Question........

Suggested Solutions	Marks	Marker's Comments
$\int_0^a \sqrt{a^2-x^2}\,dx$ is a quarter of a circle radius a		Poorly done for a standard question.
ii) $= \int_0^a \sqrt{a^2-x^2}\,dx = \frac{1}{4}\pi a^2$	1	
$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \therefore \quad y^2 = b^2\left[1 - \frac{x^2}{a^2}\right]$		Many students used trig. substitutions which wasn't needed
$y = \pm \frac{b}{a}\left[a^2 - x^2\right]$		
$\therefore$ area $= \frac{4b}{a}\int\sqrt{a^2-x^2}\,dx$		
$= \frac{1}{4} \times \pi a^2 \times \frac{b}{a} \times 4$	1	
$= \underline{\pi ab}$		
(b) 	3	1 for e 1 for asymptotes and directrices 1 for graph
$16 = 9(e^2-1)$ $= 1 + \frac{16}{9}$ $= \frac{25}{9}$ $\therefore x = \frac{9}{5}$ or $-\frac{9}{5}$ $y = \pm\frac{4}{3}x$ $S(5,0) \quad S'(-5,0)$		

MATHEMATICS Extension 2: Question 9		
Suggested Solutions	Marks	Marker's Comments

b(ii)



$x = 3\sec\theta \qquad y = 4\tan\theta$

$\frac{dx}{d\theta} = 3\sec\theta\tan\theta \qquad \frac{dy}{d\theta} = 4\sec^2\theta.$

$\frac{dy}{dx} = \frac{4\sec^2\theta}{3\sec\theta\tan\theta}$

$= \frac{4\sec\theta}{3\tan\theta}$ — 1 — Cannot quote formula need to derive it

$y - 4\tan\theta = \frac{4\sec\theta}{3\tan\theta}(x - 3\sec\theta)$

$3y\tan\theta - 12\tan^2\theta = 4x\sec\theta - 12\sec^2\theta$

$4x\sec\theta - 3y\tan\theta = 12\sec^2\theta - 12\tan^2\theta$

$= 12(\sec^2\theta - \tan^2\theta)$

$= 12$ since $\sec^2\theta - \tan^2\theta = 1$ — 1 — Since show question need to make it clear what has happened to $\sec^2\theta$ and $\tan^2\theta$.

tangent $4x\sec\theta - 3y\tan\theta = 12$

MATHEMATICS Extension 2: Question 9

Suggested Solutions	Marks	Marker's Comments
Tangent at P		
$4x\sec\theta - 3y\tan\theta = 12$		
Let $y = \frac{4}{3}x$		
$4x\sec\theta - 3\times\frac{4}{3}x\tan\theta = 12$		
$4x\sec\theta - 4x\tan\theta = 12$		
$4x(\sec\theta - \tan\theta) = 12$		
$x = \frac{3}{\sec\theta - \tan\theta}$		
$y = \frac{4}{3}\times\frac{3}{\sec\theta - \tan\theta}$		
$= \frac{4}{\sec\theta - \tan\theta}$.	1	
~~when~~ For R at $y = -\frac{4}{3}x$		
$4x\sec\theta + 4x\tan\theta = 12$		
$x = \frac{3}{\sec\theta + \tan\theta}$		many students lost negative which made next part difficult
$y = \frac{-4}{\sec\theta + \tan\theta}$	1	
midpoint of RQ.		
$\left(\frac{\frac{3}{\sec\theta - \tan\theta} + \frac{3}{\sec\theta + \tan\theta}}{2}, \frac{\frac{4}{\sec\theta - \tan\theta} - \frac{4}{\sec\theta + \tan\theta}}{2}\right)$		
$\left(\frac{\frac{3\sec\theta + 3\tan\theta + 3\sec\theta - 3\tan\theta}{2}}{2}, \frac{4\sec\theta + 4\tan\theta - 4\sec\theta + 4\tan\theta}{2}\right)$		Many students fudged answer
$\left(\frac{\frac{6\sec\theta}{2}}{2}, \frac{8\tan\theta}{2}\right)$		
$(3\sec\theta, 4\tan\theta)$ which is point P		

MATHEMATICS Extension 2: Question 9

Suggested Solutions	Marks	Marker's Comments
iv) Area of triangle $= \frac{1}{2} \times d_{QR} \times P_{\text{dist } O \text{ to } QR}$.		
$d_{QR} = \sqrt{\left(\frac{3}{\sec\theta - \tan\theta} - \frac{3}{\sec\theta + \tan\theta}\right)^2 + \left(\frac{4}{\sec\theta - \tan\theta} + \frac{4}{\sec\theta + \tan\theta}\right)^2}$		
$= \sqrt{9\left(\frac{\sec\theta + \tan\theta - \sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta}\right) + 16\left(\frac{\sec\theta + \tan\theta + \sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta}\right)}$		
$= \sqrt{36\tan^2\theta + 64\sec^2\theta}$		
$= 2\sqrt{9\tan^2\theta + 16\sec^2\theta}$	1	
Perp dist $= \frac{\lvert 4x\sec\theta - 3y\tan\theta - 12 \rvert}{\sqrt{16\sec^2\theta + 9\tan^2\theta}}$		
$= \frac{\lvert -12 \rvert}{\sqrt{16\sec^2\theta + 9\tan^2\theta}}$		Other method using
$\therefore$ Area $= \frac{1}{2} \times \frac{12}{\sqrt{16\sec^2\theta + 9\tan^2\theta}} \times 2\sqrt{16\sec^2\theta + 9\tan^2\theta}$		$\frac{1}{2} \times OQ \times OR \times \sin$ of needed to calculate $\tan^{-1}\frac{24}{7}$.
$= 12\ u^2$		

MATHEMATICS Extension 2: Question...9.

Suggested Solutions	Marks	Marker's Comments
v) $M_{QS} = \dfrac{\dfrac{4}{\sec\theta-\tan\theta} - 0}{\dfrac{3}{\sec\theta-\tan\theta} - 5}$		
$= \dfrac{4}{3-5(\sec\theta-\tan\theta)}$		
$M_{RS} = \dfrac{-4}{3-5(\sec\theta+\tan\theta)}$	1	1 for both gradients
$\tan\theta = \left\lvert \dfrac{\dfrac{4}{3-5(\sec\theta-\tan\theta)} + \dfrac{4}{3-5(\sec\theta+\tan\theta)}}{1 - \dfrac{16}{3-5(\sec\theta-\tan\theta)(3-5(\sec\theta+\tan\theta)}} \right\rvert$		
$= \left\lvert \dfrac{12-20\sec\theta-20\tan\theta+12-20\sec\theta+20\tan\theta}{9-15(\sec\theta+\tan\theta)-15(\sec\theta-\tan\theta)+25(\sec^2\theta-\tan^2\theta)-16} \right\rvert$		
$= \left\lvert \dfrac{24-40\sec\theta}{9+25-16-30\sec\theta} \right\rvert$		
$= \left\lvert \dfrac{4}{3}\dfrac{(6-10\sec\theta)}{(6-10\sec\theta)} \right\rvert$		
$= 1\frac{4}{3}$		
Angle is obtuse		
$\therefore \angle QSR = 180 - 53^\circ 8'$		
$= 126^\circ 52'$		