YEAR 12 ASSESSMENT

MARCH 1999

4 UNIT ADDITIONAL PAPER

Time Allowed: 3 hours

All questions may be attempted

All questions are of equal value

In every question, all necessary work should be shown

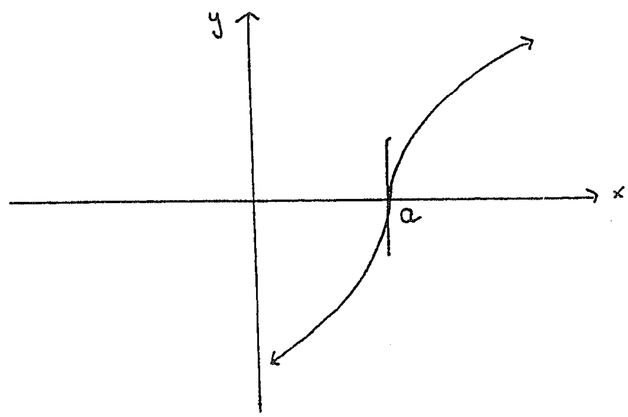
Marks may not be awarded for careless or badly arranged work

Approved calculators may be used

Begin each question on a new page

QUESTION 1. **MARKS**

a) Consider the graph of $y = f(x)$ sketched below. 4



A vertical tangent exists at $x = a$.

Sketch:
i) $y = f'(x)$
ii) $y = f''(x)$
iii) $y = \int f(x)dx$

b)Given $F(x) = \dfrac{x^2 - 1}{x^2 + 1}$ sketch the following: 11

i) $y = F(x)$

ii) $[F(x)]^2 = \dfrac{x^2 - 1}{x^2 + 1}$

iii) $y = \dfrac{|x+1|(x-1)}{x^2 + 1}$

iv) $y = [F(x)]^2$

v) $y = e^{F(x)}$

QUESTION 2

15

a)i) Express $1, \omega, \omega^2$ (the cube roots of unity) in Modulus/Argument form and verify that $1.\omega.\omega^2 = 1$ and $1 + \omega + \omega^2 = 0$.

ii) Show that $1, \omega, \omega^2$ are equally spaced around the unit circle. Hence prove that $|1 - \omega| = |\omega - \omega^2| = |\omega^2 - 1|$.

iii) If the points representing z_1, z_2 and z_3 are the roots of $z^3 = 1$ and form an equilateral triangle taken in anticlockwise direction prove that $z_1 + \omega z_2 + \omega^2 z_3 = 0$.

b) If ω, ω^2 are two of the roots of $P(x) = x^3 + px^2 + qx + r$ show that if $p = q$ then $p = q = r + 1$.

c) You are given that $|z| < 1$ and $I(z) > 0$. Show by using a diagram that $0 < \arg(z + 2) < \frac{\pi}{6}$.

QUESTION 3

a) 9

i) Use De Moivre's theorem to express $\cos 3\theta$ and $\sin 3\theta$ in terms of $\cos\theta$ and $\sin\theta$. Hence show that :

$$\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

ii) Find the roots of the equation $x^3 - 3x^2 - 3x + 1 = 0$. in trigonometric form.

iii) Hence express $x^3 - 3x^2 - 3x + 1$ as a product of one linear factor and one quadratic factor.

b) If β is a positive repeated root of $P(x) = x^3 - 3ax + b$ show that $b^2 = 4a^3$ 6

QUESTION 4

a) 4

i) Find the eccentricity, the equations of the directrices and the co-ordinates of the focii of the ellipse with equation $7x^2+16y^2=112$.

ii) Sketch the ellipse showing on your diagram the above information and the larger of the auxiliary circles.

b) Set up the integrals that give: 6

i) The area of a quadrant of the circle with equation $x^2+y^2=16$.

ii) The area of the quadrant of the ellipse $7x^2+16y^2=112$.

iii) Show that the integral in b) (ii) is $\frac{b}{a}$ times the integral in b) (i) and deduce the area of the ellipse from the known area of the above circle. Hence write down a general formula for the area of an ellipse.

c) Show that $x > \dfrac{3\sin x}{2+\cos x}$ for $x > 0$. 5

QUESTION 5

a) Evaluate the following integrals. 11

i) $\int_{-1}^{1} x\sin x \, dx$

ii) $\int_{-2}^{2} x^3 \cos x \, dx$

iii) $\int \frac{(1+\sqrt{u})^{\frac{1}{2}}}{\sqrt{u}} \, du$

iv) $\int_1^2 \frac{d}{dx}(\ell^x \ln x)dx$

v) $\int \frac{5\sin 2x}{2+3\cos^2 x}dx$ using the substitution $u = 2+\cos^2 x$

b) 4

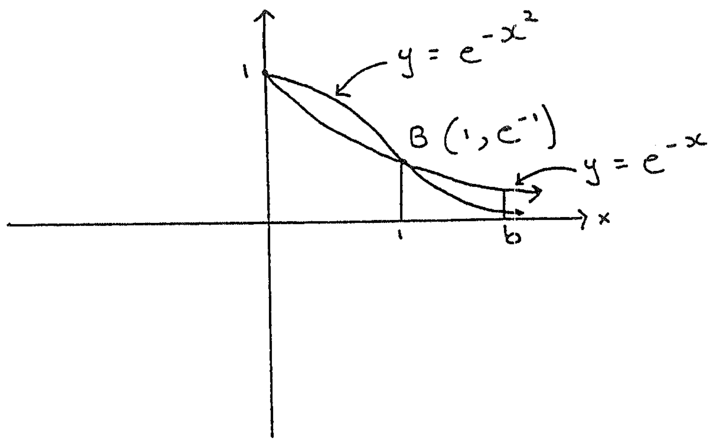
i) Express $\frac{5x-3}{(x+1)(x-3)}$ as a sum of its partial fractions.

ii) Hence evaluate $\int \frac{5x-3}{(x+1)(x+3)}dx$

QUESTION 6

a) 5

Below are drawn the graphs of $y = e^{-x^2}$ and $y = e^{-x}$



i) Write down an expression for $\int_1^b e^{-x} dx$

ii) As $b \to \infty$ write down the value of $\int_1^b e^{-x} \; dx$

iii) Show that as $b \to \infty$ $\int_1^b e^{-x^2} \; dx < e^{-1} - e^{-b} < e^{-1}$

b) Consider the equation $y = (x-1)^{\frac{2}{3}}$ 10

i) Show that $x = 1$ is not a turning point of this function.

ii) For what values of x is the function either increasing or decreasing?

iii) Show that at $x = 1$ there is no point of inflexion.

iv) For what values of x is the function either concave down or up?

v) Taking $x_1 = 0.5$ use Newton's method with two applications to show that the required root cannot be found.

vi) By sketching the graph of $y = (x-1)^{\frac{2}{3}}$ give reasons with the aid of your diagram as to why Newton's method is not appropriate in this case.

QUESTION 7

15

a) Write down the co-ordinates in parametric form of a point P lying on the ellipse with equation $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$

b) Show that the equation of the tangent at P is $bx\cos\theta + ay\sin\theta = ab$,and hence or otherwise derive the equation of the normal at P.

c) The tangent and normal at P cut the y axis at A and B respectively. Find the co-ordinates of A and B.

d) Show that the focus S lies on the circumference of the semicircle which has diameter AB.

QUESTION 8

a) 3

Let $\alpha, \beta, \gamma, \delta$ be the roots of $8x^4 - 6x + 1 = 0$. Find the equation whose roots are $3\alpha, 3\gamma, 3\beta, 3\delta$

b) 6

Show that the equation $10x^4 + 27x^3 - 110x^2 + 27x + 10 = 0$, may be written in the form $10(x^2 + \frac{1}{x^2}) + 27(x + \frac{1}{x}) - 110 = 0$ and hence show that the equation has real roots.

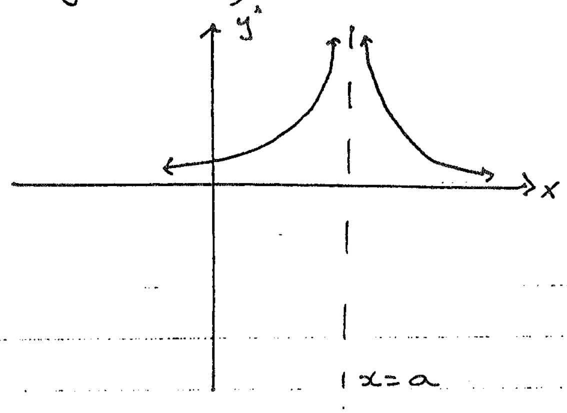
c) 6

By finding the turning points on the curve $y = x^3 + 2x^2 + x + k$ and by the use of a suitable graph, show that the polynomial has two roots for $0 \le k \le -\frac{4}{27}$.

For what values of k does the polynomial have only one root?

Question 1

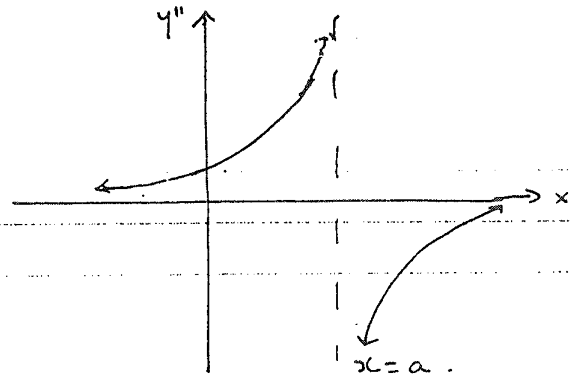
a) i) $y = f'(x)$



$\frac{1}{2}$: assymptote

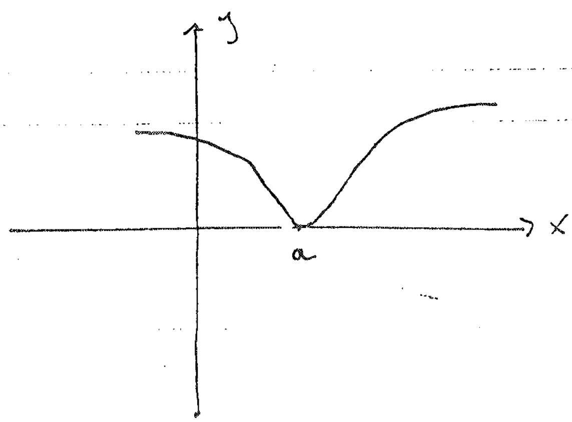
$\frac{1}{2}$ graph

ii) $y = f''(x)$



1

iii) $y = f(x)$

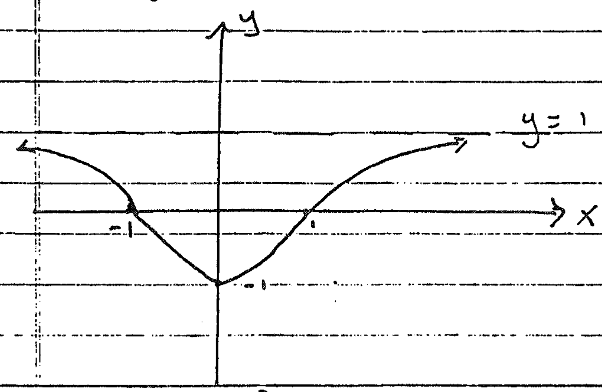


2

1 t.pt at a

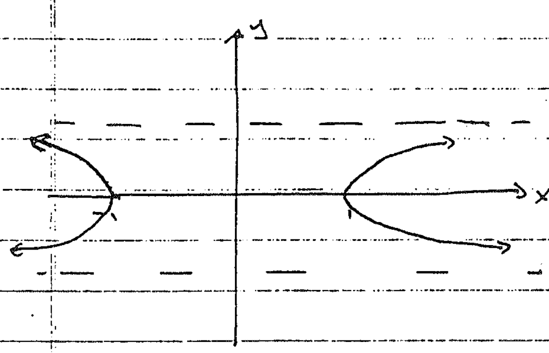
1 for both halfs

b) i) $y = F(x)$



2

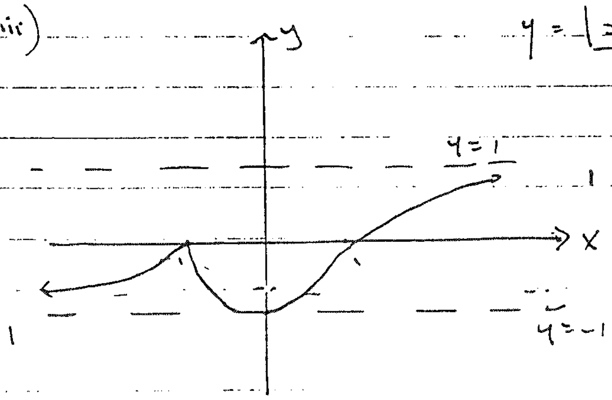
ii) $\left[F(x)\right]^2 = \dfrac{x^2 - 1}{x^2 + 1}$



1 symmetry

1 graph

iii)



1

1

$y = \dfrac{|x+1|\,(x-1)}{x^2+1}$

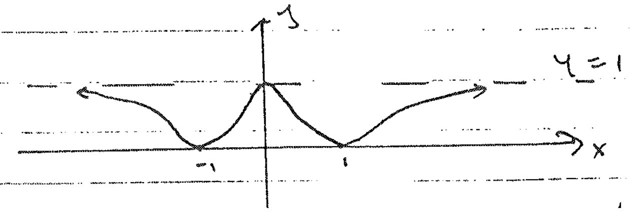
for $x > -1$

$y = \dfrac{(x+1)(x-1)}{x^2+1}$

ie original graph.

for $x < -1$ $y = -\dfrac{(x+1)(x-1)}{x^2+1}$.

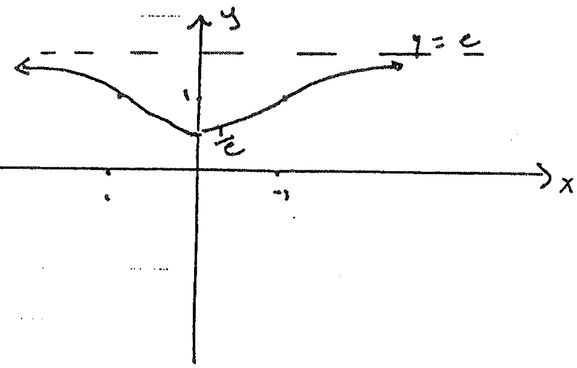
iv)



$y = \left[F(x)\right]^2$

1 for symmetry

v) $y = e^{F(x)}$



1 for assymptote
1 for t. pt
1 for graph.

Question 2: Aw L

a) i) $1,\ -\frac{1}{2}+\frac{\sqrt{3}}{2}i,\ -\frac{1}{2}-\frac{\sqrt{3}}{2}i$ are roots of $z^3-1=0$

$\omega = -\frac{1}{2}+\frac{\sqrt{3}}{2},\quad \omega^2 = -\frac{1}{2}-\frac{\sqrt{3}}{2}i$

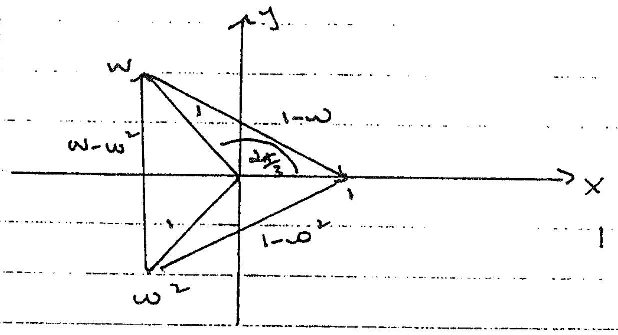
$1+\omega+\omega^2 = \frac{-b}{a} = 0$

Aw 1.

$1.\omega.\omega^2 = \frac{-d}{a}$

$= --1 = 1$

ii) modulus and Argument of each root is 1 and $\frac{2\pi}{3}$ respectively.



1

From the diagram we can see the vectors $1-\omega$, $\omega-\omega^2$ $1-\omega^2$ 1

These vectors are also the sides at the equilateral triangle formed by joining the cube root of unity All sides of an equilateral triangle are equal in length

$\therefore |1-\omega| = |\omega-\omega^2| - |\omega^2-1|$

iii) Let $z_1 = 1$, $z_2 = \omega$ and $z_3 = \omega^2$

$\therefore\ z_1 + \omega z_2 + \omega^2 z_3 = 1+\omega^2+\omega^3$ 1

$= 0$ from (a)

b) $\omega^3 + p\omega^2 + q\omega + r = 0$

$\omega^6 + p\omega^4 + q\omega^2 + r = 0$ 1

ie $1 + p\omega^2 + q\omega + r = 0$

$1 + p\omega + q\omega^2 + r = 0$

if $p = q \Rightarrow 1 + p\omega^2 + p\omega + r = 0$ — ①

$1 + p\omega^2 + p\omega + r = 0$ — ② 1

① + ② $2 + 2p\omega^2 + 2p\omega + r = 0$ 1

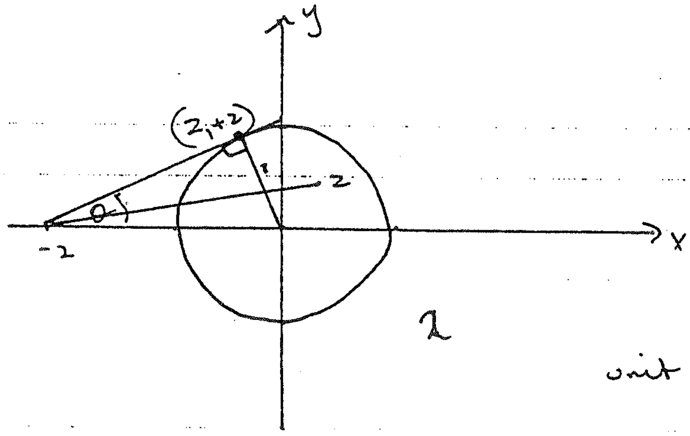
$\therefore \quad 2 + 2p(-1-\omega) + 2p\omega + 2r = 0$

$2 - 2p - 2p\omega + 2p\omega + 2r = 0$

$2p = 2 + 2r$

$p = 1 + r.$ 1

c).



2

since $\operatorname{Im}(z) > 0$
then $\arg z > 0$ 1

z can also lie on the radius of the given unit circle such that when the radius touches tangent from -2 we obtain the maximum argument θ. 1

$\sin\theta = \frac{1}{2}$

1

$\theta = \frac{\pi}{6} \Rightarrow 0 < \arg(z+2) < \frac{\pi}{6}$

Question 3

a) i) $(\cos\theta + i\sin\theta)^3 = \cos^3\theta + 3\cos^2\theta . i\sin\theta + 3\cos\theta . i^2\sin^2\theta + i^3\sin^3$

$= \cos^3\theta + 3\cos^2\theta . i\sin\theta - 3\cos\theta\sin^2\theta - i\sin^3$

Using De Moivre's theorem

$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta$

$\therefore \cos 3\theta = \cos^3\theta - 3\cos\theta(1-\cos^2\theta)$

$\sin 3\theta = 3(1-\sin^2\theta)\sin\theta - \sin^3\theta$

ie $\cos 3\theta = \cos^3\theta - 3\cos\theta + 3\cos^3\theta$

$= 4\cos^3\theta - 3\cos\theta$ 1

$\sin 3\theta = 3\sin\theta - 3\sin^3\theta - \sin^3\theta$

$= -4\sin^3\theta + 3\sin\theta$ 1

$\therefore \tan\theta = \dfrac{\sin 3\theta}{\cos 3\theta} = \dfrac{-4\sin^3\theta + 3\sin\theta}{4\cos^3\theta - 3\cos\theta}$

Divide numerator and denominator by $4\cos^3\theta$

$\dfrac{\frac{3}{4}\sec^2\theta\tan\theta - \tan^3\theta}{1 - \frac{3}{4}\sec^2\theta}$ 1

$= \dfrac{\frac{3}{4}(1+\tan^2\theta)\tan\theta - \tan^3\theta}{1 - \frac{3}{4}(1+\tan^2\theta)}$

$= \dfrac{\frac{3}{4}\tan\theta + \frac{3}{4}\tan^3\theta - \tan^3\theta}{1 - \frac{3}{4} - \frac{3}{4}\tan^2\theta}$

$= \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^3\theta}$ 1

1

If $\dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^3\theta} = 1$ we get $\tan^3\theta - 3\tan^2\theta - 3\tan\theta + 1 =$

let $x = \tan\theta$.

$\therefore$ roots of the equation occur when $\tan 3\theta = 1$

ie $3\theta = n\pi + \frac{\pi}{4}$

when $n=0$ $\quad \theta = \frac{\pi}{12}$

$n=1$ $\quad \theta = \frac{5\pi}{12}$

$n=-1$ $\quad \theta = -\frac{\pi}{4}$

$\therefore$ Roots are $x = \tan\frac{\pi}{12}, \tan\frac{5\pi}{12}, \tan\frac{-\pi}{12}$ 1

iii) $\tan\frac{\pi}{12} = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$$= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3} \cdot \frac{\pi}{4}} = \frac{\sqrt{3}-1}{1+\sqrt{3}}$$

$= 2 - \sqrt{3}$ 1

$\tan\frac{5\pi}{12} = \tan\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$$

$= 2 + \sqrt{3}$

$\therefore x^3 - 3x^2 - 3x + 1 = (x+1)(x - (2-\sqrt{3}))(x - (2+\sqrt{3}))$ 1

$\therefore x^3 - 3x^2 - 3x + 1 = (x+1)(x^2 - 4x - 1)$ 1

b) $\beta^3 - 3a\beta + b = 0 \quad -①$ 1

$3\beta^2 - 3a = 0 \quad -②$ 1

$6\beta = 0$ Now $\beta \neq 0$ because $P(0) = b \neq 0$.

$\therefore$ From ② $\beta^2 = a$

$\beta = a^{\frac{1}{2}}$ $\quad \beta > 0$. 1

sub into ① $\beta^{3/2} - 3a^{3/2} + b = 0$ 1

$-2a^{3/2} = -b$ 1

$4a^3 = b^2$ 1

Question 4

a) i) $7x^2 + 16y^2 = 112$

$$\frac{x^2}{16} + \frac{y^2}{7} = 1$$

$a = 4, \quad b = \sqrt{7}$

$b^2 = a^2(1 - e^2)$

$\frac{7}{16} = 1 - e^2$

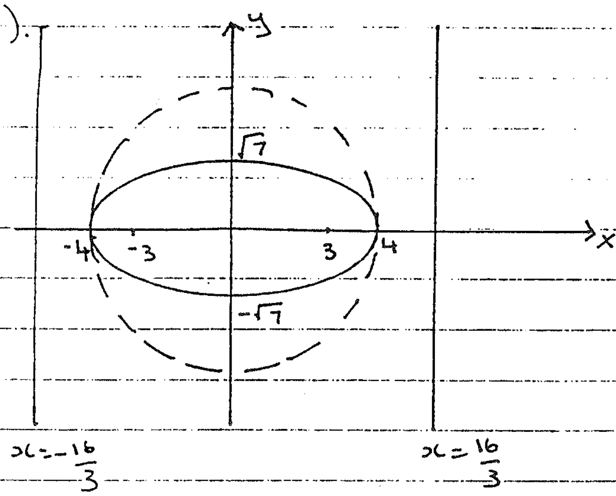
$e^2 = \frac{9}{16}$

$e = \frac{3}{4} \quad e > 0.$ 1

Co-ordinates of foci $(\pm 3, 0)$ 1

Equation of directrices: $x = \pm \frac{16}{3}$ 1

ii)



1

b) i) $\int_0^4 \sqrt{16 - x^2}\, dx$ 1

ii) $\int_0^4 \sqrt{\frac{112 - 7x^2}{4}}\, dx$ 1

iii) $\int_0^4 \sqrt{\frac{112 - 7x^2}{4}}\, dx = \frac{1}{4}\int_0^4 \sqrt{7}\sqrt{16 - x^2}\, dx$ 1

$= \frac{\sqrt{7}}{4}\int_0^4 \sqrt{16 - x^2}\, dx$

$= \frac{b}{a}\int_0^4 \sqrt{16 - x^2}\, dx$ 1

$\therefore$ Area of ellipse $= \frac{\sqrt{7}}{4} \times \pi \times 16$

$= 4\sqrt{7}\,\pi.$ 1

General formula for the area of an ellipse is πab 1

c) Let $f(x) = x - \dfrac{3\sin x}{2+\cos x} \qquad x > 0$

$$f'(x) = 1 - \left\{ \frac{(2+\cos x)(3\cos x) - 3\sin x \times -\sin x}{(2+\cos x)^2} \right\}$$ 1

$$= 1 - \left\{ \frac{6\cos x + 2\cos^2 x + 3\sin^2 x}{(2+\cos x)^2} \right\}$$

$$= 1 - \left\{ \frac{6\cos x + 2\cos^2 x + 3 - 3\cos^2 x}{(2+\cos x)^2} \right\}$$

$$= 1 - \left\{ \frac{6\cos x - \cos^2 x - 3}{(2+\cos x)^2} \right\}$$ 1

$$= \frac{(2+\cos x)^2 - 6\cos x + \cos^2 x + 3}{(2+\cos x)^2}$$

$$= \frac{2\cos^2 x - 2\cos x + 7}{(2+\cos x)^2}$$ 1

Denominator is a perfect square $\therefore$ always > 0
Applying the discriminant to the nomerator we have

$$\Delta = 4 \pm 4 \times 2 \times 7$$
$$= 4 - 56$$ 1
$$< 0$$

$\therefore$ numerator is positive for all value of x

$\therefore \; x - \dfrac{3\sin x}{2+\cos x} > 0$ for $x > 0$ 1

Question 5

a) i) $\int_{-1}^{1} x\sin x \, dx = 2\int_{0}^{1} x\sin x \, dx$ 2 even function

ii) $\int_{-2}^{2} x^3 \cos x = 0$ 2 odd function

iii) $\int \frac{(1+\sqrt{u})^{\frac{1}{2}}}{\sqrt{u}} \, du$

$= 2 \frac{(1+\sqrt{u})^{3/2}}{\frac{3}{2}} + c$ 2 -1 per mistake

$= \frac{4}{3}(1+\sqrt{u})^{3/2} + c$

iv) $\int_{+1}^{2} \frac{d}{dx}(e^{x} \ln x) \, dx$

$= \left[e^{x} \ln x \right]_{+1}^{2}$ 1

$= e^2 \ln 2 - 0$

$= e^2 \ln 2$ 1

v) $\int \frac{5\sin 2x}{2+3\cos^2 x} \, dx$

$u = 2 + \cos^2 x$

$\frac{du}{dx} = -2\cos x \sin x$

$= -\sin 2x$ 1

$\therefore dx = \frac{du}{-\sin 2x}$

$= -\int \frac{5}{2+3u-6} \, du$ 1

$= -5 \int \frac{du}{3u-4} \, du$

$= -\frac{5}{3} \ln(3u-4) + c.$ 1

$= -\frac{5}{3} \ln(6+3\cos^2 x - 4)$

$= -\frac{5}{3} \ln(2+3\cos^2 x) + c.$

b) $\frac{5x-3}{(x+1)(x-3)} = \frac{A}{x+1} + \frac{B}{x-3}$ 1

$5x-3 = A(x-3) + B(x+1)$

if $x = 3$

$12 = 4B$

$B = 3.$ 1

if $x = -1$

$-8 = -4A$

$A = 2$ 1

$\therefore \frac{5x-3}{(x+1)(x-3)} = \frac{2}{x+1} + \frac{3}{x-3}$

$\therefore \int \frac{5x-3}{(x+1)(x-3)} = 2\ln(x+1) + 3\ln(x-3) + C$ 1

Question 6

a) i) $\int_1^b e^{-x}\,dx = \left[-e^{-x}\right]_1^b$

$= -e^{-b} + e^{-1}$ 1

ii) As $b \to \infty$ $\int_1^b e^{-x}\,dx = e^{-1}$ as $-e^{-b} = -\frac{1}{e^b}$ 1

$\to 0$ as $b \to \infty$

iii) From the graph $\int e^{-x^2}\,dx < \int e^{-x}\,dx$ 1

$< +e^{-1} - e^{-b}$ 1

$< e^{-1}$ as $b \to \infty$ 1

from solution in (ii)

$\therefore \int e^{-x^2} < +e^{-b} + e^{-1} < e^{-1}$

b) $y = (x-1)^{2/3}$

i) $y' = \frac{2}{3}(x-1)^{-\frac{1}{3}}$

$= \frac{2}{3(x-1)^{\frac{1}{3}}}$ 1

$x \neq \phi$ the function does not have a turning point 1

ii) $x > 1$ function is increasing

$x < 1$ function is decreasing 1

iii) $y'' = \frac{-2}{9(x-1)^{4/3}}$ 1

$x \neq 1$ No pts of inflexion. 1

iv) $x > 1$ function is concave down

$x < 1$ function is concave up. 1

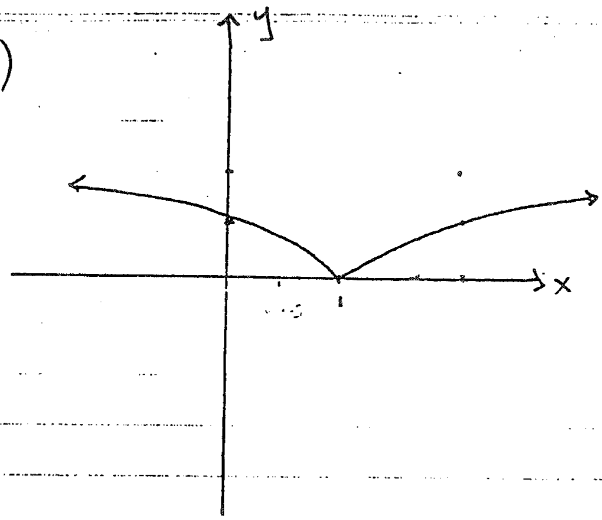
v) $x_2 = 0.5 - \frac{f(0.5)}{f'(0.5)}$

$x_2 = 0.5 -$

$x_3 = 1.25 - \frac{f(1.25)}{f'(1.25)}$

$x_3 = 2.583$ 1

vi)



The function although continuous at $x=1$ is <u>not</u> <u>differentiable</u> at $x=1$. Although the root is $x=1$ Newton's method is not appropriate, as you can see from the graph that although the line is tangental to one part of the graph it is <u>not</u> tangental to the other part.

7

Question 7

a. $P(a\cos\theta,\ b\sin\theta)$

b. Gradient of tangent: $\dfrac{2x}{a^2} + \dfrac{2y}{b^2} \cdot \dfrac{dy}{dx} = 0$ 1

$$\frac{2y}{b^2} \cdot \frac{dy}{dx} = -\frac{2x}{a^2}$$

1

$$\frac{dy}{dx} = -\frac{b^2}{a^2} \cdot \frac{x}{y}$$

1

At $x = a\cos\theta$; $y = b\sin\theta$.

$$\frac{dy}{dx} = \frac{-b^2}{a^2} \frac{a\cos\theta}{b\sin\theta}$$

$$= -\frac{b\cos\theta}{a\sin\theta}.$$

1

$$\frac{y - b\sin\theta}{x - a\cos\theta} = \frac{-b\cos\theta}{a\sin\theta}$$

$$ay\sin\theta - ab\sin^2\theta = -bx\cos\theta + ab\cos^2\theta$$

$$ay\sin\theta + bx\cos\theta = ab\cos^2\theta + ab\sin^2\theta$$

$$ay\sin\theta + bx\cos\theta = ab.$$

1

Equation of normal

$$\frac{y - b\sin\theta}{x - a\cos\theta} = \frac{a\sin\theta}{b\cos\theta}$$

1

$$yb\cos\theta - b^2\sin\theta\cos\theta = ax\sin\theta - a^2\sin\theta\cos\theta.$$

$$by\cos\theta - ax\sin\theta = b^2\sin\theta\cos\theta - a^2\sin\theta\cos\theta$$

$$= \sin\theta\cos\theta(b^2 - a^2)$$

1

Dividing by $\sin\theta\cos\theta$

$$\frac{by}{\sin\theta} - \frac{ax}{\cos\theta} = b^2 - a^2$$

1

ie $\dfrac{ax}{\cos\theta} - \dfrac{by}{\sin\theta} = a^2 - b^2$

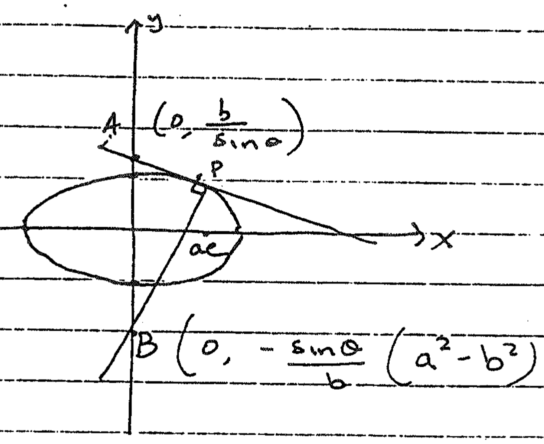
c. when $y = 0 \Rightarrow x = \dfrac{a}{\cos\theta}$

when $x = 0 \Rightarrow y = \dfrac{b}{\sin\theta}$

$A\left(0, \dfrac{b}{\sin\theta}\right)$ 1

For normal

a)



If ae lies on the circumference of a semi-circle with AB then the angle subtended by AB at (ae, 0) must be a right-angle.

ie Gradient of BS × Gradient of AS = −1. 1

∴ Gradient of BS $= \dfrac{-\sin\theta}{-aeb}(a^2-b^2)$ 1

Gradient of AS $= \dfrac{-b}{ae\sin\theta}$ 1

∴ Gradient of BS × Gradient of AS $= \dfrac{-b}{ae\sin\theta} \times \dfrac{\sin\theta}{aeb}(a^2-b^2)$

$= \dfrac{-1}{a^2e^2} \cdot a^2-b^2$

$e^2 = 1 - \dfrac{b^2}{a^2}$ 1

$\therefore \dfrac{-1}{a^2\left(1-\frac{b^2}{a^2}\right)} \cdot a^2-b^2 = \dfrac{-1}{a^2-b^2} \times a^2-b^2$

$= -1$ 1

∴ Angle subtended by AB at ae is a right-angle
∴ ae lies on the circumference of the semi-circle with diameter AB.

Question 8

a) Let $x = 3\alpha$

$\alpha = \frac{x}{3}$ 1

$\therefore$ Polynomial becomes $8\left(\frac{x}{3}\right)^4 - 6\left(\frac{x}{3}\right) + 1 = 0$ 1

$\frac{8x}{81} - 2x + 1 = 0$

$8x - 162x + 81 = 0$ 1

b). Expand $10\left(x^2 + \frac{1}{x^2}\right) + 27\left(x + \frac{1}{x}\right) - 110 = 0$

$10x^2 + \frac{10}{x^2} + 27x + \frac{27}{x} - 110 = 0$ 1

$10x^4 + 10 + 27x^3 + 27x - 110x^2 = 0$

$10x^4 + 27x^3 - 110x^2 + 27x + 10 = 0$ $

Let $m = x + \frac{1}{x}$ 1

$m^2 = \left(x + \frac{1}{x}\right)^2 - 2$ 1

ie $x^2 + \frac{1}{x^2} = m^2 + 2$ 1

$\therefore$ $10(m^2 + 2) + 27m - 110 = 0$ 1

$10m^2 + 20 + 27m - 110 = 0$

$10m^2 + 27m - 90 = 0$

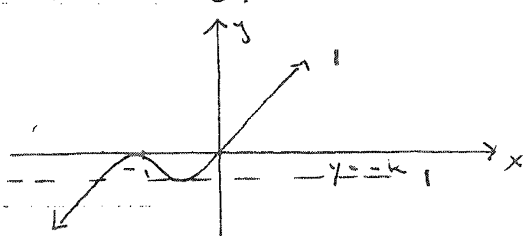
$\Delta = 27^2 - 4 \times 27 \times -90$

> 0 1

$\therefore$ Equation has real roots.

c) Consider the equation $x^3 + 2x^2 + x + k = 0$ and rewrite this expression as $x^3 + 2x^2 + x = -k$.

If $k > 0$.



$\cdot$ $x^3 + 2x^2 + x = x(x+1)^2$

Let $y = x^3 + 2x^2 + x$

$y' = 3x^2 + 4x + 1$

$= 3x^2 + 3x + x + 1$

$y'' = 6x + 4$

when $x = -\frac{1}{3}$

$y'' > 0$

$\therefore$ min t. pt at $\left(-\frac{1}{3}, -\frac{4}{27}\right)$ 1

if $k > 0$ there are two roots as seen from graph

in the domain $0 \leq k \leq -\frac{4}{27}$

. If $k \leq -\frac{4}{27}$ there is only one root 1

if $k < 0$ there is only one root when $k > 0$. 1