Name: ____________________________ Student Number: ________________________

LORETO KIRRIBILLI

2016
HSC ASSESSMENT TASK 2
TERM 1 ASSESSMENT BLOCK

Mathematics Extension 2

Wednesday 23rd March (Morning Session)

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen
- Board-approved calculators may be used
- A Reference Sheet is provided.
- In Questions 11–16, show relevant mathematical reasoning and/or calculations

Total marks – 100

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Section II

90 marks

Attempt Questions 11–16

Allow about 2 hours 45 minutes for this section

Outcomes to be Assessed:

A student:

E2 chooses appropriate strategies to construct arguments and proofs in both concrete and abstract settings.

E3 uses the relationship between algebraic and geometric representations of complex numbers and of conic sections.

E4 uses efficient techniques for the algebraic manipulation required in dealing with questions such as those involving conic sections and polynomials.

E6 combines the ideas of algebra and calculus to determine the important features of the graphs of a wide variety of functions.

E9 communicates abstract ideas and relationships using appropriate notation and logical argument.

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1-10.

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1 What is $-\sqrt{3}+i$ expressed in modulus-argument form?

(A) $\sqrt{2}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

(B) $2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

(C) $\sqrt{2}\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

(D) $2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

2 Which of the following expressions is equivalent to $\frac{2i}{1-2i}=?$

(A) $-\frac{4}{3}-\frac{2}{3}i$

(B) $-\frac{4}{5}+\frac{2}{5}i$

(C) $\frac{4}{5}+\frac{2}{5}i$

(D) $\frac{4}{3}-\frac{2}{3}i$

3 The eccentricity of the hyperbola $\frac{x^2}{4k^2}-\frac{y^2}{k^2}=1$, where k is a positive constant is?

(A) $\frac{\sqrt{3}}{2}$

(B) 2

(C) $\frac{\sqrt{5}}{2}$

(D) $\sqrt{5}$

4 The equation $8x^3-54x+k$ has a double root. What are the possible values of k?

(A) ±4

(B) ±9

(C) ±40.5

(D) ±54

5 If z represents a variable point on the argand diagram, which best represents the locus of $|z-4+i|-|z+4-i|=0$?

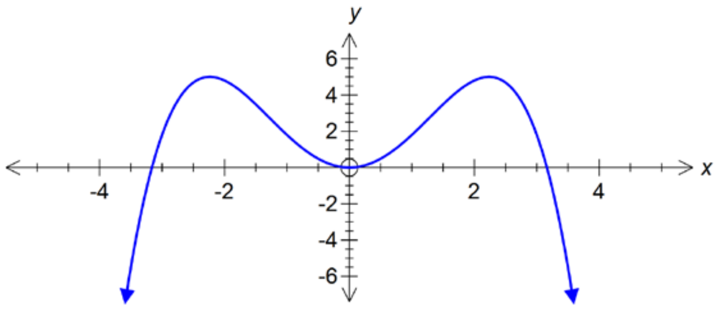
(A) a hyperbola

(B) an ellipse

(C) a circle

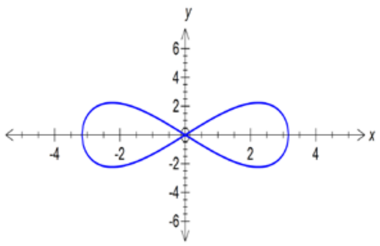
(D) a line

6 The diagram below shows the graph of the function $y = f(x)$.

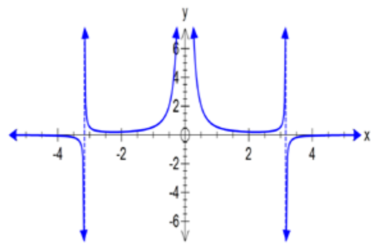


Which of the following is the graph of $y = \sqrt{f(x)}$?

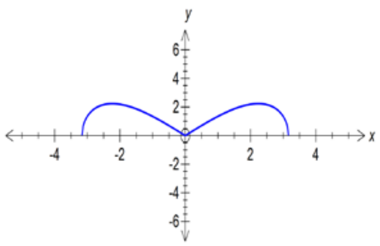
(A)



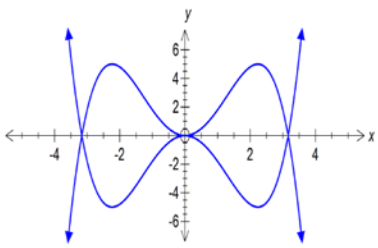
(B)



(C)



(D)



7 The polynomial $P(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_0$ is an odd polynomial with at least 1 multiple root. Julie was asked to give some facts about the curve and replied

I $y = P(x)$ must pass through the origin

II If $P'(a) = 0$ then $P(-a) = 0$

Which of Julie's statements must always be correct?

(A) I only

(B) II only

(C) both I and II

(D) neither I nor I

8 What is the number of asymptotes of the graph of $y = \dfrac{2x^3}{x^2 - 1}$?

(A) 1

(B) 2

(C) 3

(D) 4

9 If ω is a complex cubic root of unity, which of the following statements is **NOT** correct?

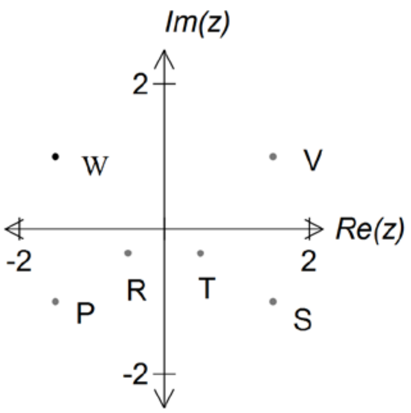
(A) ω^2 is the other complex root

(B) $\omega^2 + \omega = -1$

(C) $\omega^6 - \omega = \omega^2$

(D) $\bar{\omega}$ is also a root

10



The point W on the Argand diagram above represents a complex number w where $|w| = 1.5$. The complex number w^{-1} is best represented by which point?

(A) P

(B) R

(C) S

(D) T

Section II

90 marks Attempt Questions 11–16

Allow about 2 hours and 45 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/ or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet.

(a) Let $z = 4 - i$ and $w = 2 + 2i$.

Express in the form $x + iy$, where x and y are real numbers:

(i) $\overline{iz}$ **1**

(ii) zw **1**

(iii) $\dfrac{z}{w}$ **2**

(b) The complex number w is given by $w = -1 + i\sqrt{3}$.

(i) Show that $w^2 = 2\overline{w}$. **2**

(ii) Evaluate $|w|$ and $\arg(w)$. **2**

(iii) Show that w is a root of the equation $w^3 - 8 = 0$. **1**

Question 11 continues on the next page

(c) The polynomial $P(x)$ leaves a remainder of 6 when divided by $(x-1)$ **2**

and a remainder of 2 when divided by $(x-3)$. Find the remainder when $P(x)$ is divided by $(x-1)(x-3)$.

(d) *P* is a point on the Argand diagram representing the complex number $Z = 1+i$. The points *Q*, *R* and *T* represent $Q = iZ$, $R = -Z$ and $T = \frac{1}{Z}$.

(i) Find the values of *Q*, *R* and *T*, expressing each in the form $x+iy$ with *x* and *y* real. **2**

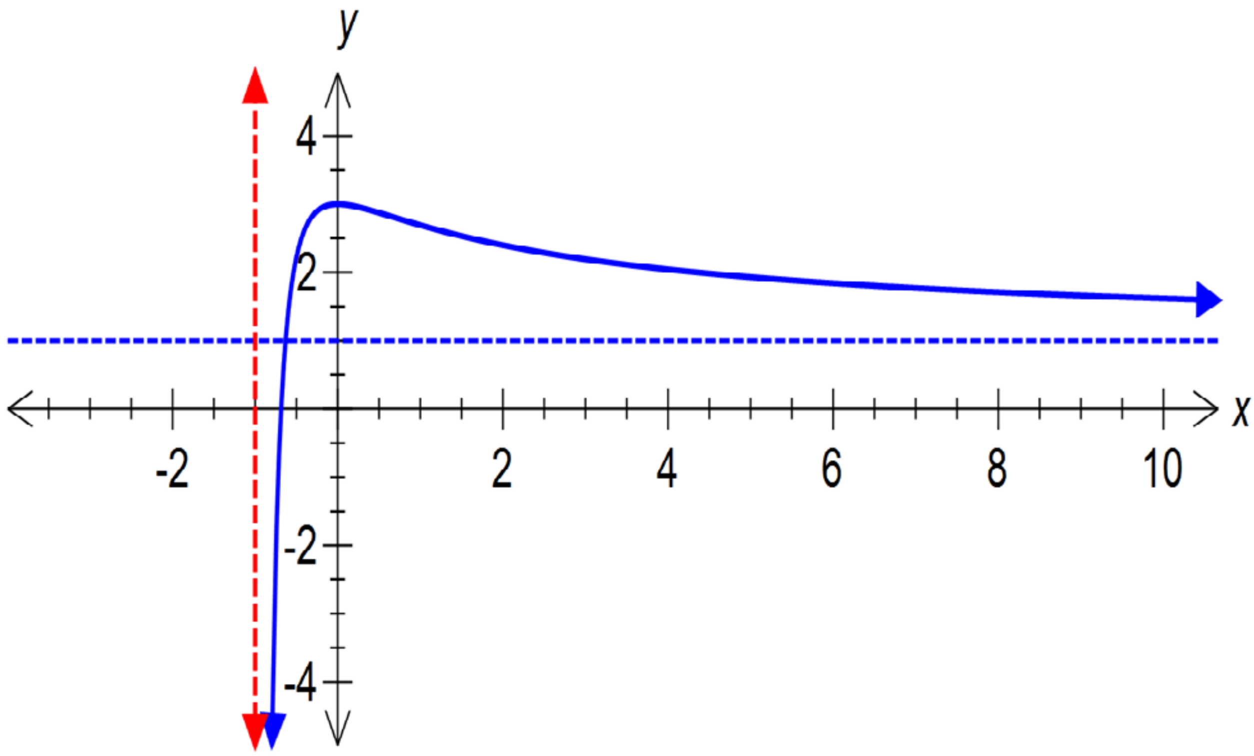
(ii) Locate *P*, *Q*, *R* and *T* on the Argand diagram. What is the best description of the quadrilateral *PQRT?* **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet.

(a) The diagram shows the graph of $y = f(x)$.

$x = -1$ and $y = 1$ are asymptotes and $(0, 3)$ is a maximum turning point.



Draw separate one-third page sketches of the graphs of the following:

(i) $y = f(|x|)$ **1**

(ii) $|y| = f(x)$ **1**

(iii) $y = [f(x)]^2$ **2**

(iv) $y^2 = f(x)$ **2**

(v) $y = e^{f(x)}$ **2**

Question 12 continues on the next page

Question 12 (continued)

(b) The letters of the word EQUATIONS are arranged in a row from left to right.

(i) Find the probability that the vowels occur in alphabetical order reading from left to right. **1**

(ii) Find the probability that no two vowels are next to each other. **2**

(c) Solve $\dfrac{|x-1|-2}{12+x-x^2} \geq 0$. **4**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet.

(a) On the Argand diagram, let $A = \sqrt{3} + i$

(i) Draw a clear sketch to show the important features of the curve defined by $|z - A| = 2$. **1**

(ii) For z on this curve find the maximum value of $|z|$. **1**

(ii) For z on this curve find the range of values of $\arg(z)$. **2**

(b) The equation of the ellipse, E, is $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$.

The point P on the ellipse is (x_1, y_1).

(i) Find the eccentricity of the ellipse. **1**

(ii) Find the coordinates of the foci and the equations of the directrices of the ellipse. **2**

(iii) Show that the equation of the tangent at P is $\dfrac{xx_1}{25} + \dfrac{yy_1}{9} = 1$. **2**

(iv) Let the tangent at P meet the directrix at a point J. Show that $\angle PSJ$ is a right angle where S is the corresponding focus. **3**

(c) Solve the equation $x^4 - 5x^3 - 9x^2 + 81x - 108 = 0$ given that it has a triple root. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet.

(a) Express $\dfrac{x-8}{\left(x^2+2\right)\left(x-2\right)}$ in the form $\dfrac{Ax+B}{x^2+2}+\dfrac{C}{x-2}$ **3**

(b) Consider $f\left(x\right)=\sin x+\cos x$

(i) Find A and B such that $\sin x+\cos x=A\sin\left(x+B\right)$ **2**

where $A>0$ and $0\le B\le\dfrac{\pi}{2}$

(ii) Sketch $f\left(x\right)=\sin x+\cos x$ for $-2\pi\le x\le 2\pi$. **2**

(iii) Hence, or otherwise sketch $y=\dfrac{1}{f\left(x\right)}$ for $-2\pi\le x\le 2\pi$. **2**

(iv) Sketch $y=\dfrac{f\left(x\right)}{x}$. **2**

(c) The equation $x^3-3x^2+9=0$ has roots α,β and γ.

(i) Find the equation with roots α^2,β^2 and γ^2. **2**

(ii) Find the value of $\alpha^2+\beta^2+\gamma^2$ and hence evaluate $\alpha^3+\beta^3+\gamma^3$. **2**

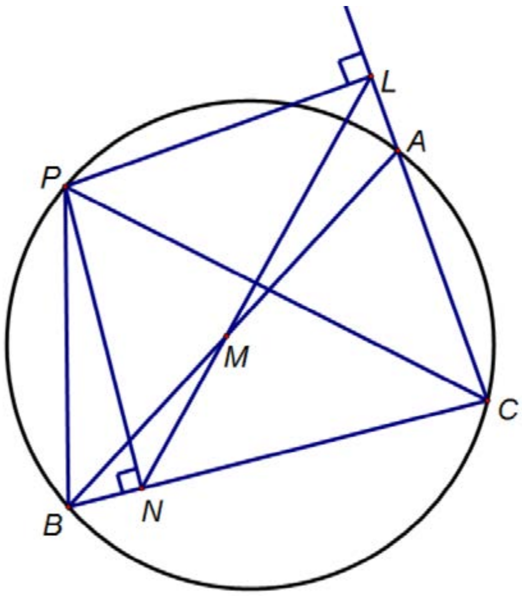
End of question 14

Question 15 (15 marks) Use a SEPARATE writing booklet.

(a) For the relation $x^3 + 3y^2 = 6xy - 3x$, find $\dfrac{dy}{dx}$ in terms of x and y. **2**

(b) Sketch the region on the Argand diagram defined by $(z-3+i)(\overline{z}-3-i) \le 9$. **3**

(c)



ABC is an acute angled triangle inscribed in a circle. *P* is a point on the minor arc *AB* such that *PL* is perpendicular to *CA* produced. *N* is a point on BC such that *PN* is perpendicular to *BC*. *LN* cuts *AB* at *M*.

Copy or trace the diagram into your writing booklet.

(i) Explain why *PNCL* is a cyclic quadrilateral. **1**

(ii) Show that $\angle PBM = \angle PNM$. **2**

(iii) Show that *PM* is perpendicular to *AB*. **3**

Question 15 continues on the next page

Question 15 (continued)

(d) A and B are on the curve $y = x^4 + 4x^3$ at $x = \alpha$ and $x = \beta$ respectively.

The line $y = mx + b$ is a tangent to the curve at both the points A and B.

(i) The zeroes of the equation $x^4 + 4x^3 - mx - b = 0$ are α, α, β and β. **1**

Explain this result.

(ii) Find the values of m and b. **3**

End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet.

(a) If ω is a complex root of the equation $x^3 = 1$.

(i) Show that the other complex root is ω^2. **1**

(ii) Show that $1+\omega+\omega^2 = 0$. **1**

(iii) Find in its simplest form, the cubic equation whose roots are 3, $2\omega+\omega^2$ and $2\omega^2+\omega$. **3**

(b) The points $P\left(a\cos\theta, b\sin\theta\right)$ and $Q\left(-a\sin\theta, b\cos\theta\right)$ lie on the ellipse E,

Given by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(i) Show that if O is the origin then $OP^2+OQ^2=a^2+b^2$. **1**

The equations of the tangents at P and Q are:

$\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$ and $\frac{-x\sin\theta}{a}+\frac{y\cos\theta}{b}=1$ respectively.

Do not prove these.

(ii) Show that the point of intersection of T of the two tangents at P and Q is given by $T\left(a\left(\cos\theta-\sin\theta\right), b\left(\cos\theta+\sin\theta\right)\right)$. **3**

(iii) Show that the locus of T is given by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$. **2**

(iv) If α is the angle between the tangents at P and Q show that **4**

$$\tan\alpha = 2\frac{\sqrt{1-e^2}}{e^2\sin 2\theta}$$

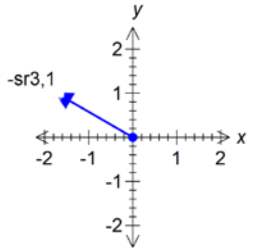
End of paper.

Student Number | | | | | | | | |

MATHEMATICS EXTENSION 2

MULTIPLE-CHOICE ANSWER SHEET

1	A ○	B ○	C ○	D ○
2	A ○	B ○	C ○	D ○
3	A ○	B ○	C ○	D ○
4	A ○	B ○	C ○	D ○
5	A ○	B ○	C ○	D ○
6	A ○	B ○	C ○	D ○
7	A ○	B ○	C ○	D ○
8	A ○	B ○	C ○	D ○
9	A ○	B ○	C ○	D ○
10	A ○	B ○	C ○	D ○

1. $-\sqrt{3}+1 \quad \lvert z \rvert = 2, \quad \arg(z) = \frac{5\pi}{6}$ 	D
2. $\frac{2i}{1-2i} = \frac{2i(1+2i)}{(1-2i)(1+2i)}$ $= \frac{2i-4}{1+4}$ $= \frac{-4}{5} + \frac{2i}{5}$	B
3. $e^2 = \frac{a^2+b^2}{a^2}$ $= \frac{4k^2+k^2}{4k^2}$ $= \frac{5\cancel{k^2}}{4\cancel{k^2}}$ $e = \frac{\sqrt{5}}{2}$	C
4. $f(x) = 8x^3 - 54x + k$ $f'(x) = 24x^2 - 54$ for a double root f"(x)=0 $x = \pm 1.5$ $f(1.5) = 8(\pm 1.5)^3 - 54(\pm 1.5) + k$ $= \mp 54 + k$ $\therefore k = \pm 54$	D
5. $\lvert z-(4-i) \rvert = \lvert z-(-4+i) \rvert$ refers to the distance of z from 2 points is equal therefore Z lies on the perpendicular bisector	D
6.	C
7. As P(x) is a continuous function statement 1 must be true. Statement 11 is true for the multiple root, however not all stationary points are necessarily on the x-axis.	A
8. $y = \frac{2x^3}{x^2-1}$ $= 2x - \frac{2x}{x^2-1}$ $Asymptotes: x=1, x=-1, y=2x$	C

9. $If\ \ \omega\ is\ a\ complex\ root\ \omega^3=1$ $\therefore\left(\omega^2\right)^3=\left(\omega^3\right)^2=1\ is\ also\ a\,root$ $Sum\,of\ roots=0$ $\therefore 1+\omega+\omega^2=0$ $\therefore\ \omega+\omega^2=-1$ $\therefore C\ is\ not\ correct\ as$ $\omega^6-\omega=\omega^2$ $\omega+\omega^2=\omega^6$ $=1$ $\neq -1$	C
10 $w=1.5cis\theta$ $\dfrac{1}{w}=\dfrac{1}{1.5}cis\left(-\theta\right)$ $\therefore R$	B
$11a(i)\quad iz=i\left(4-i\right)$ $=4i+1$ $\overline{iz}=1-4i$	1 correct solution
$(ii)\quad zw=\left(4-i\right)\left(2+2i\right)$ $=8+8i-2i+2$ $=10+6i$	1 correct solution
$(iii)\quad \dfrac{z}{w}=\dfrac{4-i}{2+2i}\times\dfrac{2-2i}{2-2i}$ $=\dfrac{8-8i-2i-2}{4+4}$ $=\dfrac{6-12i}{8}$ $=\dfrac{3-6i}{4}$	2 correct solution 1 multiplying by conjugate or equivalent
$b)i)\quad w^2=\left(-1+i\sqrt{3}\right)^2$ $=1-2i\sqrt{3}-3$ $=-2-2i\sqrt{3}$ $2\overline{w}=2\left(-1-i\sqrt{3}\right)$ $=-2-2i\sqrt{3}$ $=w^2$	2 Correct solution 1 correctly finding LHS or RHS
$ii)\quad \left\|w\right\|=2\qquad \arg(w)=\dfrac{2\pi}{3}$ Draw a diagram	2 both correct 1 one correct

$iii)\ w^3-8=\left(2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)\right)^3-8$ $=\left(2^3\left(\cos\frac{6\pi}{3}+i\sin\frac{6\pi}{3}\right)\right)-8$ $=8(1+0i)-8$ $=0$ So w is a root of the equation	1 Correct show
$c)\quad P(x)=(x-1)(x-3)Q(x)+R(x)$ $=(x-1)(x-3)Q(x)+ax+b \qquad as\ \deg R(x)<2$ $P(1)=6$ $6=(1-1)(1-3)Q(1)+a+b$ $a+b=6$ $P(3)=2$ $2=(3-1)(3-3)Q(3)+3a+b$ $3a+b=2$ $a+b=6$ $2a=-4$ $a=-2,\quad b=8$ $R(x)=-2x+8$	2 correct solution 1 correct attempt at finding a and b
$d)\quad P=Z=1+i$ $Q=i(1+i)$ $=-1+i$ $R=-(1+i)$ $=-1-i$ $T=\frac{1}{1+i}\times\frac{1-i}{1-i}$ $=\frac{1-i}{2}$ $=\frac{1}{2}-\frac{i}{2}$	2 correct solution 1 two correct solutions

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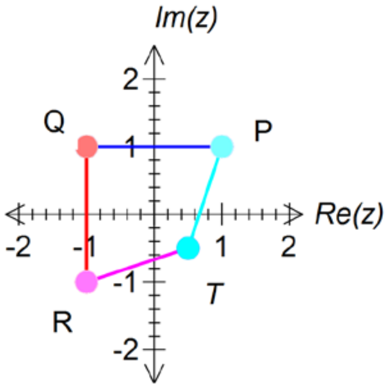
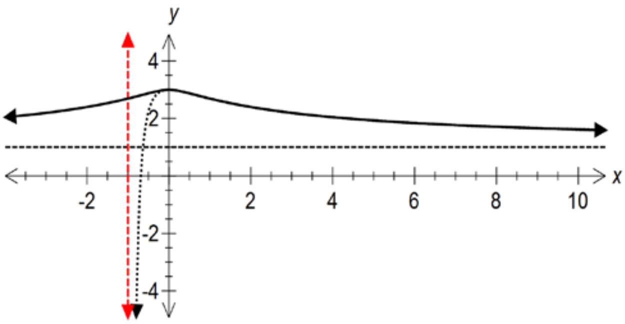
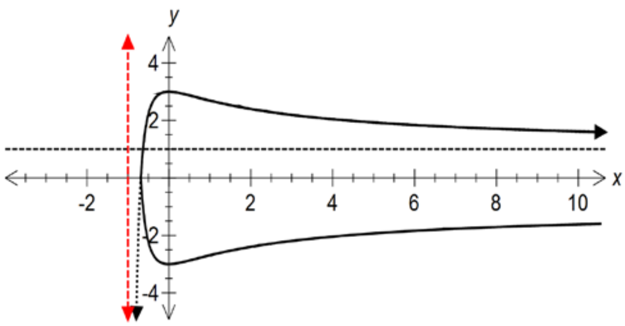
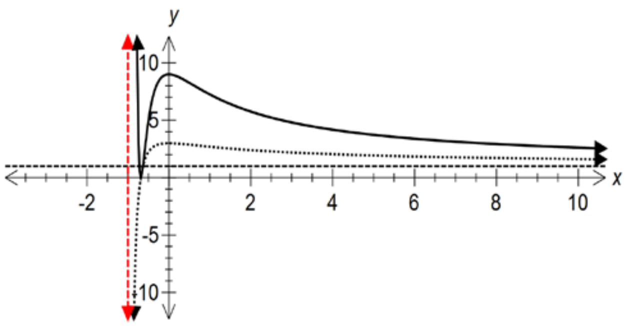
<tr><td>d) ii) PQRT is a kite

</td><td>2 correct solution

1 correct attempt</td></tr>
<tr><td>Question 12
a) i)


It reflects across the y axis</td><td>1 correct solution</td></tr>
<tr><td>

It reflects above the x axis to below the x axis</td><td>1 correct solution</td></tr>
<tr><td>
</td><td>2 correct graph

1 correct attempt showing most features</td></tr>
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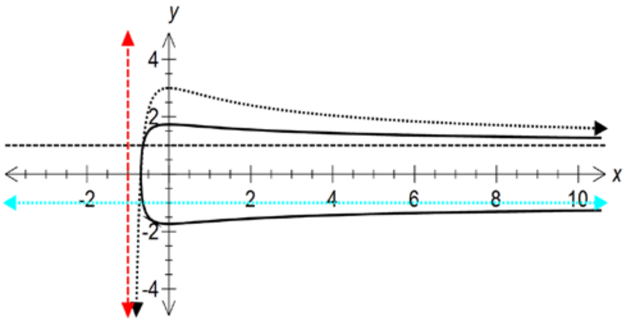

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<td>2 correct solution

1. attempt to find square root or reflect with correct domain</td>
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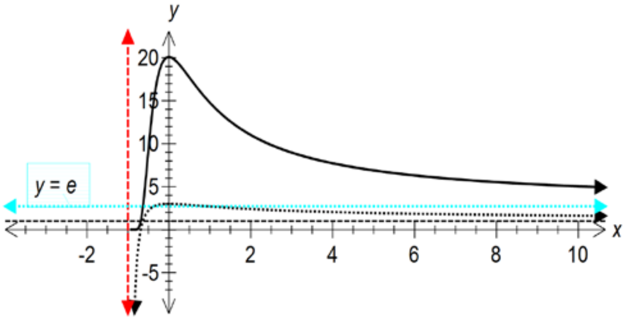

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<td>2 correct solution

1 correct attempt showing some features</td>
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b) i) There are 5! Ways to organise the vowels only one of the arrangements is in alphabetical order.

$$\therefore P(\text{ vowels are in alphabetical order}) = \frac{1}{5!} = \frac{1}{120}$$

</td>
<td>1 correct solution</td>
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ii) If no two vowels are next to each other order must be VCVCVCVCV
Order the vowels 5! Ways
Order the consonants 4! Ways
No restriction 9! Ways

$$\therefore P(\text{ no vowels next to each other}) = \frac{4!5!}{9!} = \frac{1}{126}$$

</td>
<td>2 correct solution

1 correct attempt</td>
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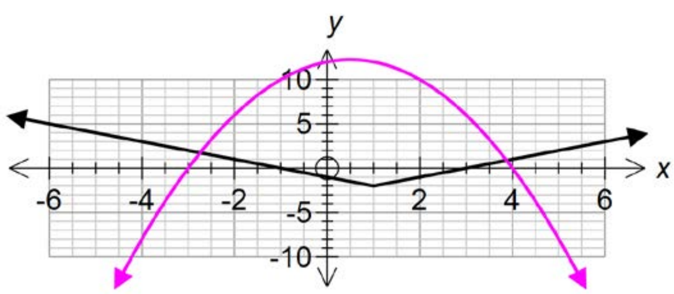
c) $\dfrac{|x-1|-2}{12+x-x^2} \geq 0$ $\boxed{x \neq 4, -3}$

Sketch

y=$|x-1|-2$ and y=$12+x-x^2$



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$\frac{|x-1|-2}{12+x-x^2}$ will be ≥ 0 when both graphs are positive or both graphs are negative. From the graph both are positive for:
$-3 < x \leq -1$ or $3 \leq x < 4$

Algebraically it is easier to use critical points method

Numerator =0

$$|x-1|-2=0$$
$$|x-1|=2$$
$$x-1=\pm 2$$
$$x=-1,3$$

denominator=0

$$12+x-x^2=0$$
$$(4-x)(3+x)=0$$
$$x=4,-3$$

Need to test each region
For *x<-3 test x=-4*
For *-3<x<-1* *test x=-2*
For -1<x<3 test x=0
For 3<x<4 test x=3.5
For x>4 test x=5

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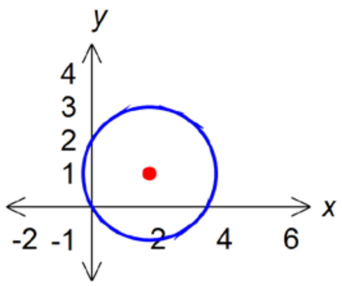
13 a)

Z lies on a circle centre $(\sqrt{3},1)$ and radius 2.



</td><td>1 correct sketch</td></tr>
<tr><td>

a)ii) maximum value of |z| will be when z lies on diameter of the circle that passes through (0,0)

|z|=4

</td><td>1 correct solution</td></tr>
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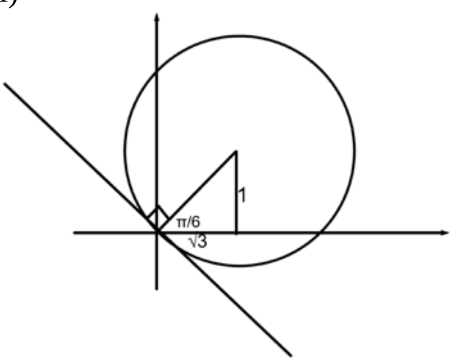
iii)

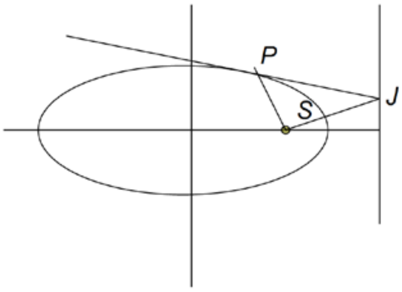


$$\frac{\pi}{6}-\frac{\pi}{2}<\arg z<\frac{\pi}{6}+\frac{\pi}{2}=$$
$$-\frac{\pi}{3}<\arg z<\frac{2\pi}{3}$$

</td><td>2 correct solution

1 correct attempt</td></tr>
</table>

$b)i)\quad e^2=\dfrac{a^2-b^2}{a^2}$ $=\dfrac{25-9}{25}$ $e=\dfrac{4}{5}$	1 correct solution
$ii)\quad Foci:(\pm ae,0)=(\pm 4,0)$ $Diretrices: x=\pm\dfrac{a}{e}$ $=\dfrac{\pm 25}{4}$	2 correct solution 1 either foci or directrices correct
$iii)\quad \dfrac{x^2}{25}+\dfrac{y^2}{9}=1$ Differentiate implicitly: $\dfrac{2x}{25}+\dfrac{2y}{9}.\dfrac{dy}{dx}=0$ $\dfrac{dy}{dx}=\dfrac{-9x}{25y}$ $At\ P(x_1,y_1),\quad \dfrac{dy}{dx}=\dfrac{-9x_1}{25y_1}$ Equation of the tangent : $y-y_1=\dfrac{-9x_1}{25y_1}(x-x_1)$ $25yy_1-25y_1^{\ 2}=-9xx_1+9x_1^{\ 2}$ $9xx_1+25yy_1=9x_1^{\ 2}+25y_1^{\ 2}$ $\dfrac{xx_1}{25}+\dfrac{yy_1}{9}=\dfrac{x_1^{\ 2}}{25}+\dfrac{y_1^{\ 2}}{9}$ $\dfrac{xx_1}{25}+\dfrac{yy_1}{9}=1$ as P is a point on the ellipse.	
iv)	



$$At \;\; J \quad x=\frac{a}{e}=\frac{25}{4}$$

$$\frac{\left(\frac{25}{4}\right)x_1}{25}+\frac{yy_1}{9}=1$$

$$\frac{x_1}{4}+\frac{yy_1}{9}=1$$

$$\frac{yy_1}{9}=1-\frac{x_1}{4}$$

$$y=\frac{9}{y_1}\left(\frac{4-x_1}{4}\right)$$

$$J\left(\frac{25}{4},\frac{9}{y_1}\left(\frac{4-x_1}{4}\right)\right)$$

Gradient of $PS\, m=\dfrac{y_1-0}{x_1-4}$

Gradient JS

$$m=\frac{\frac{9}{4y_1}\left(4-x_1\right)-0}{\frac{25}{4}-4}\times\frac{4y_1}{4y_1}$$

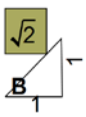
$$=\frac{9\left(4-x_1\right)}{\left(25-16\right)y_1}$$

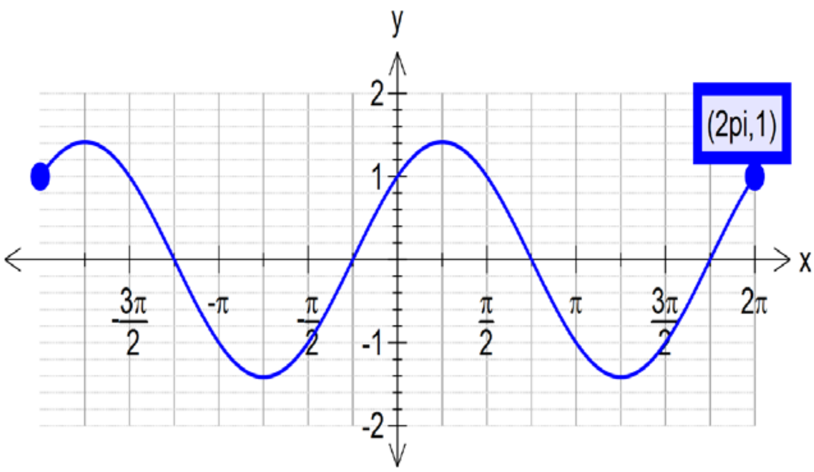
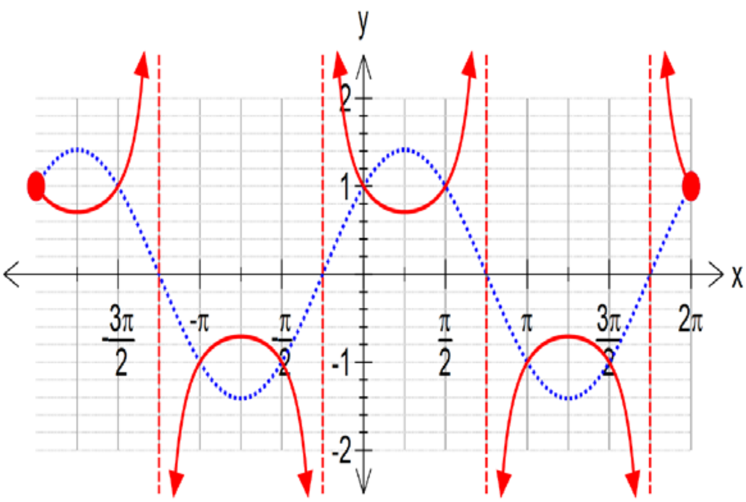
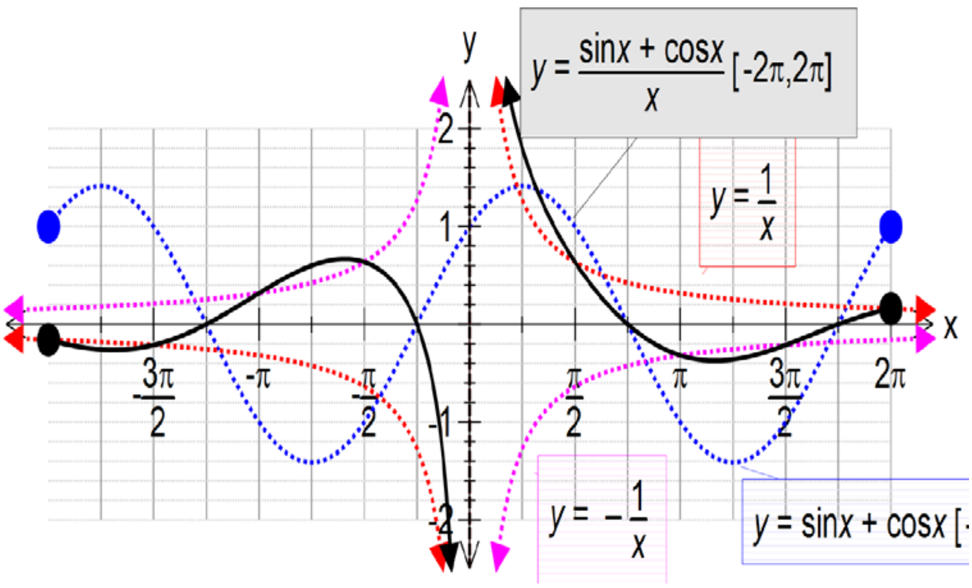
$$=\frac{\left(4-x_1\right)}{y_1}$$

$$Gradient\ PS\times GradientJS=\frac{y_1}{\left(x_1-4\right)}\times\frac{\left(4-x_1\right)}{y_1}$$

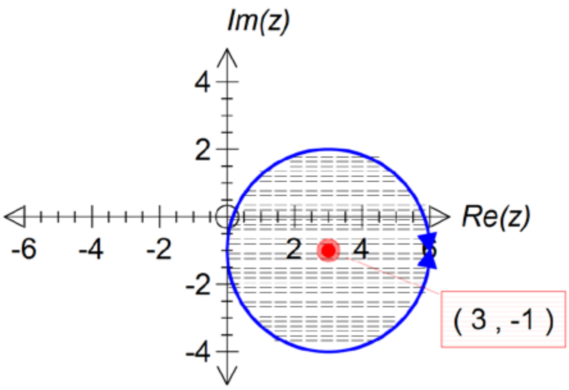
$$=-1$$

$$\therefore \angle PSJ=90^0$$

$c)\quad Let\ P(x)=x^4-5x^3-9x^2+81x-108$ $P'(x)=4x^3-15x^2-18x+81$ $P''(x)=12x^2-30x-18$ if P(x) has a triple root then that root solves $P9x)=P'(x)=P''(x)=0$ $P''(x)=0$ $12x^2-30x-18=0$ $2x^2-5x-3=0$ $(2x+1)(x-3)=0$ $x=3,\frac{-1}{2}$ $P'(3)=P(3)=0$ (should show but too lazy to type in). $\therefore x=3$ is the triple root. Now sum of roots $=\frac{-b}{a}$ $3+3+3+\alpha=5$ $\alpha=-4$ $\therefore$ Solution to $x^4-5x^3-9x^2+81x-108=0$ is $x=3,3,3$ and -4	
Question 14	
$a)\quad \frac{x-8}{(x^2+2)(x-2)}=\frac{Ax+B}{(x^2+2)}+\frac{C}{x-2}$ $x-8=(Ax+B)(x-2)+C(x^2+2)$ When $x=2$ $\quad -6=O+C(4+2)$ $C=-1$ when $x=0$ $\quad -8=B(-2)-1(2)$ $B=3$ When $x=1$ $\quad 1-8=(A+3)(-1)-1(3)$ $-7=-A-3-3$ $A=1$ $\therefore \frac{x-8}{(x^2+2)(x-2)}=\frac{x+3}{(x^2+2)}-\frac{1}{x-2}$	3 correct solution 2 correct attempt minor error 1 finding A, B or C
$b)i)\quad \sin x+\cos x=A\sin(x+B)$ $=A\sin x\cos B+A\cos x\sin B$ $\therefore A\cos B=1,\quad A\sin B=1$  $\therefore A=\sqrt{2},\quad B=\frac{\pi}{4}$ $\sin x+\cos x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$	2 correct solution 1 correctly finding A or B

	2 correct graph showing all major features 1 correct attempt to graph showing some features
	2 correct graph showing all major features 1 correct attempt to graph showing some features
iv) 	2 correct graph showing all major features 1 correct attempt to graph showing some features

$c)\ \ \alpha$ is a root of $x^3-3x^2+9=0$ $\therefore \alpha^3-3\alpha^2+9=0$ $Let\ \ \alpha=\sqrt{X}\qquad \therefore X=\alpha^2$ $Then\ \left(\sqrt{X}\right)^3-3\left(\sqrt{X}\right)^2+9=0$ $X\sqrt{X}-3X+9=0$ $X\sqrt{X}=3X-9$ $\left(X\sqrt{X}\right)^2=(3X-9)^2$ $X^2.X=9X^2-54X+81$ $X^3-9X^2+54X-81=0$ $\therefore \alpha^2,\beta^2,\gamma^2$ are the roots of $x^3-9x^2+54x-81=0$	2 correct equation 1 correct approach
$ii)\ \ \alpha^2,\beta^2,\gamma^2$ are the roots of $x^3-9x^2+54x-81=0$ $\therefore \alpha^2+\beta^2+\gamma^2=\frac{-b}{a}$ $=9$ α,β,γ are the roots of $x^3-3x^2+9=0$ $\therefore \alpha^3-3\alpha^2+9=0$ $\beta^3-3\beta^2+9=0$ $\underline{\gamma^3-3\gamma^2+9=0}$ $\left(\alpha^3+\beta^3+\gamma^3\right)-3\left(\alpha^2+\beta^2+\gamma^2\right)+27=0$ $\left(\alpha^3+\beta^3+\gamma^3\right)-3(9)+27=0$ $\alpha^3+\beta^3+\gamma^3=0$	2 correct solution 1 finding $\alpha^2+\beta^2+\gamma^2$ and attempting the hence
Question 15	
$a)\ \ x^3+3y^2=6xy-3x$ *Differentiate implicitly* $3x^2+6y\frac{dy}{dx}=6x\frac{dy}{dx}+6y-3$ $(6x-6y)\frac{dy}{dx}=3x^2-6y+3$ $\frac{dy}{dx}=\frac{3x^2-6y+3}{6x-6y}$ $=\frac{x^2-2y+1}{2x-2y}$	2 correct solution 1 correct attempt

$b)\ \ (z-3+i)(\overline{z}-3-i)\le 9$ Let $z=x+iy$ $((x-3)+(y+1)i)((x-3)-i(y+1))\le 9$ Difference of two squares $(x-3)^2+(y+1)^2\le 9$ Interior of a circle centre (3,-1) and radius 3 	2 correct graph 1 correct attempt to find locus
$c)i)\ \ \angle PNB=\angle PLC=90^0$ $\therefore$ The exterior angle is equal to the interior opposite angle $\therefore$ $PNCL$ is a cyclic quadrilateral	1 correct reason
$ii)$ In $PBCA$ $\angle PBM=\angle PCA$ (angles in the same segment are equal) $In\ \ PNCL$ $\angle PCL=\angle PNL$(angles in the same segment are equal) $\therefore \angle PBM=\angle PNM$ (both equal to $\angle PCA$)	2 correct show 1
$iii)$ Construct PM $\angle PBN=\angle PNM$ (from ii) $\therefore$ PMNB is a cyclic quadrilateral as B and N lie on the same side of the interval PB and the angles subtended at these two points by the interval are equal, then the two points and the endpoints of the interval are concyclic. $\therefore$ ∠PMN=∠PNB $=90^0$ (angles in the same segment are equal) $\therefore$ PM is perpendicular to AB.	3 correct show 2 correct show with minor error 1 correct approach

$d)i)$ At the point of intersection of the curve and the line $y = x^4 + 4x^3$ *and* $y = mx + b$ *Then* $x^4 + 4x^3 = mx + b$ $\therefore x^4 + 4x^3 - mx - b = 0$ *As* $y = mx + b$ is a tangent at both $x = \alpha$ and $x = \beta$ then both α and β must be double roots. Thus the zeroes of $x^4 + 4x^3 - mx - b = 0$ are α, α, β and β.	1 explantion
$ii)$ $x^4 + 4x^3 - mx - b = 0$ has roots $\alpha, \alpha, \beta, \beta$ Sum of roots $= \frac{-b}{a}$ $\alpha + \alpha + \beta + \beta = -4$ $\alpha + \beta = -2$ Sum of roots two at a time= $\frac{c}{a}$ $\alpha^2 + \alpha\beta + \alpha\beta + \alpha\beta + \alpha\beta + \beta^2 = 0$ $\left(\alpha^2 + 2\alpha\beta + \beta^2\right) + 2\alpha\beta = 0$ $\left(\alpha + \beta\right)^2 + 2\alpha\beta = 0$ $2\alpha\beta = -4$ $\alpha\beta = -2$ Now product of the roots $= \frac{e}{a}$ $\alpha\alpha\beta\beta = -b$ $\left(\alpha\beta\right)^2 = -b$ $b = -4$ Summing three at a time $= \frac{-d}{a}$ $\alpha^2\beta + \alpha^2\beta + \alpha\beta^2 + \alpha\beta^2 = m$ $\alpha\beta\left(2\alpha + 2\beta\right) = m$ $\therefore m = -2\left(2 \times -2\right)$ $m = 8$ $\therefore m = 8$ and $b = -4$	
Question 16	
$a)i)$ *If* ω is a complex root of $x^3 = 1$ then $\omega^3 = 1$ *Now* $\left(\omega^2\right)^3 = \left(\omega^3\right)^2 = 1$ $\therefore \omega^2$ is also the other complex root of $x^3 = 1$	1 show

$ii)$ The complex roots of $x^3-1=0$ are ω and ω^2. The real root is 1. The sum of the roots $=\dfrac{-Coefficient\ of\ x^2}{Coefficient\ of\ x^3}$ $\therefore 1+\omega+\omega^2=0$	1 show
$iii)\quad For\ \ 3, 2\omega+\omega^2, 2\omega^2+\omega$ Sum of roots 1 at a time = $3+2\omega+\omega^2+2\omega^2+\omega$ $=3\left(1+\omega+\omega^2\right)$ $=0$ *Sum* of roots 2 at a time= $3\left(2\omega+\omega^2\right)+3\left(2\omega^2+\omega\right)+\left(2\omega+\omega^2\right)\left(2\omega^2+\omega\right)$ $=6\omega+3\omega^2+6\omega^2+3\omega+4\omega^3+2\omega^2+2\omega^4+\omega^3$ $=6\omega+3\omega^2+6\omega^2+3\omega+4+2\omega^2+2\omega+1$ *as* $=11\left(\omega+\omega^2\right)+5$ $=-11+5 \qquad As\ \ 1+\omega+\omega^2=0$ $=-6$ Sum of roots 3 at a time = $3\left(2\omega+\omega^2\right)\left(2\omega^2+\omega\right)$ $=3\left(4\omega^3+2\omega^2+2\omega^4+\omega^3\right)$ $=3\left(4+2(\omega^2+\omega)+1\right)$ $=3(4-2+1)$ $=9$ $\therefore 3, 2\omega+\omega^2, 2\omega^2+\omega$ solve the cubic equation $x^3-0x^2-6x-9=0$	3 correct equation 2 1
$b)i)\quad OP^2+OQ^2=\left(a\cos\theta\right)^2+\left(b\sin\theta\right)^2+\left(-a\sin\theta\right)^2+\left(b\cos\theta\right)^2$ $=a^2\cos^2\theta+b^2\sin^2\theta+a^2\sin^2\theta+b^2\cos^2\theta$ $=a^2\left(\cos^2\theta+\sin^2\theta\right)+b^2\left(\cos^2\theta+\sin^2\theta\right)$ $=a^2+b^2$	1 show

At T $\dfrac{x\cos\theta}{a}+\dfrac{y\sin\theta}{b}=1 \qquad \times\cos\theta$ $\dfrac{-x\sin\theta}{a}+\dfrac{y\cos\theta}{b}=1 \qquad \times\sin\theta$ $\therefore \dfrac{x\cos^2\theta}{a}+\dfrac{y\sin\theta\cos\theta}{b}=\cos\theta$ $\dfrac{-x\sin^2\theta}{a}+\dfrac{y\cos\theta\sin\theta}{b}=\sin\theta$ $\therefore \dfrac{x\cos^2\theta}{a}+\dfrac{x\sin^2\theta}{a}=\cos\theta-\sin\theta$ $\dfrac{x}{a}\left(\cos^2\theta+\sin^2\theta\right)=\cos\theta-\sin\theta$ $x=a\left(\cos\theta-\sin\theta\right)$ $\therefore AT\ T \quad \dfrac{x\cos\theta}{a}+\dfrac{y\sin\theta}{b}=1$ $\dfrac{\cancel{a}\left(\cos\theta-\sin\theta\right)\cos\theta}{\cancel{a}}+\dfrac{y\sin\theta}{b}=1$ $\dfrac{y\sin\theta}{b}=1-\cos^2\theta+\sin\theta\cos\theta$ $\dfrac{y\sin\theta}{b}=\sin^2\theta+\sin\theta\cos\theta$ $\dfrac{y\sin\theta}{b}=\sin\theta(\sin\theta+\cos\theta)$ $y=b\ (\sin\theta+\cos\theta)$ $\therefore T\left(a\left(\cos\theta-\sin\theta\right),b\ (\sin\theta+\cos\theta)\right)$	

$iii)$At T $x = a\left(\cos\theta - \sin\theta\right)$ $y = b\left(\cos\theta + \sin\theta\right)$ $\therefore \left(\cos\theta - \sin\theta\right)^2 = \left(\frac{x}{a}\right)^2$ $\left(\cos\theta + \sin\theta\right)^2 = \left(\frac{y}{b}\right)^2$ $\therefore \cos^2\theta - 2\sin\theta\cos\theta + \sin^2\theta = \frac{x^2}{a^2}$ $\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = \frac{y^2}{b^2}$ $\therefore 1 - 2\sin\theta\cos\theta = \frac{x^2}{a^2}$ $1 + 2\sin\theta\cos\theta = \frac{y^2}{b^2}$ $\therefore 2 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	

iv) Tangent at P $\quad \dfrac{x\cos\theta}{a}+\dfrac{y\sin\theta}{b}=1$

$$\frac{y\sin\theta}{b}=1-\frac{x\cos\theta}{a}$$

$$y=\frac{b}{\sin\theta}\left(1-\frac{x\cos\theta}{a}\right)$$

$$m_P=\frac{-b\cos\theta}{a\sin\theta}$$

Tangent at Q $\quad \dfrac{-x\sin\theta}{a}+\dfrac{y\cos\theta}{b}=1$

$$\frac{y\cos\theta}{b}=1+\frac{x\sin\theta}{a}$$

$$y=\frac{b}{\cos\theta}\left(1+\frac{x\sin\theta}{a}\right)$$

$$m_Q=\frac{b\sin\theta}{a\cos\theta}$$

$$\tan\alpha=\frac{m_Q-m_P}{1+m_Qm_P}$$

$$=\frac{\dfrac{b\sin\theta}{a\cos\theta}-\dfrac{-b\cos\theta}{a\sin\theta}}{1+\dfrac{b\sin\theta}{a\cos\theta}\cdot\dfrac{-b\cos\theta}{a\sin\theta}}$$

$$=\frac{\dfrac{b\sin^2\theta+b\cos^2\theta}{a\cos\theta\sin\theta}}{1-\dfrac{b^2}{a^2}}$$

$$=\frac{b}{a\cos\theta\sin\theta}\cdot\frac{a^2}{a^2-b^2}$$

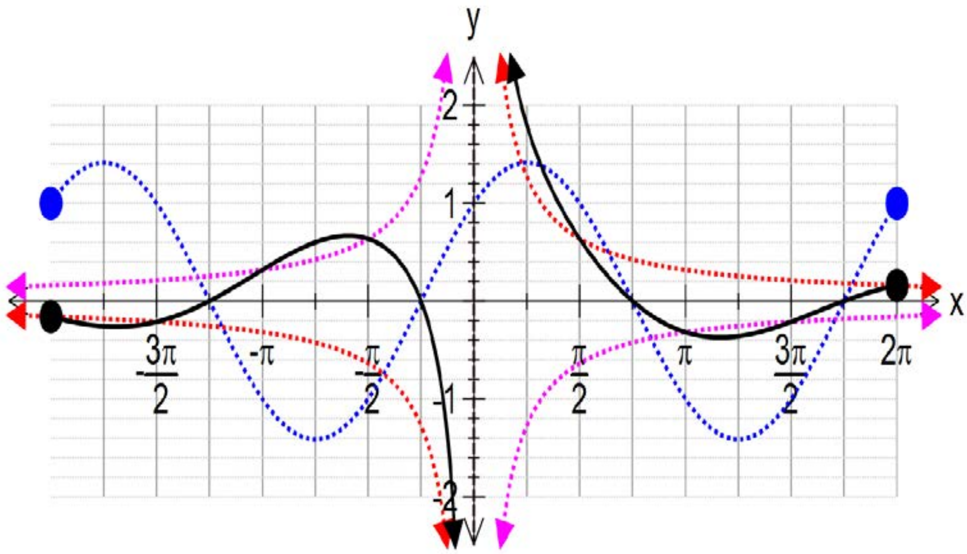
$$=\frac{2b}{a\sin 2\theta}\cdot\frac{a^2}{a^2-a^2\left(1-e^2\right)} \qquad as \quad \begin{matrix}\sin 2\theta=2\text{s}\\ b^2=a^2\big(\end{matrix}$$

$$=\frac{2b}{a\sin 2\theta}\cdot\frac{1}{1-1\left(1-e^2\right)}$$

$$=\frac{2b}{a\sin 2\theta}\cdot\frac{1}{e^2}$$

$$=\frac{2a\sqrt{1-e^2}}{ae^2\sin 2\theta} \qquad as\ b^2=a^2\left(1-e^2\right)$$

$$=\frac{2\sqrt{1-e^2}}{e^2\sin 2\theta}$$



y
x
2
1
-1
-2
$-\frac{3\pi}{2}$
$-\pi$
$-\frac{\pi}{2}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π