The Scots College

HSC Mathematics Extension 2

Pre Trial Examination

April 2013

General Instructions

- Working time : 2 hours + 5 minutes reading time.
- Write using blue or black pen
- Board approved calculators may be used (Non Graphic)
- All necessary working should be shown in every question
- Standard Integrals Table attached
- Answer each question on a SEPARATE answer booklet

TOTAL MARKS: 75

SECTION I

7 marks

- Attempt Questions 1-7
- Answer on the Multiple Choice answer sheet provided.
- Allow about 10 minutes for this section.

SECTION II

68 marks

- Attempt questions 8 – 11
- Answer on the booklets provided, unless otherwise instructed. Start a new booklet for each question.
- Allow about 1 hours & 50 minutes for this section

WEIGHTING: 30 %

Section I

7 marks

Attempt Questions 1-7

Allow about 10 minutes for this section

Use the multiple choice answer sheet for questions 1 – 7.

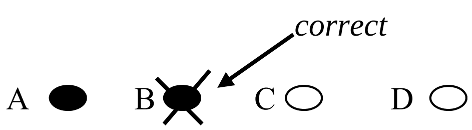
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample

$2 + 4 = ?$ (A) 2 (B) 6 (C) 8 (D) 9

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

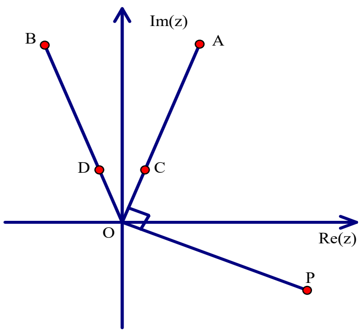
If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:



1 Let $z^2 = \sqrt{3}i$, what is the value of z^{8n+4}?

A) -3^{2n+1} B) $(-3)^{8n+2}$ C) $(-3)^{6n+4}$ D) 3^{2n+1}

2 The point P represents the complex number ω where $\omega\overline{\omega} = 4$.
Which of the points A, B, C, or D shown in the Argand diagram could represent the complex number $i\omega^{-1}$?



3 For the curve with equation $y = [f(x)]^n$:

(A) The x intercept(s) of $y = f(x)$ correspond to the stationary points of the curve.
(B) The curve exhibits point symmetry about the origin.
(C) The curve does not exist for values for which $f(x) < 0$.
(D) More information is needed about the curve to determine further properties.

4 The roots of the equation $x^3 - 3x + 3 = 0$ are $\alpha, \beta\ and\ \gamma$. What is the value of $\alpha^2 + \beta^2 + \gamma^2$.

A) - 6 B) - 3 C) 3 D) 6

5 The Cartesian equation of the locus of complex number z, such that $|z + 3| + |z - 3| = 12$, isj

(A) $\frac{x^2}{36} - \frac{y^2}{27} = 1$
(B) $\frac{x^2}{36} + \frac{y^2}{27} = 1$
(C) $\frac{x^2}{27} - \frac{y^2}{36} = 1$
(D) $\frac{x^2}{27} + \frac{y^2}{36} = 1$

6 The foci of the ellipse $25\ x^2 + 9\ y^2 = 225$ have the coordinates

A) $(\pm 4, 0)$ B) $(\pm\sqrt{34}, 0)$ C) $(0, \pm 4)$ D) $(0, \pm\sqrt{34})$

7 The eccentricity of the hyperbola $x^2 - y^2 = 4$ is

A) 2 B) $\frac{1}{2}$ C) $\frac{1}{\sqrt{2}}$ D) $\sqrt{2}$

End of Section I

Section II

Total Marks (68)
Attempt Questions 8 – 11.
Allow about 1 hours & 50 minutes for this section.

Answer all questions, starting each question on a new answer booklet with your name and question number at the top of the page.
All necessary working should be shown in every question.

Question 8 (Marks 17) Answer on a new booklet.

a) Evaluate i^{2054} [1]

b) Let $z = \frac{9+2i}{2+i}$. [4]

i) Simplify $(9+2i)(\overline{2+\ i})$

ii) Express z in the form $a + ib$, where a and b are real numbers.

iii) Hence, or otherwise, find $|z|$ and $\arg(z)$.

c) If $z = \cos\theta + i\sin\theta$ [2]
Show that $1 + z = 2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)$.

d) Let $z = \cos\theta + i\sin\theta$. [4]
Given that $z^n + \frac{1}{z^n} = 2\cos n\theta$
and that $z^n - \frac{1}{z^n} = 2\,\mathrm{i}\sin n\theta$
Show that $sin^4\theta + cos^4\theta = \frac{1}{4}(\cos 4\theta + 3)$

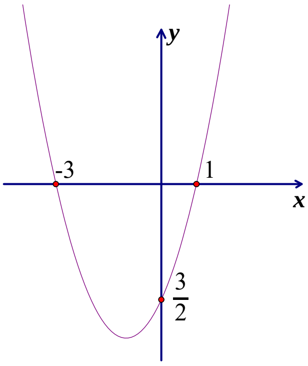
e) Express $(3+2i)(5+4i)$ and $(3-2i)(5-4i)$ in the form $x + iy$. [3]
Hence express $7^2 + 22^2$ as a product of prime factors.

f) On an Argand diagram, shade in the region containing all points representing the complex number z, such that [3]

$$|z - (2+2i)| \le 2 \quad and \quad |z - (2+2i)| \le |z|$$

Question 9 (Marks 17) Answer on a new booklet.

a) [7]



The sketch above shows the parabola
$y = f(x)$ where $f(x) = \frac{1}{2}(x^2 + 2x - 3)$

Without the use of calculus, draw separate sketches of the following, showing all important features such as intercepts, asymptotes and turning points.

i) $y^2 = f(x)$

ii) $y = e^{f(x)}$

iii) $y = tan^{-1} f(x)$

iv) $y = \frac{1}{2}(x+3)|x-1|$

b) Consider the graph of the function $y = \frac{x^2}{x+2}$ [7]

i) Find the equations of the asymptotes for the graph.
ii) Find the stationary points and state their nature
iii) Draw a neat sketch of the graph, showing all important features.
iv) Hence, or otherwise, sketch the graph of $y = \frac{|x+2|}{x^2}$

c) Find the coordinates of the stationary points and determine their nature, for the curve [3]
$x^2 + xy - 2y^2 + 9 = 0$

Question 10 (Marks 17) Answer on a new booklet.

a) Consider the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$. Find [7]

i. its eccentricity

ii. the coordinates of the foci

iii. equation of the directrices

iv. equation of the asymptotes

v. the length of the latus rectum

vi. draw a neat sketch of the hyperbola, showing all the important features.

b) The point $P\ \left(4p, \frac{4}{p}\right)$, lies on the hyperbola $xy = 16$ and is in the first quadrant. The normal to the hyperbola at P meets the hyperbola again at the point $Q\ \left(4q, \frac{4}{q}\right)$. [6]

i. Find the equation of the normal at P .

ii. Show that $q = -\frac{1}{p^3}$

iii. Hence show that there is only one value of p for which the normal at P is also a normal to the hyperbola at Q. Find the coordinates of P and Q for this normal.

c) Let $C_1 = x^2 + 4y^2 - 2$ and $C_2 = 3x^2 + y^2 - 1$, and let k be a real number. [4]

i. Show that $C_1 + k\,C_2 = 0$ is the equation of the curve passing through the point of intersection of the ellipses $C_1 = 0$ and $C_2 = 0$.

ii. Find the value of k such that $C_1 + k\,C_2 = 0$ is the equation of an ellipse.

Question 11 (Marks 17) Answer on a new booklet.

a) Prove by Mathematical Induction that [4]

$\frac{(3+\sqrt{5})^n+(3-\sqrt{5})^n}{2^n}$ is a positive integer, for all positive integer values of n.

You may assume that the statement is true for $n = k$ and $n = k + 1$, and that

$$a^{n+1} + b^{n+1} = (a+b)(a^n + b^n) - ab(a^{n-1} + b^{n-1})$$

b) Show that $x^5 - 5x + 1 = 0$ cannot have a double root. [3]

c) Solve the equation [4]

$$8x^4 - 14x^3 - 69x^2 - 14x + 8 = 0$$

[6]

d) (i) Solve the equation

$$x^6 - x^3 + 1 = 0$$

(ii) Hence express $x^6 - x^3 + 1$ as a product of three real quadratic factors.

(iii) Hence show that

$$\cos\frac{\pi}{9} + \cos\frac{7\pi}{9} + \cos\frac{13\pi}{9} = 0 \text{ , and}$$

$$\cos\frac{\pi}{9} \times \cos\frac{7\pi}{9} \times \cos\frac{13\pi}{9} = \frac{1}{8}$$

End of Assessment

Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Pre-Trial HSC Examination 2013
Mathematics Extension 2

Multiple Choice Answer Sheet

Name ______________________________

Completely fill the response oval representing the most correct answer.

1.	A ⬭	B ⬭	C ⬭	D ⬭
2.	A ⬭	B ⬭	C ⬭	D ⬭
3.	A ⬭	B ⬭	C ⬭	D ⬭
4.	A ⬭	B ⬭	C ⬭	D ⬭
5.	A ⬭	B ⬭	C ⬭	D ⬭
6.	A ⬭	B ⬭	C ⬭	D ⬭
7.	A ⬭	B ⬭	C ⬭	D ⬭

Pre-Trial HSC Examination 2013
Mathematics Extension 2

Multiple Choice Answer Sheet

Name SOLUTIONS.

Completely fill the response oval representing the most correct answer.

1.	A ●	B ○	C ○	D ○	
2.	A ○	B ○	C ○	D ●	
3.	A ●	B ○	C ○	D ●	Both answers acceptable
4.	A ○	B ○	C ○	D ●	
5.	A ○	B ●	C ○	D ○	
6.	A ○	B ○	C ●	D ○	
7.	A ○	B ○	C ○	D ●	

Question ______________ Name: ____________________

Teacher: ____________________

YEAR 12 MATHEMATICS EXTENSION 2

PRE-TRIALS SOLUTIONS - 2013

Q1. $z^2 = \sqrt{3}i$

$z^{8n+4} = (z^2)^{4n+2}$

$= (\sqrt{3}i)^{4n+2}$

$= \left[(\sqrt{3}i)^2\right]^{2n+1}$

$= (-3)^{2n+1}$

(A)

2. $i\omega^{-1} = \frac{i}{\omega} = \frac{i\bar{\omega}}{\omega\bar{\omega}} = \frac{i\bar{\omega}}{4}$

(D)

3. (A)

4. $\alpha^2+\beta^2+\gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta+\beta\gamma+\gamma\alpha)$

$= 0 - (-3)$

$= 6$

(D)

Question ______________ Name: ____________________

Teacher: ____________________

Q5 $|z-3|+|z+3| = 12$

$a = 6 \quad a^2 = 36$

$ae = 3, \quad e = \frac{1}{2}$

$b^2 = 36(1-\frac{1}{4})$

$= 27$

B

Q6 $25x^2+9y^2 = 225$

$\frac{x^2}{9}+\frac{y^2}{25} = 1$

$a^2 = 25 \quad b^2 = 9$

$9 = 25(1-e^2)$

$1-e^2 = \frac{9}{25}$

$e^2 = 1-\frac{9}{25}$

$= \frac{16}{25}$

$e = \frac{4}{5} \quad ae = 4$

C

Q7 D

Question ______________ Name: ____________________

Teacher: ____________________

SECTION II

Q 8.

(a) i^{2054}

$= i^{4(513)+2}$

$= i^{2}$

$= -1$

(b) (i) $(9+2i)(\overline{2+i})$

$= (9+2i)(2-i)$

$= 18+2-9i+4i$

$= 20-5i$

(ii) $z = \frac{9+2i}{2+i}$

$= \frac{(9+2i)(2-i)}{(2+i)(2-i)}$

$= \frac{20-5i}{5} = 4-i$

(iii) $|z| = \sqrt{16+1} = \sqrt{17}$

$\arg(z) = \tan^{-1}(-1/4)$

Question ______________ Name: ____________________

Teacher: ____________________

(c) $Z = \cos\theta + i\sin\theta$

$1 + Z = 1 + \cos\theta + i\sin\theta$

$= 2\cos^2\theta/_2 + 2i\sin\theta/_2\cos\theta/_2$

$= 2\cos\theta/_2\left(\cos\theta/_2 + i\sin\theta/_2\right)$

(d) $Z^n + \frac{1}{Z^n} = 2\cos n\theta$

$Z^n - \frac{1}{Z^n} = 2i\sin n\theta$

let $n = 1$

$(2i\sin\theta)^4 = \left(Z - \frac{1}{Z}\right)^4$

$16\sin^4\theta = Z^4 - 4Z^2 + 6 - \frac{4}{Z^2} + \frac{1}{Z^4}$ — (1)

$(2\cos\theta)^4 = \left(Z + \frac{1}{Z}\right)^4$

$16\cos^4\theta = Z^4 + 4Z^2 + 6 + \frac{4}{Z^2} + \frac{1}{Z^4}$

$16(\sin^4\theta + \cos^4\theta) = 2Z^4 + 12 + \frac{2}{Z^4}$

$= 2(2\cos 4\theta) + 12$

$= 4(\cos 4\theta + 3)$

$\therefore \sin^4\theta + \cos^4\theta = \frac{1}{4}(\cos 4\theta + 3)$

Question ____________ Name: ____________________

Teacher: ____________________

(e) $(3+2i)(5+4i) = 15 - 8 + 12i + 10i$

$= 7 + 22i$

$(3-2i)(5-4i) = 15 - 8 - 12i - 10i$

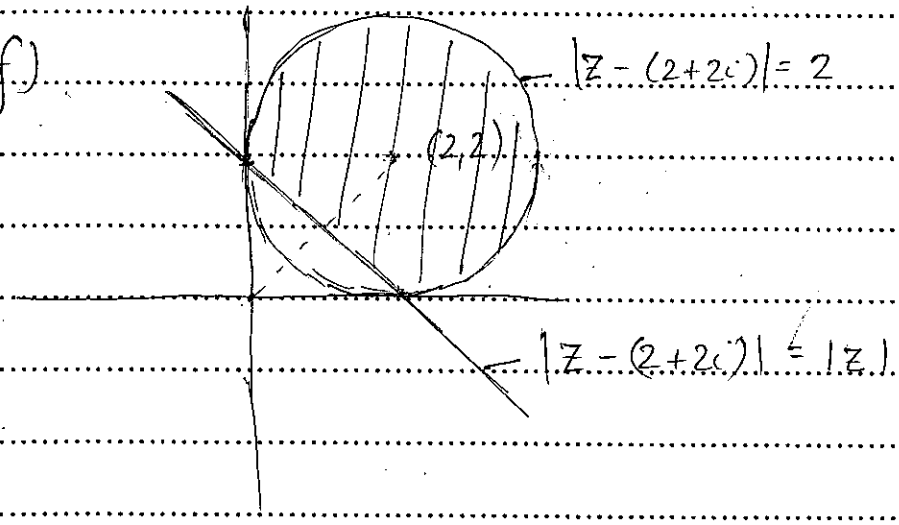
$= 7 - 22i$

$(3+2i)(5+4i)(3-2i)(5-4i) = (7+22i)(7-22i)$

$(9+4)(25+16) = 7^2 + 22^2$

or $7^2 + 22^2 = 13 \times 41$

(f)

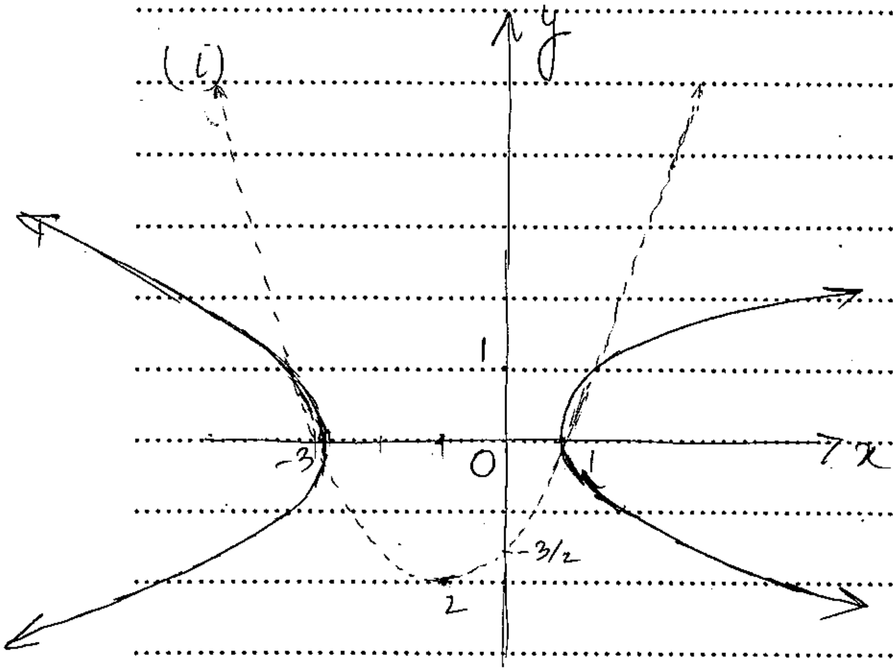


Question ______________ Name: ____________________

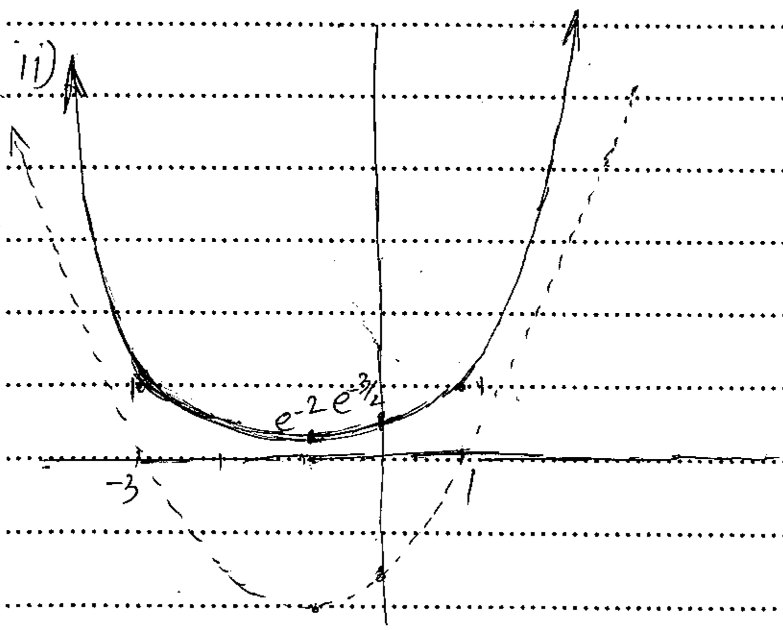
Teacher: ____________________

Q. 9

(i)



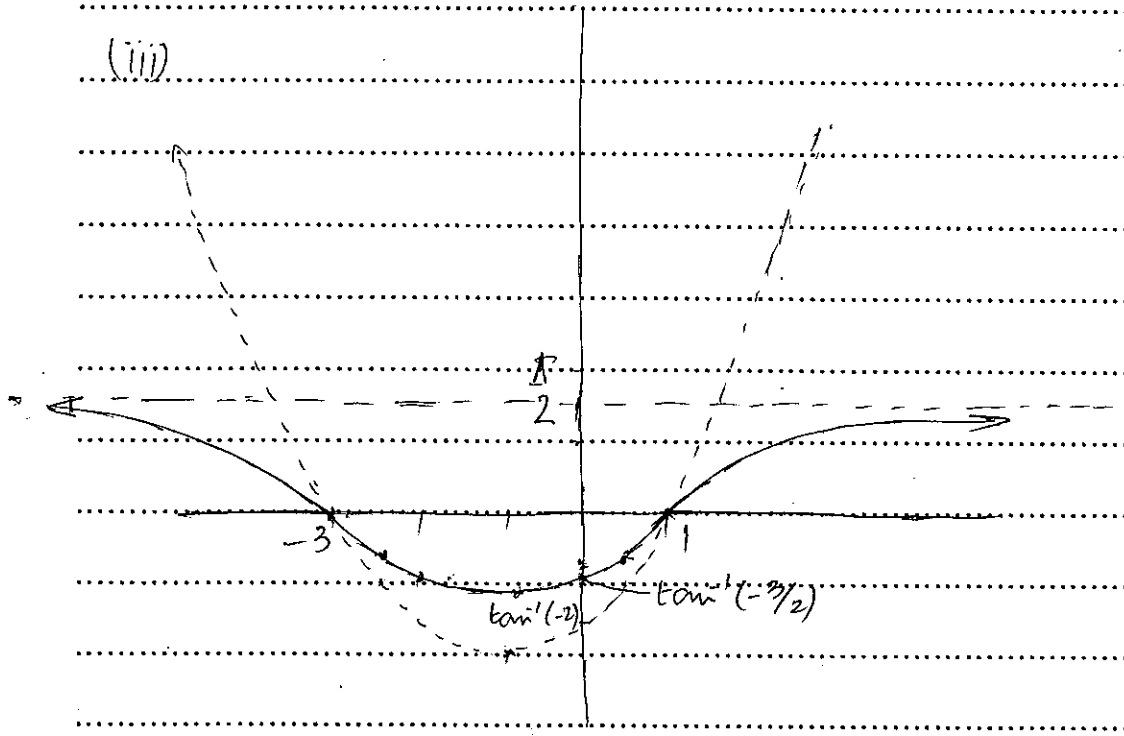
(ii)



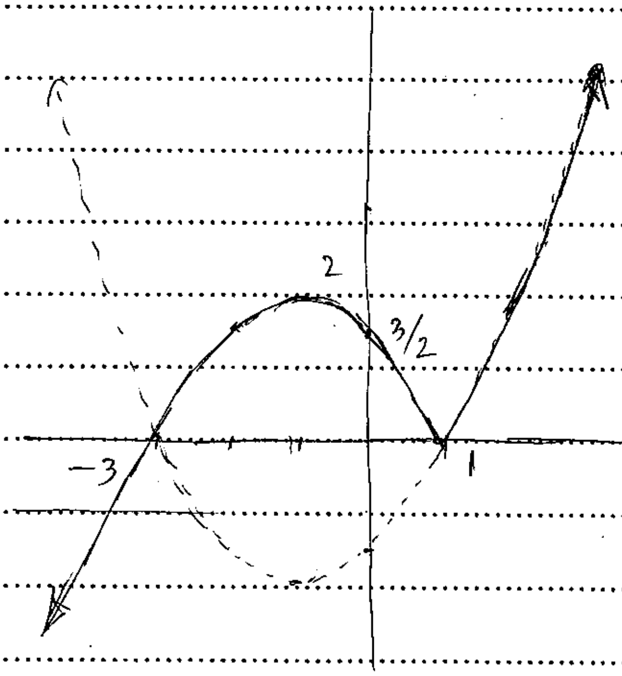
Question ______________ Name: ____________________

Teacher: ____________________

(iii)



(iv)



$y = \frac{1}{2}(x+3)|(x-1)|$

$= \frac{1}{2}(x+3)(x-1)$ for $x \geq 1$

$= -\frac{1}{2}(x+3)(x-1)$ for $x < 1$

Question ______________ Name: ____________________

Teacher: ____________________

9(b)

$$y = x^2/x+2$$

(i) $$y = \frac{x^2 - 4 + 4}{x+2}$$

$$= x - 2 + \frac{4}{x+2}$$

Oblique asymptote at $y = x - 2$

Vertical asymptote $x = -2$

(ii) $$\frac{dy}{dx} = \frac{(x+2)(2x) - x^2(1)}{(x+2)^2}$$

$$= \frac{2x^2 + 4x - x^2}{(x+2)^2}$$

$$= \frac{x^2 + 4x}{(x+2)^2}$$

$$\frac{dy}{dx} = 1 - \frac{4}{(x+2)^2}$$

$$\frac{d^2y}{dx^2} = \frac{8}{(x+2)^3}$$

for stationary points $dy/dx = 0$

$$1 = \frac{4}{(x+2)^2} \quad (x+2)^2 = 4$$

$$x + 2 = \pm 2$$

$$x = 0 \text{ or } -4$$

$$y = 0 \text{ or } -8$$

When $x = 0$, $\frac{d^2y}{dx^2} > 0$

$\therefore (0,0)$ is min pt

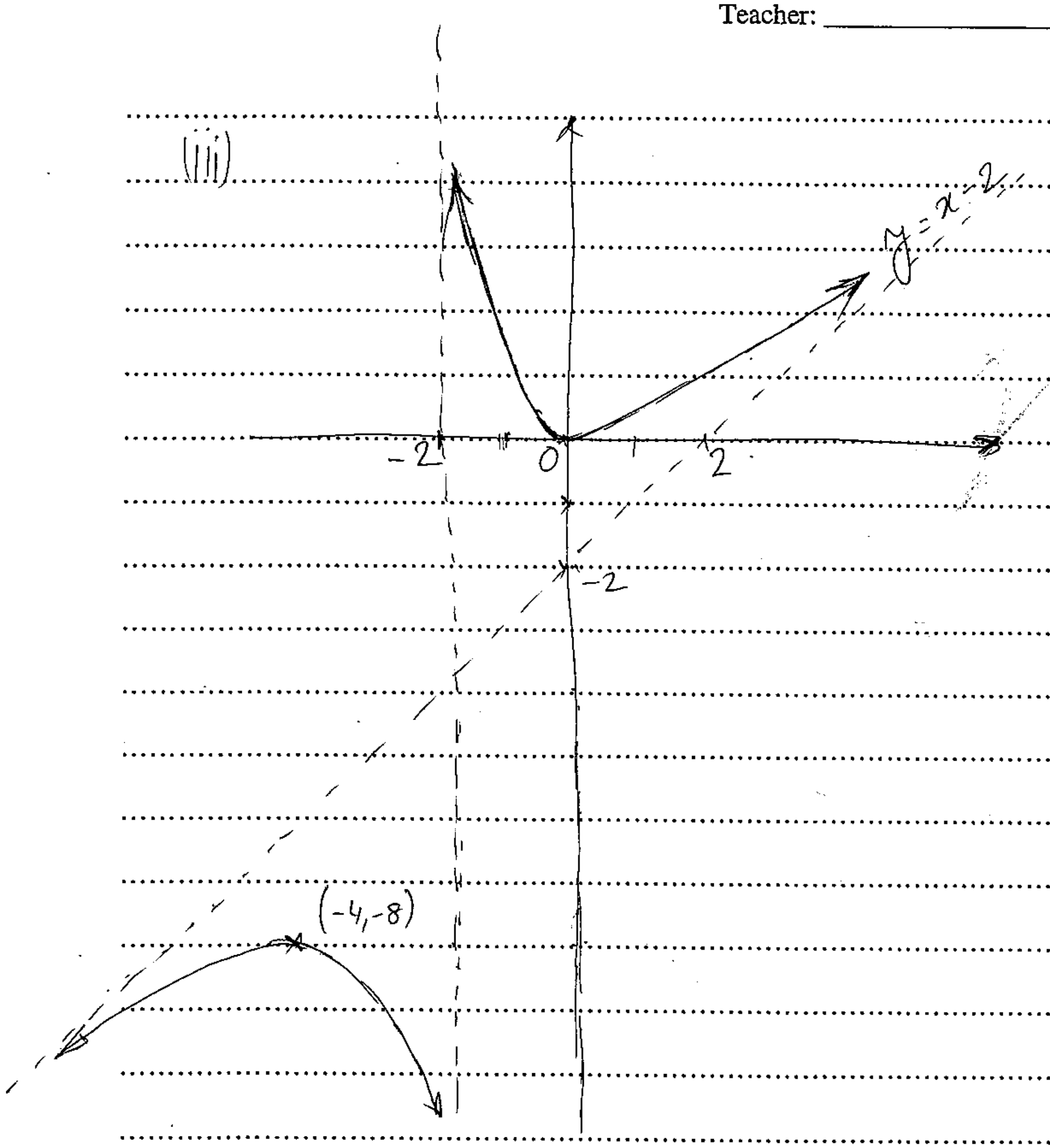
$x = -4$, $\frac{d^2y}{dx^2} < 0$

$\therefore (-4,-8)$ is max pt

Question ______________ Name: ____________________

Teacher: ____________________

(iii)

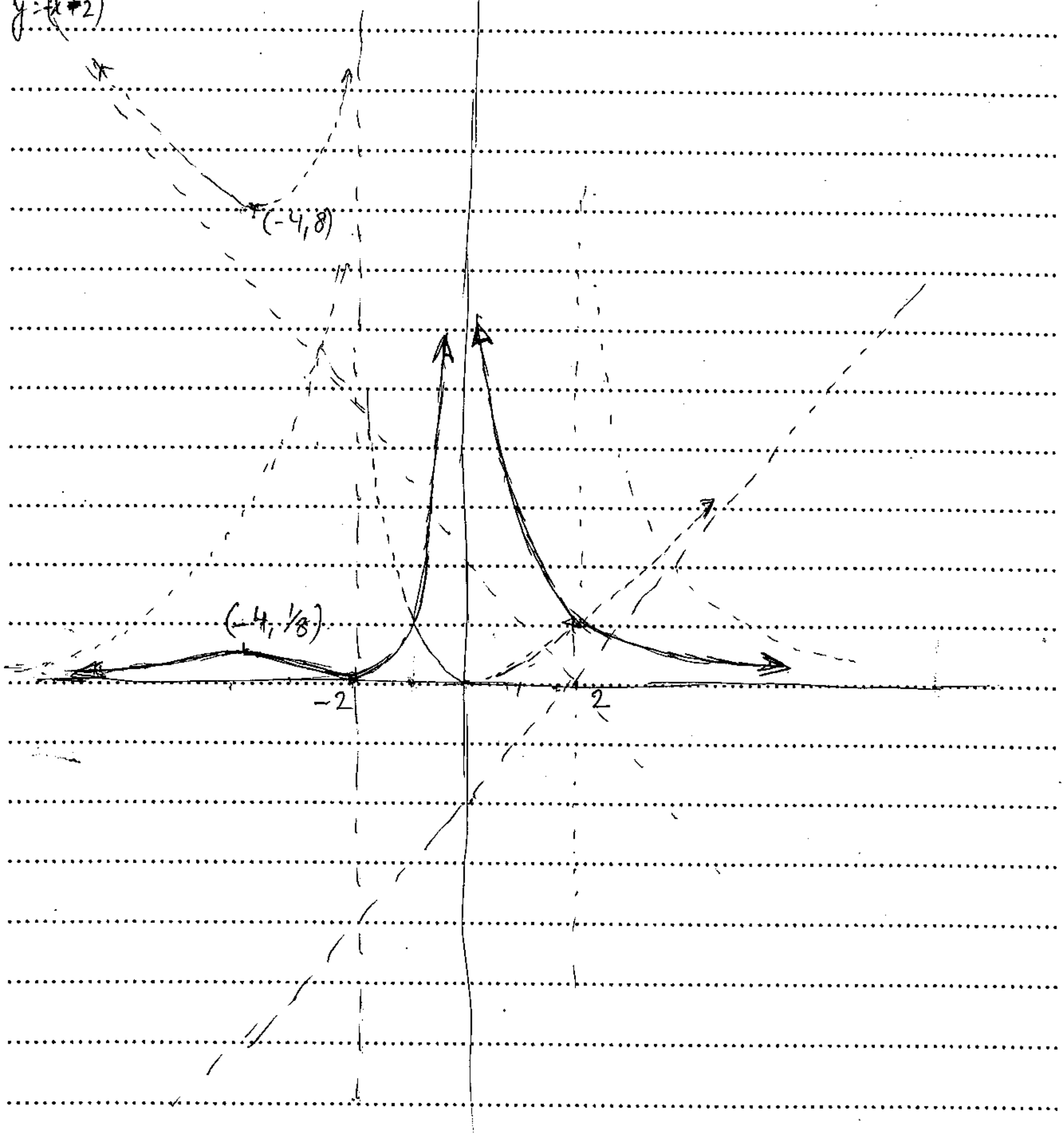


Question ______________ Name: ______________________

Teacher: ______________________

(iv)

y = (x+2)



Question ______________ Name: ____________________

Teacher: ____________________

9(c)

$$x^2 + xy - 2y^2 + 9 = 0$$

Differentiating w.r.t. x

$$2x + y + xy' - 4y.y' = 0$$

$$2x + y = (4y - x)y'$$

$$y' = \frac{2x+y}{4y-x}$$

For st. points $y' = 0$

$$2x + y = 0$$

$$y = -2x$$

Sub into $x^2 + xy - 2y^2 + 9 = 0$

$$x^2 - 2x^2 - 2(4x^2) + 9 = 0$$

$$-9x^2 + 9 = 0$$

$$x^2 = 1$$

$x = 1$, or $x = -1$

$y = -2$ or $y = 2$

$$y'' = \frac{(4y-x)(2+y') - (2x+y)(4y'-1)}{(4y-x)^2}$$

When $x = 1$, $y = -2$, $y' = 0$

$$y'' = \frac{(-8-1)(2+0) - 0}{(-8-1)^2}$$

< 0 $\therefore$ (1,-2) is a max pt

When $x = -1$, $y = 2$, $y' = 0$

$$y'' = \frac{(8+1)(2+0) - 0}{(8+1)^2}$$

> 0

$\therefore$ $(-1, 2)$ is a min. pt.

Question ______________ Name: ____________________

Teacher: ____________________

10

(a) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$a^2 = 16, \quad b^2 = 9$

$9 = 16(e^2 - 1)$

$e^2 - 1 = 9/16$

$e^2 = 1 + 9/16 = 25/16$

(i) $e = 5/4$

(ii) foci : $ae = 4 \times 5/4$

$= 5$

$\therefore$ foci: $(\pm 5, 0)$

(iii) Equation of directrices: $x = \pm a/e$

$x = \pm 16/5$

(iv) Asymptotes

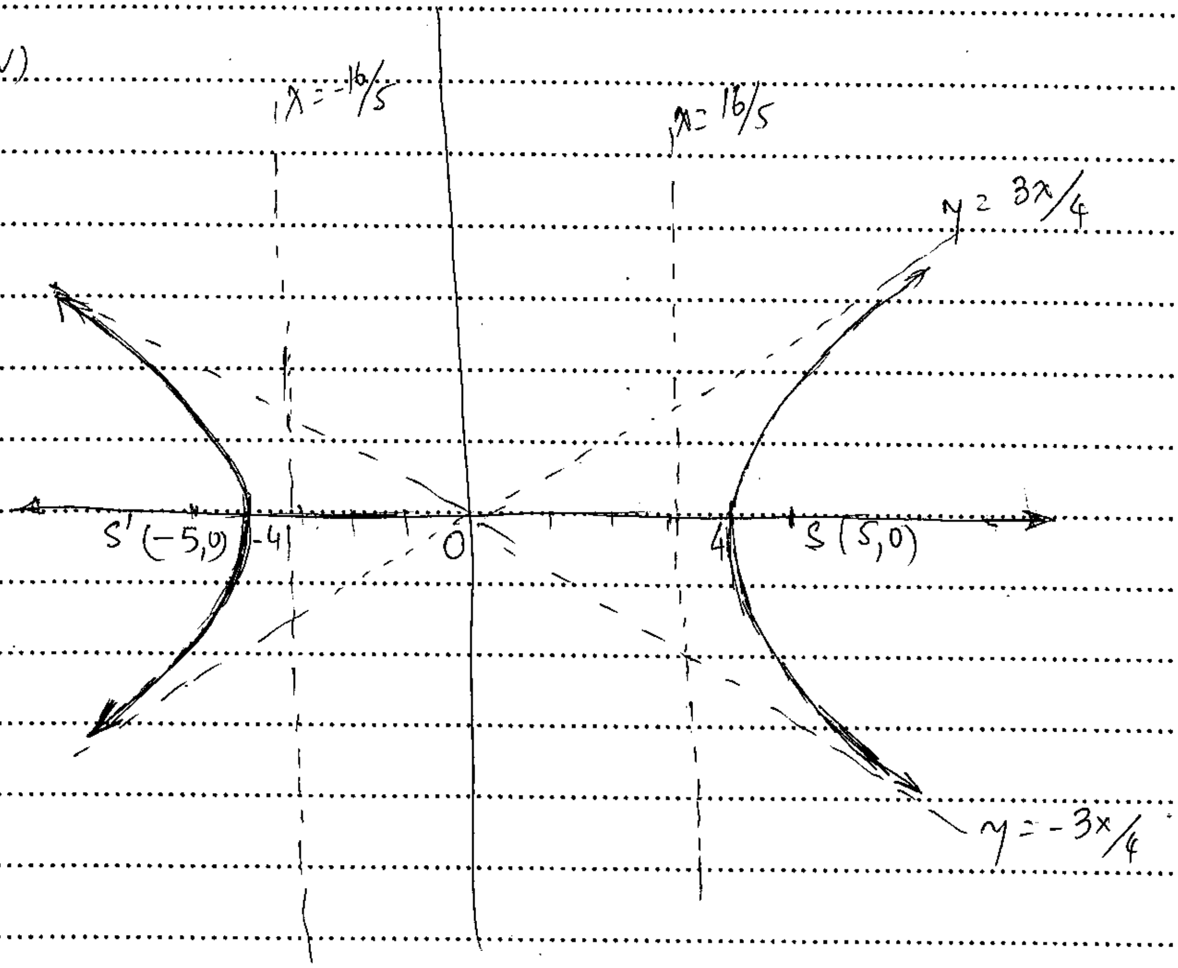
$y = \pm \frac{bx}{a}$

$y = \pm \frac{3x}{4}$

Question ______________ Name: ____________________

Teacher: ____________________

(v)



Question ______________ Name: ____________________

Teacher: ____________________

10 (b)

$xy = 16$

(i) $y = 16/x$

$\frac{dy}{dx} = -16/x^2$

at P, $m_t = -16/16p^2 = -1/p^2$

$m_N = p^2$

Equation of normal is

$y - 4/p = p^2(x - 4p)$

or $py - 4 = p^3(x - 4p)$

or $p^3x - py = 4p^4 - 4$

(ii) Sub $(4q, 4/q)$ into the equation of the normal

$p^3(4q) - \frac{4p}{q} = 4p^4 - 4$

$\cancel{4}p^3q^2 - \cancel{4}p = \cancel{4}p^4q - 4q$

$p^3q^2 - p^4q = p - q$

$p^3q(q - p) = p - q \Rightarrow p^3q = -1 \therefore q = -1/p^3$

Question ______________ Name: ____________________

Teacher: ____________________

(iii) M_N at $Q = q^2 = M_N$ at P.

$\therefore \quad p^2 = q^2$

$= \left(\frac{-1}{p^3}\right)^2$

or $p^2 = \frac{1}{p^6}$

$p^8 = 1$

$p^4 = 1$ or -1 (rej)

$p^2 = 1$ or -1 (rej)

$\underline{\underline{p = 1}}$ or -1 (rej as P is in 1st quadrant)

$\therefore \quad P(4, 4) \quad , \quad Q(-4, -4)$

Question ______________ Name: ____________________

Teacher: ____________________

(c) $C_1 \equiv x^2+4y^2-2$ $C_2 \equiv 3x^2+y^2-1$

(i) The pt of intersection of C_1 and C_2 must satisfy $C_1 = 0$ and $C_2 = 0$

$\therefore$ It will satisfy

$$C_1 + k\,C_2 = 0$$

$\therefore$ $C_1 + kC_2 = 0$ is the required equation

(ii) $x^2+4y^2-2 + k(3x^2+y^2-1) = 0$

or $(1+3k)x^2 + (4+k)y^2 - 2 - k = 0$

or $\dfrac{x^2}{(2+k)/_{1+3k}} + \dfrac{y^2}{\frac{2+k}{4+k}} = 1$

For an ellipse

$\dfrac{2+k}{1+3k} > 0$, $\dfrac{2+k}{4+k} > 0$ and $\dfrac{2+k}{1+3k} \neq \dfrac{2+k}{4+k}$

$(2+k)(1+3k) > 0$ $(2+k)(4+k) > 0$ $1+3k \neq 4+k$

$k < -2$ or $k > -1/3$ $k > -2$ or $k < -4$ $2k \neq 3$

$k \neq 3/2$

$\therefore$ $k > -1/3$, $k < -4$, $k \neq 3/2$

Question ______________ Name: ______________________

Teacher: ____________________

Q 11

(a) $\dfrac{(3+\sqrt{5})^n + (3-\sqrt{5})^n}{2^n}$ is a positive integer

Step 1 prove true for $n=1$ and $n=2$

$\dfrac{(3+\sqrt{5})^1 + (3-\sqrt{5})^1}{2^1} = \dfrac{6}{2} = 3$

$\dfrac{(3+\sqrt{5})^2 + (3-\sqrt{5})^2}{2^2} = \dfrac{9+5+6\sqrt{5}+9+5-6\sqrt{5}}{4}$

$= 28/4$

$= 7$

Hence true for $n=1$ and $n=2$

Step 2 Assume true for $n=k$ and $n=k+1$

$\dfrac{(3+\sqrt{5})^k + (3-\sqrt{5})^k}{2^k} = p$ and $\dfrac{(3+\sqrt{5})^{k+1} + (3-\sqrt{5})^{k+1}}{2^{k+1}} = q$

where p and q are positive integers

Question ______________ Name: ____________________

Teacher: ____________________

Step 3 Prove true for $n = k+2$

ie $\dfrac{(3+\sqrt{5})^{k+2} + (3-\sqrt{5})^{k+2}}{2^{k+2}}$ is an integer

$$\frac{(3+\sqrt{5})^{k+2} + (3-\sqrt{5})^{k+2}}{2^{k+2}} = \frac{(3+\sqrt{5}+3-\sqrt{5})\left((3+\sqrt{5})^{k+1} + (3-\sqrt{5})^{k+1}\right) - (3+\sqrt{5})(3-\sqrt{5})\left((3+\sqrt{5})^{k} + (3-\sqrt{5})^{k}\right)}{2^{k+2}}$$

$$= \frac{6\left(q \cdot 2^{k+1}\right) - (9-5)(p \cdot 2^{k})}{2^{k+2}}$$

$$= \frac{3q \cdot 2^{k+2} - p \cdot 2^{k+2}}{2^{k+2}}$$

$$= 3q - p$$

which is an integer since p & q are positive integers.

Hence true for $n = k+2$

Step 4: Therefore by the principle of mathematical induction, the statement is true for all positive integers n.

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(b) $x^5 - 5x + 1 = 0$

let $P(x) = x^5 - 5x + 1$

$P'(x) = 5x^4 - 5 = 0$

$x^4 = 1$

$x = 1, -1, i, -i$

$P(1) = 1 - 5 + 1 \neq 0$

$P(-1) = -1 + 5 + 1 \neq 0$

$P(i) = i - 5i + 1 \neq 0$

$P(-i) = -i + 5i + 1 \neq 0$

$\therefore$ No root of $P'(x) = 0$ is a root of $P(x) = 0$

$\therefore$ No double root.

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(c) $8x^4 - 14x^3 - 69x^2 - 14x + 8 = 0$

$\div x^2$

$8x^2 - 14x - 69 - \frac{14}{x} + \frac{8}{x^2} = 0$

or $8\left(x^2 + \frac{1}{x^2}\right) - 14\left(x + \frac{1}{x}\right) - 69 = 0$

let $t = x^{\cancel{2}} + \frac{1}{x^{\cancel{2}}}$

$t^2 = x^2 + 2 + \frac{1}{x^2} \Rightarrow t - 2 = x^2 + 1/x^2$

$\therefore 8(t^2 - 2) - 14(t) - 69 = 0$

or $8t^2 - 14t - 85 = 0$ $\quad$ 8×85

$8t^2 - 34t + 20t - 85 = 0$ $\quad$ $= 4 \times 2 \times 17 \times 5$

$2t(4t - 17) + 5(4t - 17) = 0$ $\quad$ $= 34, 20$

or $(2t + 5)(4t - 17) = 0$

$\therefore t = -5/2$ or $17/4$

$x + \frac{1}{x} = -\frac{5}{2}$ $\qquad$ $x + \frac{1}{x} = \frac{17}{4}$

$2x^2 + 2 = -5x$ $\qquad$ $4x^2 + 4 = 17x$

or $2x^2 + 5x + 2 = 0$ $\qquad$ $4x^2 - 17x + 4 = 0$

$(2x + 1)(x + 2) = 0$ $\qquad$ $(4x - 1)(x - 4) = 0$

$x = -1/2$ or -2 $\qquad$ $x = 1/4$ or 4

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(d) $x^6 - x^3 + 1 = 0$

$x^3 = \frac{1 \pm \sqrt{1-4}}{2}$

$= \frac{1 \pm \sqrt{3}i}{2}$

$x^3 = \frac{1 + \sqrt{3}i}{2}$ or $x^3 = \frac{1 - \sqrt{3}i}{2}$

$= cis\ \pi/3$ $= cis\ -\pi/3$

$x_k = cis\ \frac{2k\pi + \pi/3}{3}$, $k = 0, 1, 2$ $x^3 = cis\ \frac{-2k\pi - \pi/3}{2}$, $k = 0, 1, 2$

$x_1 = cis\ \pi/9$ $x_4 = cis\ -\pi/9$

$x_2 = cis\ 7\pi/9$ $x_5 = cis\ -7\pi/9$

x_3 $cis\ 13\pi/9$ $x_6 = cis\ -13\pi/9$

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(ii) $\alpha+\beta = 2\cos\frac{\pi}{9}$ $\quad \alpha\beta = 1$

$\gamma+\delta = 2\cos\frac{7\pi}{9}$ $\quad \gamma\delta = 1$

$e+f = 2\cos\frac{13\pi}{9}$ $\quad ef = 1$

$\therefore x^6 - x^3 + 1 = \left(x^2 - 2x\cos\frac{\pi}{9}+1\right)\left(x^2 - 2x\cos\frac{7\pi}{9}+1\right)\left(x^2 - 2x\cos\frac{13\pi}{9}+1\right)$

(iii) Equating coeff of x^5

$0 = -2x\cos\frac{\pi}{9} - 2x\cos\frac{7\pi}{9} - 2x\cos\frac{13\pi}{9}$

$\therefore \cos\frac{\pi}{9} + \cos\frac{7\pi}{9} + \cos\frac{13\pi}{9} = 0$

Equating coeff of x^3

$-1 = -2\cos\frac{\pi}{9} - 2\cos\frac{7\pi}{9} - 2\cos\frac{13\pi}{9} - 8\cos\frac{\pi}{9}\cos\frac{7\pi}{9}\cos\frac{13\pi}{9}$

$-1 = 0 - 8\cos\frac{\pi}{9}\cos\frac{7\pi}{9}\cos\frac{13\pi}{9}$

$\therefore \cos\frac{\pi}{9}\cos\frac{7\pi}{9}\cos\frac{13\pi}{9} = \frac{1}{8}$