THE SCOTS COLLEGE

MATHEMATICS EXTENSION II

YEAR 12 PRETRIAL

23TH MARCH 2015

GENERAL INSTRUCTIONS

- Reading time – 5 minutes
- Working time - 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided
- Show all necessary working in Section II

WEIGHTING

30%

TOTAL MARKS

70

SECTION I (7 MARKS)

- Answers to be recorded on the multiple choice answer sheet provided
- Clearly label your answer sheet with your student number
- Allow about 10 minutes for this section

SECTION II (63 MARKS)

- Questions 8 - 11
- Answers to be recorded in the answer booklets provided
- *Each question must be completed in a new answer booklet.*
- Label each answer booklet with your student number and the question number attempted. Clearly indicate the booklet order if more than one booklet is used for a question. E.g. Book 1 of 2 and 2

Section 1

Question 1

The equation of a conic is $25x^2 - 16y^2 = -400$. The eccentricity of the conic is given by

A. $e = \frac{\sqrt{14}}{5}$ **B**. $e = \frac{\sqrt{41}}{15}$ **C**. $e = \frac{\sqrt{21}}{5}$ **D**. $e = \frac{\sqrt{41}}{5}$

Question 2

If $e^{x+y} = xy$ then $\frac{dy}{dx} =$

A. $\frac{y^{1-x}}{x^{y-1}}$

B. $\frac{y(x-1)}{x(y-1)}$

C. $\frac{y(1-x)}{x(y-1)}$

D. $\frac{x(1-y)}{y(x-1)}$

Question 3

If $z^2 = 4cis\left(\frac{4\pi}{3}\right)$ then $z =$

A. $1 - \sqrt{3}i \ or \ -1 + \sqrt{3}i$

B. $3 + i \ or \ -3 - i$

C. $-1 + \sqrt{3}i \ or \ -1 - \sqrt{3}i$

D. $3 - i \ or -3 + i$

Question 4

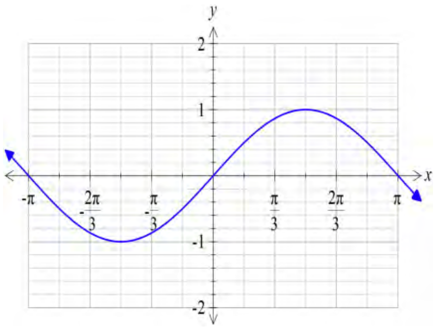
If $P(z) = z^3 - 2z^2 + 4z - 8, and\ z \in C$ then a linear factor of $P(z)$ is

A. $z + 2i$ **B.** $z + 2$ **C.** $2i$ **D.** 2

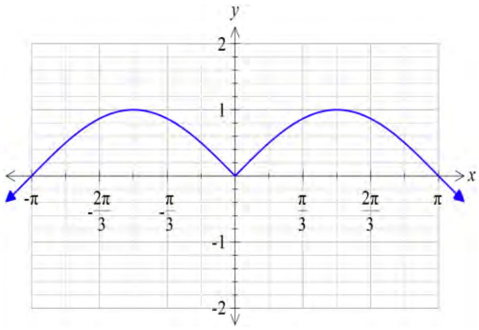
Question 5

Which one of the graphs below represents $|y| = |\sin x|$?

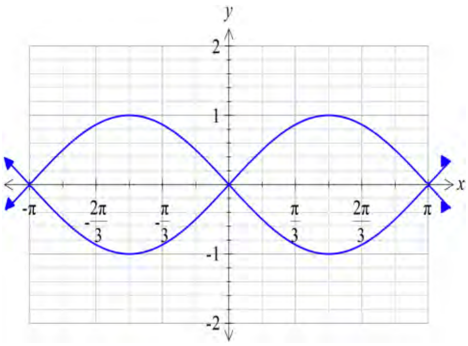
A.



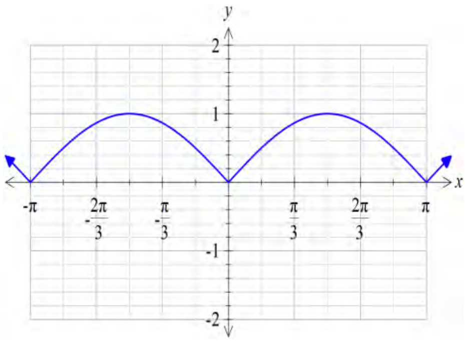
B.



C.



D.



Question 6

Let α, β, γ be the roots of the equation $x^3 + 4x^2 - 3x + 1 = 0$. The equation with roots $\alpha^{-1}, \beta^{-1}, \gamma^{-1}$ is

A. $x^3 + 4x^2 - 3x + 1 = 0$

B. $x^3 - 3x^2 + 4x + 1 = 0$

C. $x^3 - 8x^2 + 12x + 3 = 0$

D. $x^3 - 4x^2 + 8x - 3 = 0$

Question 7

The line $y = mx + c$ will touch the hyperbola $xy = k$ if and only if

A. $c^2 - 4mk = 0$

B. $c^2 + 4mk = 0$

C. $k^2 + 4mc = 0$

D. $m^2 + 4ck = 0$

Section 2

Question 8 (Marks 13)

a) Given the complex numbers $z_1 = \frac{p}{1+\ 2i}$ and $z_2 = \frac{q}{1+\ i}$ where p and q are real, find p and q if $z_1 - \ z_2\ = 4i$. [3]

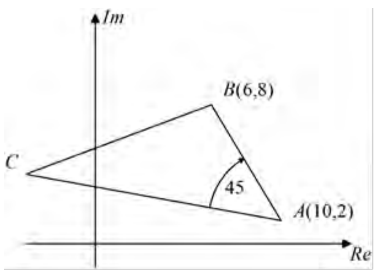
b) Show that $z^n + \ z^{-n} = 2\cos n\theta$ where $z = \cos\theta\ +\ i\sin\theta$. [4]
Hence or otherwise show that

$$cos^4\theta = \frac{\cos 4\theta\ +\ 4\cos 2\theta\ +\ 3}{8} .$$

c) The locus of a point (x, y) , which moves in the complex plane is represented by $|z\ -\ 3i|\ =\ 2.$ [3]

i) Sketch the locus on an Argand diagram.
ii) Show that the minimum value of $\arg z$ is $cos^{-1}\left(\frac{2}{3}\right)$.
iii) Find the modulus of z when P is in the position of minimum argument.

d) [3]



Triangle ABC is drawn on the Argand plane, where $\angle BAC = 45^o$, A represents the complex number $10 + 2i$ and B represents $6 + 8i$.

If the length of the side AC is twice the length of AB then find the complex number that point C represents.

Question 9 [marks 10]

(a) Consider the function $(x) = \frac{e^x - 1}{e^x + 1}$. [7]

i) Show that the function is odd.

ii) Show that the function is always increasing.

iii) Find $f'(0)$.

iv) Sketch $f(x)$ showing any asymptotes.

v) Use your graph to find the values of k for which $\frac{e^x - 1}{e^x + 1} = kx$ has 3 real solutions.

(b) Given $f(x) = 1 - x^2$, without using any calculus, draw neat sketches of the following curves showing intercepts, asymptotes and turning points. The sketches should be about half a page each. [3]

i) $y = \frac{1}{f(x)}$

ii) $y = e^{f(x)}$

Question 10 [marks 21]

a) i) Show that the point $P(a\cos\theta, b sin\theta)$ lies on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. [6]

ii) Find the equation of the tangent to above ellipse at P.

iii) Q is a point on the circle $x^2 + y^2 = a^2$ having the same x value as P. Write down the coordinates of Q.

iv) Show that the tangents at P and Q (provided $\neq \frac{\pi}{2}$) meet on the $x - axis$.

b) The hyperbola has equation $\frac{x^2}{16} - \frac{y^2}{9} = 1$. [6]

i) Sketch the hyperbola showing the coordinates of its foci, the equation of it's directrices and asymptotes.

ii) $P(4\sec\theta, 3\tan\theta)$, is a point on the hyperbola. Perpendiculars form P to the asymptotes meet these lines in M and N . Prove that $PM.PN$ is independent of the position of P.

c) The point $P\left(cp, \frac{c}{p}\right)$ lies on the rectangular hyperbola $xy = c^2$ in the first quadrant. The tangent to the hyperbola at the point P, crosses the $x - axis$ at the point A and the $y - axis$ at the point B. [9]

i) Find the equation of the tangent to the hyperbola at the point P.

ii) Show that the equation to the normal to the hyperbola at the point P is

$$p^3x - py = cp^4 - c$$

iii) If the normal at P meets the other branch of the hyperbola at the point Q, determine the coordinates of Q.

iv) Show that the area of the triangle ABQ is

$$c^2\left(p^2 + \frac{1}{p^2}\right)^2$$

v) Prove that the area of this triangle is a minimum when $p = 1$.

Question 11 [marks 19]

a) For what values of r is $z - ri$ a factor of $P(z) = z^4 - z^3 + 9z^2 - 4z + 20$? [4]

Hence or otherwise, solve $P(z) = 0,\ r \neq 0$ over the set of Complex numbers.

b) Given that $P(x) = x^3 + 3px + q$ has a factor of $(x - k)^2$, [4]

i) Show that $p = -k^2$

ii) Find q in terms of k.

iii) Hence verify that $4p^3 + q^2 = 0$.

c) If α, β and γ are the roots of $x^3 - x^2 - 4x + 1 = 0$ find the equation whose roots are $\alpha + \beta - \gamma$, $\beta + \gamma - \alpha$, and $\gamma + \alpha - \beta$. [4]

Hence evaluate $(\alpha + \beta - \gamma)(\beta + \gamma - \alpha)(\gamma + \alpha - \beta)$.

d) Given $\frac{3-x}{(1+6x)(1+2x)^2} = \frac{c_1}{(1+6x)} + \frac{c_2}{1+2x} + \frac{c_3}{(1+2x)^2}$, [3]

Find c_1, c_2 and c_3 .

e) Show that if the equation $x^n - ax^2 + b = 0$ has a multiple root, then [4]

$$n^n\, b^{n-2} = 4a^n(n-2)^{n-2} .$$

End of Exam

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

CANDIDATE NUMBER: ______________________

SECTION I – MULTIPLE CHOICE ANSWER SHEET (7 MARKS)

Mark the correct answer by filling in the circle. To make a correction, neatly place a cross over the circle and then fill in the correct circle.

EXAMPLE:	A	B	C	D
	○	⊗	●	○

	A	**B**	**C**	**D**
Question 1	○	○	○	○
Question 2	○	○	○	○
Question 3	○	○	○	○
Question 4	○	○	○	○
Question 5	○	○	○	○
Question 6	○	○	○	○
Question 7	○	○	○	○

Solutions Ext 2

Multi Choice

1	D
2	C
3	C
4	A
5	C
6	B
7	B

Q8

(a) $z_1 = \frac{p}{1+2i}$, $z_2 = \frac{q}{1+i}$

$z_1 = \frac{p}{1+2i} \times \frac{1-2i}{1-2i}$ $\quad$ $z_2 = \frac{q}{1+i} \times \frac{1-i}{1+i}$

$= \frac{p-2pi}{5}$ $\quad$ $= \frac{q-qi}{2}$

$z_1 - z_2 = 4i$

$\frac{p-2pi}{5} - \frac{q-qi}{2} = 4i$

$2p - 4pi - 5q + 5qi = 40i$

$2p - 5q = 0 \quad — (1) \quad \times 2.$

$5q - 4p = 40 \quad — (2)$

use (1) in (2)

$5q - 10q = 40$

$-5q = 40$

$q = -8$ ✓

$\therefore p = +\frac{5(-8)}{2}$

$= \frac{-40}{2}$

$= -20$ ✓

(b) Let $z = \cos\theta + i\sin\theta$

$\therefore z^{-1} = \cos\theta - i\sin\theta$

$z + z^{-1} = 2\cos\theta \quad — (1)$

Now $z^2 + z^{-2} = (\cos\theta + i\sin\theta)^2 + (\cos\theta - i\sin\theta)^2$

$= \cos^2\theta + 2i\sin\theta\cos\theta - \sin^2\theta$

$+ \cos^2\theta - 2i\sin\theta\cos\theta - \sin^2\theta$

$= 2(\cos^2\theta - \sin^2\theta)$

$= 2\cos 2\theta$

Illy

$\therefore z^n + z^{-n} = 2\cos n\theta$

From (1) $(z + z^{-1})^4 = 2^4\cos^4\theta$

$= 16\cos^4\theta$

Expanding L.H.S.

$z^4 + 4z^3z^{-1} + 6z^2z^{-2} + 4zz^{-3} + z^{-4} = 16\cos^4\theta$

$\Rightarrow (z^4 + z^{-4}) + 4z^2 + 6 + 4z^{-2} = 16\cos^4\theta$

$\Rightarrow 2\cos 4\theta + 4(z^2 + z^{-2}) + 6 = 16\cos^4\theta$

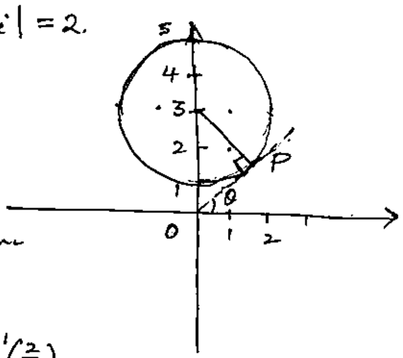
$\Rightarrow 2\cos 4\theta + 4 \times 2\cos 2\theta + 6 = 16\cos^4\theta$

$\Rightarrow \cos 4\theta + 4\cos 2\theta + 3 = 8\cos^4\theta$

$\Rightarrow \cos^4\theta = \frac{\cos 4\theta + 4\cos 2\theta + 3}{8}$

Q8

(c). $|z - 3i| = 2.$



From diagram

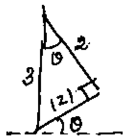
$\cos\theta = \frac{2}{3}$

$\theta = \cos^{-1}\left(\frac{2}{3}\right)$

i.e. $\arg z = \cos^{-1}\left(\frac{2}{3}\right)$

$|z| = \sqrt{3^2 - 2^2}$

$= \sqrt{5}$.



(d) $\vec{AB} = (6+8i) - (10+2i)$

$= -4 + 6i$

Now $\vec{OA} + \vec{AC} = \vec{OC}$

$\vec{AC} = 2 \text{ cis } 45° \times \vec{AB}$

$= 2\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)(-4+6i)$

$= \sqrt{2}(1+i)(-4+6i)$

$= \sqrt{2}(-10+2i)$

$\therefore \vec{OC} = (10+2i) + \sqrt{2}(-10+2i)$

$= 10(1-\sqrt{2}) + 2i(1+\sqrt{2})$ ✓

Q9.

a) i) $f(x) = \frac{e^x - 1}{e^x + 1}$

$f(-x) = \frac{e^{-x} - 1}{e^{-x} + 1}$

$= \frac{\frac{1}{e^x} - 1}{\frac{1}{e^x} + 1}$

$= \frac{\frac{1 - e^x}{e^x}}{\frac{1 + e^x}{e^x}}$

$= \frac{1 - e^x}{1 + e^x}$

$= - \frac{e^x - 1}{e^x + 1}$

$= - f(x)$

$\therefore$ Function is odd.

ii) $f'(x) = \frac{(e^x+1)e^x - (e^x-1)e^x}{(e^x+1)^2}$

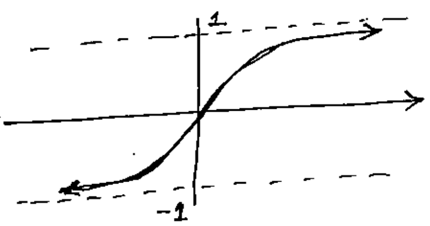
$= \frac{2e^x}{(e^x+1)^2}$

> 0 for all x, $e^x > 0$

$\therefore$ function is an increasing function.

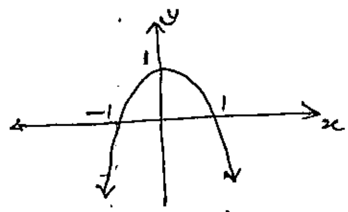
iii) $f'(0) = \frac{2e^0}{(e^0+1)^2} = \frac{1}{2}$

iv)

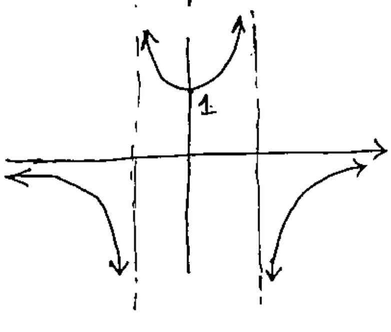


v) $0 < k < 1/2$

b) $f(x) = 1 - x^2$

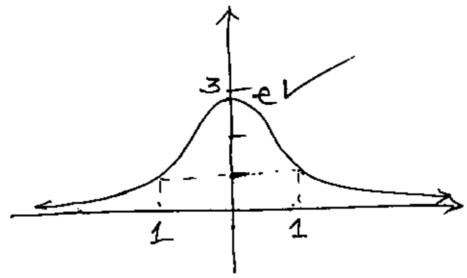


(i)



asymptotes ✓
graph ✓

(ii)



Q10

a) (i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$L.H.S = \frac{a^2\cos^2\theta}{a^2} + \frac{b^2\sin^2\theta}{b^2}$

$\cos^2\theta + \sin^2\theta$

$= 1 \quad = RHS.$

∴ P lies on the ellipse

ii) At P, $x = a\cos\theta$, $y = b\sin\theta$

$\frac{dx}{d\theta} = -a\sin\theta \qquad \frac{dy}{d\theta} = b\cos\theta.$

$\frac{dy}{dx} = -\frac{b\cos\theta}{a\sin\theta}$

∴ tangent at P is:

$y - b\sin\theta = -\frac{b\cos\theta}{a\sin\theta}(x - a\cos\theta)$

$a\sin\theta y - ab\sin^2\theta = -b\cos\theta x + ab\cos^2\theta$

$a\sin\theta y + b\cos\theta x = ab(\sin^2\theta + \cos^2\theta)$

$a\sin\theta y + b\cos\theta x = ab$ ✓ ← (A)

iii) $Q = (a\cos\theta, a\sin\theta)$

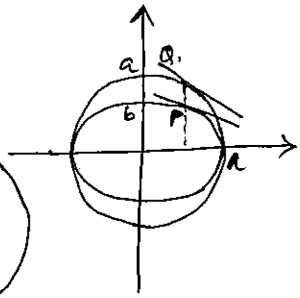
∵ $x^2 + y^2 = a^2$

$a^2\cos^2\theta + y^2 = a^2$

$y^2 = a^2(1 - \cos^2\theta)$

$y^2 = a^2\sin^2\theta$

$y = a\sin\theta$



iv) at Q.

$x = a\cos\theta \qquad y = a\sin\theta$

$\frac{dx}{d\theta} = -a\sin\theta \qquad \frac{dy}{d\theta} = a\cos\theta$

$\frac{dy}{dx} = -\frac{\cos\theta}{\sin\theta}$

∴ tangent at Q is:

$y - a\sin\theta = -\frac{\cos\theta}{\sin\theta}(x - a\cos\theta)$

$\sin\theta y - a\sin^2\theta = -\cos\theta x + a\cos^2\theta$

$\cos\theta x + \sin\theta y = a(\sin^2\theta + \cos^2\theta)$

$\cos\theta x + \sin\theta y = a$ ✓— (B)

$y = 0$ in (A) $\Rightarrow \quad x = \frac{a}{\cos\theta}$

$y = 0$ in B $\Rightarrow \quad x = \frac{a}{\cos\theta}.$

Both tangents have the same x-intercept, hence they meet on x-axis.

Q10 (b)

i) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

$\frac{9}{16} = e^2 - 1$

$e^2 = \frac{25}{16}$

$e = \frac{5}{4}$ ✓

$S(\pm ae, 0) = (\pm 5, 0)$

directrices

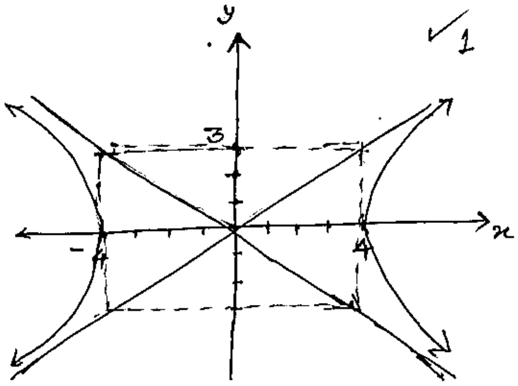
$x = \pm \frac{a}{e} = \pm \frac{16}{5}$

$= \pm 3\frac{1}{5}$ ✓

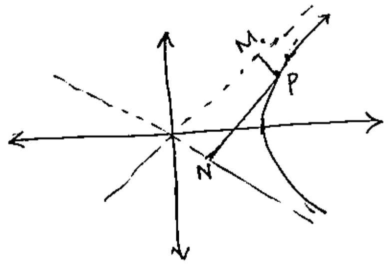
asymptotes.

$y = \pm \frac{b}{a} x$

$= \pm \frac{3}{4} x$. ✓



ii)



$P(4\sec\theta, 3\tan\theta)$

asymptotes

$3x + 4y = 0$

$3x - 4y = 0$

$PM = \frac{|12\sec\theta - 12\tan\theta|}{\sqrt{3^2+4^2}}$

$= \frac{12|\sec\theta - \tan\theta|}{5}$

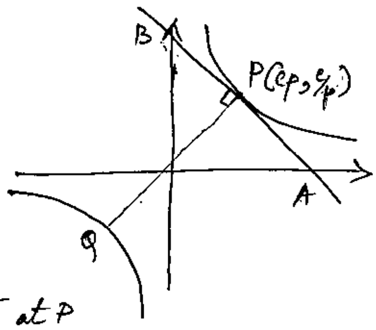
$PN = \frac{12|\sec\theta + \tan\theta|}{5}$

$PM.PN = \frac{144|\sec^2\theta - \tan^2\theta|}{25}$

$= \frac{144}{25}$

This is a constant which is independent of θ

Q10 c)



i) $xy = c^2$

$\frac{dy}{dx} = -\frac{c^2}{x^2}$

at P, $\frac{dy}{dx} = -\frac{1}{p^2}$

∴ Equation of tangent at P

is $y - \frac{c}{p} = -\frac{1}{p^2}(x - cp)$

$x + p^2 y = 2cp$ ✓

(ii) Equation of Normal

$y - \frac{c}{p} = p^2(x - cp)$

$py - c = p^3 x - cp^4$

$p^3 x - py = cp^4 - c$ ✓ — (A)

(iii) $xy = c^2$

$y = \frac{c^2}{x}$ sub in (A)

$p^3 x - p\frac{c^2}{x} = cp^4 - c$

$p^3 x^2 - pc^2 = cp^4 x - cx$

$p^3 x^2 - (cp^4 - c)x - c^2 p = 0$

$(x - cp)(p^3 x + c) = 0$

⇒ $x = -c/p^3$ at Q and $y = -cp^3$

∴ Q $\left(-\frac{c}{p^3}, -cp^3\right)$

(iv) A $(2cp, 0)$ B $\left(0, \frac{2c}{p}\right)$

$d_{AB} = \sqrt{4c^2p^2 + \frac{4c^2}{p^2}} = \frac{2c}{p}\sqrt{p^4 + 1}$

$d_{PQ} = \sqrt{\left(cp + \frac{c}{p^3}\right)^2 + \left(\frac{c}{p} + cp^3\right)^2}$

$= \frac{cp^4 + c}{p^3}\sqrt{1 + p^4}$

Area of △ $= \frac{1}{2} AB \times PQ$.

$= \frac{1}{2}\frac{2c}{p}\sqrt{p^4+1} \times \frac{cp^4 + c}{p^3}\sqrt{1 + p^4}$

$= \frac{c^2}{p^4}(p^4 + 1)^2$

$= c^2\left(p^2 + \frac{1}{p^2}\right)^2$ ✓

(v) $A = c^2\left(p^2 + \frac{1}{p^2}\right)^2$

$A = c^2\left(p^4 + 2 + \frac{1}{p^4}\right)$

$\frac{dA}{dp} = c^2\left(4p^3 - \frac{4}{p^5}\right)$

$\frac{dA}{dp} = 0 \Rightarrow p^3 = \frac{1}{p^5}$

$= p^8 = 1$

$p = 1$

$\frac{d^2A}{dp^2} = 4c^2(3p^2 + 5p^{-6}) > 0$

∴ Area is minimum.

Q11 a)

(i) $p(z) = z^4 - z^3 + 9z^2 - 4z + 20$

$P(ri) = (ri)^4 - (ri)^3 + 9(ri)^2 - 4(ri) + 20 = 0$

$\Rightarrow r^4 + r^3 i - 9r^2 - 4ri + 20 = 0$

$\Rightarrow r^4 - 9r^2 + 20 + i(r^3 - 4r) = 0$

$r^4 - 9r^2 + 20 = 0$ and $r^3 - 4r = 0$

$(r^2 - 5)(r^2 - 4) = 0$ $\quad r(r^2 - 4) = 0$

$r^2 = 5$ or $r^2 = 4$ $\quad$ ~~$r = 0$~~ or $r^2 = 4$

$r = \pm 2$ ✓ $\quad r = \pm 2$

($r^2 = 5$ satisfies only one solution)

∴ two roots are $2i, -2i$

Now $(z - 2i)(z + 2i) = z^2 + 4$

$$\begin{array}{r} z^2 - z + 5 \\ z^2+4 \overline{) z^4 - z^3 + 9z^2 - 4z + 20} \\ -z^4 \quad \pm 4z^2 \\ \hline -z^3 + 5z^2 - 4z \\ -z^3 \quad -4z \\ \hline 5z^2 \quad + 20 \\ 5z^2 \quad + 20 \\ \hline 0 \end{array}$$

∴ $P(z) = (z^2 + 4)(z^2 - z + 5)$ ✓

$z^2 - z + 5 = 0$

$\Rightarrow z = \dfrac{1 \pm \sqrt{1^2 - 4 \times 5}}{2}$

$= \dfrac{1 \pm \sqrt{-19}}{2}$

$= \dfrac{1 \pm i\sqrt{19}}{2}$ ✓

Q11 b)

i) $P(x) = x^3 + 3px + q$

has a factor $(x-k)^2$

then $P'(x) = 3x^2 + 3p$

has a factor $(x-k)$ ✓

$\therefore P'(k) = 3k^2 + 3p = 0$

$3p = -3k^2$

$p = -k^2$ ✓

ii) $P(k) = k^3 + 3pk + q = 0$

sub $p = -k^2$

$k^3 + 3(-k^2)k + q = 0$

$k^3 - 3k^3 + q = 0$

$\therefore q = 2k^3$ ✓

iii) show $4p^3 + q^2 = 0$.

sub for p and q

$L.H.S = 4(-k^2)^3 + (2k^3)^2$

$= -4k^6 + 4k^6$

$= 0$.

$= R.H.S.$ ✓

(c) $x^3 - x^2 - 4x + 1 = 0$.

We know $\alpha + \beta + \gamma = 1$

$\Rightarrow \alpha + \beta + 2\gamma - \gamma = 1$

$\alpha + \beta - \gamma = 1 - 2\gamma$

Let $y = 1 - 2x$.

$\therefore x = \frac{1-y}{2}$.

the equation is

$\left(\frac{1-y}{2}\right)^3 - \left(\frac{1-y}{2}\right)^2 - 4\left(\frac{1-y}{2}\right) + 1 = 0$ ✓

$\Rightarrow y^3 - y^2 - 17y + 9 = 0$.

OR $\Rightarrow x^3 - x^2 - 17x + 9 = 0$ ✓

Prod. of roots.

$(\alpha + \beta - \gamma)(\beta + \gamma - \alpha)(\gamma + \alpha - \beta) = -9$ ✓

Q11 d)

$$\frac{3-x}{(1+6x)(1+2x)^2} = \frac{c_1}{1+6x} + \frac{c_2}{1+2x} + \frac{c_3}{(1+2x)^2}$$

$$3-x = c_1(1+2x)^2 + c_2(1+6x)(1+2x) + c_3(1+6x).$$

$x=-\frac{1}{6} \Rightarrow \quad 3+\frac{1}{6} = c_1\left(1+2\times-\frac{1}{6}\right)^2$

$\frac{19}{6} = c_1\left(\frac{4}{6}\right)^2$

$c_1 = \frac{19\times 6}{4^2} = \frac{57}{8}$ ✓

$x=-\frac{1}{2} \Rightarrow \quad 3+\frac{1}{2} = c_3\left(1+6\times-\frac{1}{2}\right)$

$\frac{7}{2} = c_3(-2)$

$-\frac{7}{4} = c_3$ ✓

Coeff. of $x^2 \Rightarrow \quad 0 = 4c_1 + 12c_2$

$0 = 4\times\frac{57}{8} + 12c_2$

$12c_2 = -\frac{57}{2}$

$c_2 = -\frac{57}{12\times 2} = -\frac{19}{8}$ ✓

(e) $P(x) = x^n - ax^2 + b$

$P'(x) = nx^{n-1} - a2x$

Let α be a multiple root

$\alpha^n - a\alpha^2 + b = 0$ — ①

$n\alpha^{n-1} - 2a\alpha = 0$ — ②

$n\times① \Rightarrow n\alpha^n - an\alpha^2 + bn = 0$ — ③

$\alpha\times② \Rightarrow n\alpha^n - 2a\alpha^2 = 0.$ — ④

$④-③ \Rightarrow \quad a\alpha^2(-2+n) - bn = 0.$

$$\alpha^2 = \frac{bn}{a(n-2)}$$

sub in ④ $\quad n\left[\frac{bn}{a(n-2)}\right]^{n/2} - 2a\left[\frac{bn}{a(n-2)}\right] = 0.$

$\Rightarrow \quad n\left[\frac{bn}{a(n-2)}\right]^{n/2} = 2a\left[\frac{bn}{a(n-2)}\right]$

squaring $\quad n^2\frac{b^n n^n}{a^n(n-2)^n} = 4a^2\frac{b^2n^2}{a^2(n-2)^2}$

$n^n b^{n-2} = 4a^n(n-2)^{n-2}$ ✓