SYDNEY BOYS
HIGH SCHOOL
MOORE PARK, SURRY HILLS

Year 12: APRIL ASSESSMENT

1998

MATHEMATICS

4 UNIT (ADDITIONAL)

Time allowed—Two hours
(Plus 5 minutes' reading time)

Examiner: P.R.Bigelow

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- The mark value of each question is shown to its right.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- *Each* section attempted is to be returned in a *separate* bundle, clearly marked Section A (Q1, Q2), Section B (Q3, Q4), Section C (Q5, Q6). Each bundle must also show your name.
- You may ask for extra Writing Booklets if you need them.

SECTION A.

QUESTION 1. Use a *separate* Writing Booklet. **Marks**

(a) Write down the exact value of $\cos^{-1}0{\cdot}5$. 1

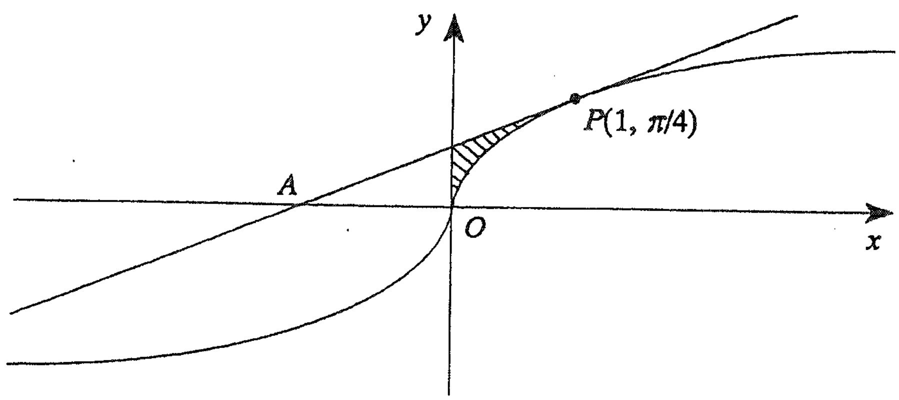
(b) State the domain and range of: 2

(i) $y = 2\sin^{-1}3x$,

(ii) $y = 3\cos^{-1}(1-x)$.

(c) Sketch $f(x) = \sin^{-1}(\cos x)$. 2

(d)



(i) Show that the tangent to $y = \tan^{-1}x$ at $P(1, \pi/4)$ is given by $x - 2y = 1 - \frac{\pi}{2}$. 2

(ii) Find the exact area (shaded) bounded by the line, the curve, and the y-axis. 2

QUESTION 2.

Marks

(a) Given that $z = \dfrac{2+3i}{1-i}$ — 3

find (i) $|z|$,

(ii) $\bar{z}$,

(iii) $z + \bar{z}$.

(b) (i) Express $z = -\sqrt{3} - i$ in mod-arg form. — 2

(ii) Hence express $\dfrac{1}{z^6}$ in the form $a + ib$.

(c) Sketch the following loci: — 2

(i) $\arg z = -\dfrac{\pi}{3}$,

(ii) $|z| = |z - 1 + i|$.

(d) (i) Sketch on an Argand diagram the locus of a point which satisfies the equation $|z - 6 - 8i| = 6$. — 1

(ii) Hence or otherwise determine the maximum values of $|z|$ and arg z for various positions of P. — 1

SECTION B.

QUESTION 3. Use a *separate* Writing Booklet.

Marks

(a) In how many ways may the letters of the word **ALGEBRA** be arranged if the vowels must fill the second, fourth, and sixth places? 2

(b) Find (i) $\int xe^{x}dx$,

(ii) $\int \cos^5 x \sin^3 x\,dx$. 4

(c) Find the value of $\displaystyle\int_0^{\sqrt{2}} \frac{x^3}{x^2+4}dx$. 2

(d) Decompose $\dfrac{2(x+1)}{(x-1)(2x-1)}$ into partial fractions 3

and hence find $\displaystyle\int_2^5 \frac{2(x+1)}{(x-1)(2x-1)}dx$.

(e) (i) Prove that if $u_n = \displaystyle\int_0^{\frac{\pi}{2}} x^n \sin x\,dx, \quad n \geq 0,$ 3

then $u_n + n(n-1)u_{n-2} = n\left(\dfrac{\pi}{2}\right)^{n-1}, \quad n \geq 2,$ and

(ii) deduce the value of $\displaystyle\int_0^{\frac{\pi}{2}} x^4 \sin x\,dx$.

QUESTION 4.

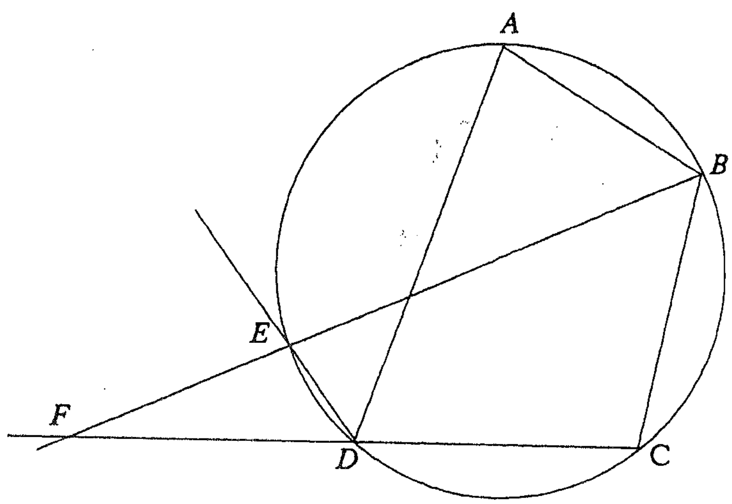
Marks

(a) Solve the equation $6z^2 + (1 + i)z - 1 + 3i = 0$. **2**

(b) Given that $(3 - i)$ is a zero of $P(z) = z^3 - 4z^2 - 2z + m$, and that m is real, find the other roots and the value of m. **3**

(c) If α, β, and γ are the roots of $x^3 + 3x - 5 = 0$, find the cubic equation whose roots are α^2, β^2, and γ^2. **2**

(d) $ABCD$ is a cyclic quadrilateral and CD is produced to F. The bisector of $A\hat{B}C$ cuts the circle at E. **3**



Prove that ED bisects $A\hat{D}F$.

SECTION C.

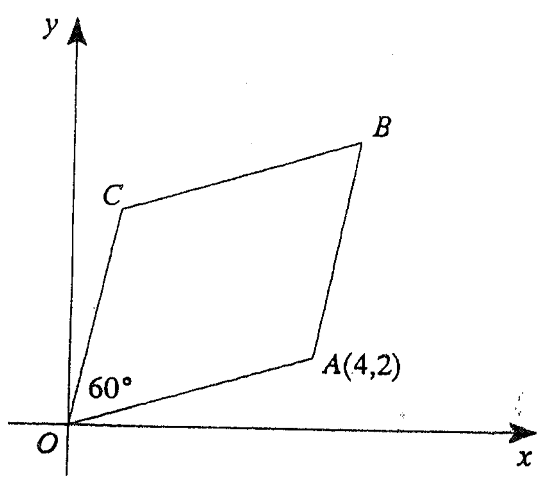
QUESTION 5. Use a *separate* Writing Booklet. **Marks**

(a) (i) By using the substitution $u = \frac{1}{x}$, or otherwise, show that **2**

$$\int_{\frac{1}{2}}^{1} \frac{\ln x}{1+x^2}\,dx = \int_{2}^{1} \frac{\ln u}{1+u^2}\,du.$$

(ii) Deduce the value of $\int_{\frac{1}{2}}^{2} \frac{\ln x}{1+x^2}\,dx$. **2**

(b)



In the diagram $OABC$ is a rhombus with A being the point (4,2) and $A\hat{O}C = 60°$. Find the coordinates of the other two points, B and C. **3**

(c) Solve $z^6 - 9z^3 + 8 = 0$ for z. **3**
Express your answer in the form $a + ib$ where $a, b \in \mathbb{R}$.

QUESTION 6.

Marks

(a) It is required to seat 7 people at a round table. Find the possible number of arrangements if 3 specified people are to sit together. **2**

(b) *ABCD* is a cyclic quadrilateral whose diagonals intersect at *M*. A circle is drawn through *A*, *B*, and *M*. Prove that the tangent at *M* to this circle is parallel to *CD*. **3**

(c) (i) Solve $z^5 = 1$ and let α be the complex root of unity with smallest positive argument. Deduce that $1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 = 0$. **6**

(ii) Find a quadratic equation whose roots are $\alpha + \alpha^4$ and $\alpha^2 + \alpha^3$.

(iii) Hence or otherwise, show that $\cos\frac{2\pi}{5} = \frac{\sqrt{5} - 1}{4}$.

THIS IS THE END OF THE PAPER