SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2011
YEAR 12 Mathematics Extension 2
HSC Task #1

Mathematics Extension 2

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each Section is to be returned in a separate bundle.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.
- Answer must be given in simplest exact form.

Total Marks – 70

- Attempt questions 1-6
- Start each new section of a separate answer booklet

Examiner: *D.McQuillan*

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1;\ x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x,\ x > 0$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax},\ a \neq 0$$

$$\int \cos ax\, dx = \frac{1}{a}\sin ax,\ \ a \neq 0$$

$$\int \sin ax\, dx = -\frac{1}{a}\cos ax,\ \ a \neq 0$$

$$\int \sec^2 ax\, dx = \frac{1}{a}\tan ax,$$

$$\int \sec ax \tan ax\, dx = \frac{1}{a}\sec ax,\ \ a \neq 0$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\ \ a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a},\ \ a > 0,\ \ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x+\sqrt{x^2-a^2}\right),\ x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\ \ x > 0$

Section A

Start each new section on a separate answer booklet.

Question 1

(a) State the exact value of $\cos^{-1}\left(-\frac{1}{2}\right)$ 1

(b) Find $\sqrt{8+15i}$. 2

(c) For the function $f(x) = \sin^{-1}(x^2)$ 6

(i) Find the stationary point and its nature.

(ii) Find the domain and range.

(iii) Hence sketch $y = f(x)$.

(d) Find real numbers A, B and C such that 5

(i) $\frac{2x^2+19x-36}{(x+3)(x-2)^2} \equiv \frac{A}{x+3} + \frac{B}{(x-2)^2} + \frac{C}{x-2}$

(ii) Hence evaluate

$$\int_3^5 \frac{2x^2+19x-36}{(x+3)(x-2)^2}\,dx$$

Question 2

(a) Solve $z^5 + 1 = 0$. 2

(b) Find $g'(a)$ where $g(x)$ is the inverse of $f(x) = x^3 + x + 1$ and $a = 1$. 3

(c) Write down a polynomial equation of the lowest possible degree that has roots $\sqrt{3} + 1$ and $2 - i$ if the polynomial has 3

(i) Complex coefficients

(ii) Rational coefficients

(d) Let $z = 2(cos\theta + isin\theta)$. 5

(i) Find $\overline{1 - z}$

(ii) Show that the real part of $\frac{1}{1-z}$ is

$$\frac{1 - 2cos\theta}{5 - 4cos\theta}$$

(iii) Express the imaginary part of $\frac{1}{1-z}$ in terms of θ.

End of Section A

Section B

Start each new section on a separate answer booklet.

Question 3

(a) Evaluate 6

(i) $\int_{-\pi}^{\pi} \sin^2(x)\ dx$

(ii) $\int_0^{\frac{\pi}{3}} \frac{d\theta}{\cos\theta - \sin\theta + 1}$

(b) Find the equation whose roots are each one more than the roots of $x^3 - 7x + 6 = 0$. 2

(c) Factorise $x^6 + 1$ into its 3

(i) real factors.

(ii) linear factors.

Question 4

(a) Let α, β and γ be the zeros of the polynomial 4

$$P(x) = 3x^3 + 7x^2 + 11x + 51.$$

(i) Find $\alpha^2 + \beta^2 + \gamma^2$.

(ii) Using part (i), or otherwise, determine how many of the zeros of $P(x)$ are real. Justify your answer.

(b) In a History examination, Rahib is asked to put five historical events; A, B, C, D and E into chronological order. 4

(i) If he knows that A occurred sometime before B and otherwise guesses his answer, what is his probability of being correct?

(ii) What is the probability of being correct if he knows that A occurred sometime before D and that D occurred sometime before B?

(c) Prove the identity $2\sin^{-1} x = \cos^{-1}(1 - 2x^2)$ where $x \geq 0$. 3

End of Section B

Section C

Start each new section on a separate answer booklet.

Question 5

(a) Let

$$C(x) = \sum_{k=0}^{5} \frac{x^k}{k!}$$

Prove that $C(x) = 0$ has no double roots. 3

(b) Sketch the locus of z such that 6

(i) $|z - 1| = 2$

(ii) $|z - 1| = Re(z)$

(iii) $\arg(z - 1) = \arg(z + 2)$

Question 6

(a) Find 7

(i) $\int e^x \cos x \ dx$

(ii) $\int \frac{1}{x^2\sqrt{x^2+4}} dx$

(iii) $\int \frac{\sqrt{1+x}}{\sqrt{1-x}} dx$

(b) A parliamentary committee of 8 is to be formed by selecting from a pool of 2 Greens, 6 Labor and 9 Liberal party members. How many committees can be formed if 5

(i) The committee must have 1 Green, 3 Labor and 4 Liberal members.

(ii) The committee must have at least 1 Green, 1 Labor and 4 Liberal members.

(iii) The committee must have 1 Green, 3 Labor and 4 Liberal members and if the Labor transport spokesman is selected on the committee then the Liberal Minister for Transport will make sure he is definitely selected on the committee.

End of Exam

(X2)

QUESTION ONE

(a) $\frac{2\pi}{3}$

(b) $\sqrt{8+15i} = a+ib$

$\therefore 8+15i = a^2-b^2+(2ab)i$

$\therefore a^2-b^2 = 8$ — ①

$2ab = 15$

now $(a^2+b^2)^2 = (a^2-b^2)^2+(2ab)^2$

$= 8^2+15^2$

$= 289$

$\therefore a^2+b^2 = 17$ — ②

From ① + ②

$2a^2 = 25$

$a^2 = \frac{25}{2}$

$a = \pm\frac{5}{\sqrt{2}}$

$= \pm\frac{5\sqrt{2}}{2}$ [NB $a>0$]

$a = \frac{5\sqrt{2}}{2}$

$2ab = 15$

$5\sqrt{2}b = 15$

$b = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$ $\therefore \sqrt{8+15i} = \frac{5\sqrt{2}}{2}+\frac{3\sqrt{2}}{2}i$

(c) (i) $f(x) = \sin^{-1}(x^2)$

$$f'(x) = \frac{2x}{\sqrt{1-x^4}}$$

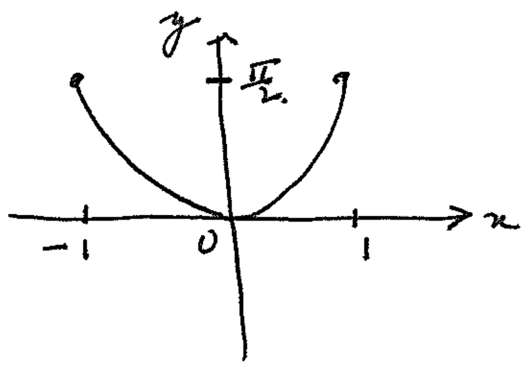
$f'(0) = 0$ & $f(0) = 0$ $\therefore$ $(0,0)$ is a st. pt.

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y'	$\frac{-4}{\sqrt{15}}$	0	$\frac{4}{\sqrt{15}}$
	<0	0	>0

$\therefore$ REL. MIN.

(ii) $D: |x| \leq 1$

$R: \ 0 \leq y \leq \frac{\pi}{2}$.



(d) (i) $$\frac{2x^2+19x-36}{(x+3)(x-2)^2} \equiv \frac{A}{x+3} + \frac{B}{(x-2)^2} + \frac{C}{x-2}$$

$$2x^2+19x-36 \equiv A(x-2)^2 + B(x+3) + C(x+3)(x-2)$$

Let $x = 2$ then $8+36-36 = 5B$ $\boxed{B = 2}$ let $x = -3$

$18-57-36 = 25A$

$\boxed{A = -3}$

let $x = 0$

$-36 = 4A + 3B - 6C$.

$-36 = -12 + 6 - 6C$

$\boxed{C = 5}$

(ii) $$\int_3^5 \frac{2x^2+19x-36}{(x+3)(x-2)^2}\,dx = \int_3^5 \frac{-3\,dx}{x+3} + \int_3^5 \frac{2}{(x-2)^2}\,dx + 5\int_3^5 \frac{dx}{x-2}.$$

$$= \Big[-3\ln(x+3) - \frac{2}{x-2} + 5\ln(x-2)\Big]_3^5$$

$$= -3\ln 8 - \frac{2}{3} + 5\ln 3 - \big[-3\ln 6 - 2 + 5\ln 1\big]$$

$$= 3\ln\frac{3}{4} + 5\ln 3 + \frac{4}{3}.$$

(x2)

QUESTION TWO.

(a) $z^5+1=0$

$z^5=-1.$ let $z=\operatorname{cis}\theta$

$(\operatorname{cis}\theta)^5=-1$

$\cos 5\theta+i\sin 5\theta=-1+0i$

$\therefore \cos 5\theta=-1$

$5\theta=2n\pi+\pi \quad , n\in Z.$

$\theta=\frac{(2n+1)\pi}{5}$

$\theta=\pm\frac{\pi}{5}, \pm\frac{3\pi}{5}, \pi.$

$\therefore$ solutions are $\operatorname{cis}\frac{\pi}{5}, \operatorname{cis}\frac{-\pi}{5}, \operatorname{cis}\frac{3\pi}{5}$

$\operatorname{cis}-\frac{3\pi}{5}, -1.$

(b) If $y=x^3+x+1.$

For inverse $x=y^3+y+1.$

now when $x=1$

$1=y^3+y+1$

$y(y^2+1)=0$

$\underline{y=0.}$

now $\frac{dx}{dy}=3y^2+1$

$\frac{dy}{dx}=\frac{1}{3y^2+1}$ if $y=0$ $\frac{dy}{dx}=1$

$\therefore \boxed{g'(1)=1.}$

(c) (i) $S_1 = \sqrt{3}+1+2-i$ $\quad S_2 = (\sqrt{3}+1)(2-i)$

$= \sqrt{3}+3-i$

$\therefore$ Polyn. is $z^2 - (\sqrt{3}+3-i)z + (\sqrt{3}+1)(2-i) = 0.$

(ii) $\left(z-(1+\sqrt{3})\right)\left(z-(1-\sqrt{3})\right)\left(z-(2-i)\right)\left(z-(2+i)\right)$

$= (z^2-2z-2)(z^2-4z+5)$

(d) (i) $\overline{1-z} = \overline{1-2\cos\theta - 2i\sin\theta}$

$= (1-2\cos\theta) + 2i\sin\theta.$

(ii) $\dfrac{1}{1-z} = \dfrac{1}{(1-2\cos\theta)-2i\sin\theta}$

$= \dfrac{1}{(1-2\cos\theta)-2i\sin\theta} \times \dfrac{(1-2\cos\theta)+2i\sin\theta}{(1-2\cos\theta)+2i\sin\theta}$

$= \dfrac{1-2\cos\theta+2i\sin\theta}{(1-2\cos\theta)^2+4\sin^2\theta}.$

$= \dfrac{1-2\cos\theta+2i\sin\theta}{1-4\cos\theta+4\cos^2\theta+4\sin^2\theta}$

$= \dfrac{1-2\cos\theta+2i\sin\theta}{1-4\cos\theta+4(\cos^2\theta+\sin^2\theta)}$

$= \dfrac{1-2\cos\theta+2i\sin\theta}{5-4\cos\theta}$

$\mathrm{Re}\left(\dfrac{1}{1-z}\right) = \dfrac{1-2\cos\theta}{5-4\cos\theta}$ & $\mathrm{Im}\left(\dfrac{1}{1-z}\right) = \dfrac{2\sin\theta}{5-4\cos\theta}.$

2011 Extension 2 Mathematics Task 1:
Solutions— Section B

3. (a) Evaluate $\boxed{6}$

(i) $\displaystyle\int_{-\pi}^{\pi} \sin^2(x)\,dx,$

Solution:
$$\begin{aligned} I &= 2\int_0^{\pi} \left(\frac{1-\cos 2x}{2}\right) dx, \\ &= \left[x - \frac{\sin 2x}{2}\right]_0^{\pi}, \\ &= \pi. \end{aligned}$$

$$\begin{aligned} \cos 2x &= 1 - 2\sin^2 x, \\ \sin^2 x &= \frac{1-\cos 2x}{2}. \end{aligned}$$

(ii) $\displaystyle\int_0^{\frac{\pi}{3}} \frac{d\theta}{\cos\theta - \sin\theta + 1}.$

Solution:
$$\begin{aligned} \text{Put } t &= \tan {}^{\theta}\!/_{2}, \\ dt &= \tfrac{1}{2}\sec^2 \tfrac{\theta}{2}\, d\theta, \\ d\theta &= \tfrac{2dt}{1+t^2}. \\ \text{When } \theta &= 0, \quad t = 0, \\ \theta &= {}^{\pi}\!/_{3}, \ t = \tfrac{\sqrt{3}}{3}. \end{aligned}$$

$$\begin{aligned} I &= \int_0^{\frac{\sqrt{3}}{3}} \frac{2\,dt}{1+t^2}\left(\frac{1}{\frac{1-t^2}{1+t^2} - \frac{2t}{1+t^2} + 1}\right), \\ &= \int_0^{\frac{\sqrt{3}}{3}} \frac{2dt}{1-t^2-2t+1+t^2}, \\ &= \int_0^{\frac{\sqrt{3}}{3}} \frac{dt}{1-t}, \\ &= \int_{1-\frac{\sqrt{3}}{3}}^{1} \frac{du}{u}, \\ &= \ln u\Big]_{1-\frac{\sqrt{3}}{3}}^{1}, \\ &= \ln\left(\frac{1}{1-\frac{\sqrt{3}}{3}}\right), \\ &= \ln\left(\frac{3}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}\right), \\ &= \ln\left(\frac{3+\sqrt{3}}{2}\right). \end{aligned}$$

$$\begin{aligned} \text{let } u &= 1-t, \\ du &= -dt. \\ \text{when } t &= \tfrac{\sqrt{3}}{3}, \ u = 1 - \tfrac{\sqrt{3}}{3}, \\ t &= 0, \quad u = 1. \end{aligned}$$

(b) Find the equation whose roots are each one more than the roots of $x^3-7x+6=0$. $\boxed{2}$

Solution: Put $x+1=y$, then

$$\begin{aligned}(y-1)^3-7(y-1)+6 &= 0,\\ y^3-3y^2+3y-1-7y+7+6 &= 0,\\ y^3-3y^2-4y+12 &= 0.\end{aligned}$$

$\therefore$ New equation is $x^3-3x^2-4x+12=0$.

(c) Factorise x^6+1 into its $\boxed{3}$

(i) real factors,

(ii) linear factors.

Solution:

$$\begin{aligned}\text{Put } x &= \operatorname{cis}\theta,\\ \text{and } x^6+1 &= 0.\\ \text{Then } \operatorname{cis}6\theta &= \operatorname{cis}(2n+1)\pi, \quad n=0,\,\pm1,\,\pm2,\,\ldots\\ \textit{i.e.},\ 6\theta &= (2n+1)\pi,\\ \theta &= \frac{\pi}{6}+\frac{n\pi}{3},\\ &= \frac{\pi}{6},\ -\frac{\pi}{6},\ \frac{\pi}{2},\ -\frac{\pi}{2},\ \frac{5\pi}{6},\ -\frac{5\pi}{6},\ \ldots\end{aligned}$$

Thus the zeroes of x^6+1 are $\operatorname{cis}\left(\pm\frac{\pi}{6}\right)$, $\pm i$, $\operatorname{cis}\left(\pm\frac{5\pi}{6}\right)$
or $\frac{\sqrt{3}}{2}\pm\frac{i}{2}$, $\pm i$, $-\frac{\sqrt{3}}{2}\pm\frac{i}{2}$.
Hence, looking first at the linear factors, we have
$(x+i)(x-i)\left(x-\operatorname{cis}\frac{\pi}{6}\right)\left(x-\operatorname{cis}\frac{-\pi}{6}\right)\left(x-\operatorname{cis}\frac{5\pi}{6}\right)\left(x-\operatorname{cis}\frac{-5\pi}{6}\right)$, or
$(x+i)(x-i)\left(x-\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\left(x-\frac{\sqrt{3}}{2}+\frac{i}{2}\right)\left(x+\frac{\sqrt{3}}{2}-\frac{i}{2}\right)\left(x+\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$.
Multiplying conjugate pairs gives the real factors:
$(x^2+1)\left(x^2-2\cos\frac{\pi}{6}x+1\right)\left(x^2-2\cos\frac{5\pi}{6}x+1\right)$, or
$(x^2+1)\left(x^2-\sqrt{3}x+1\right)\left(x^2+\sqrt{3}x+1\right)$.

4. (a) Let α, β and γ be the zeroes of the polynomial $\boxed{4}$

$$P(x)=3x^3+7x^2+11x+51.$$

(i) Find $\alpha^2+\beta^2+\gamma^2$.

Solution:

$$\begin{aligned}\alpha+\beta+\gamma &= -\frac{7}{3},\\ \alpha\beta+\beta\gamma+\gamma\alpha &= \frac{11}{3},\\ (\alpha+\beta+\gamma)^2 &= \alpha^2+\beta^2+\gamma^2+2(\alpha\beta+\beta\gamma+\gamma\alpha),\\ \therefore \alpha^2+\beta^2+\gamma^2 &= (\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha),\\ &= \frac{49}{9}-2\times\frac{11}{3},\\ &= -\frac{17}{9}.\end{aligned}$$

(ii) Using part (i), or otherwise, determine how many of the zeroes of $P(x)$ are real. Justify your answer.

Solution: Method 1—
If all zeroes were real, $\alpha^2 + \beta^2 + \gamma^2 \geqslant 0$.
$\therefore$ One of the zeroes must be complex and, as all the coefficients are real, any complex zeroes must occur in conjugate pairs.
Hence the zeroes are two complex and one real.

Solution: Method 2 (otherwise)—

$$\begin{aligned} P'(x) &= 9x^2 + 14x + 11, \\ \triangle &= 196 - 396, \\ &= -200. \end{aligned}$$

$\therefore$ There are no turning points.
As it is a cubic, there is at least one real zero, so the other two zeroes must occur in a conjugate pair.

(b) In a History examination, Rahib is asked to put five historical events, A, B, C, D and E into chronological order. $\boxed{4}$

(i) If he knows that A occurred sometime before B and otherwise guesses his answer, what is his probability of being correct?

Solution: Method 1—
There is a total of 5! ways of arranging them in order.
In only half of these is A before B, and only one of these is right.

$$\begin{aligned} \therefore P(\text{correct}) &= \frac{1}{5! \times 1/2}, \\ &= \frac{1}{60}. \end{aligned}$$

Solution: Method 2—
Choose two positions for A to be before B, then 3! permutations of C, D, and E.
So the number of ways is $\binom{5}{2} \times 3!$

$\therefore P(\text{correct}) = \dfrac{1}{60}$.

Solution: Method 3—

If A is in the first position, there are four possible positions for B.
Hence $4 \times 3!$
If A is in the second position, $3 \times 3!$
If A is in the third position, $2 \times 3!$
If A is in the fourth position, $1 \times 3!$
The total of ways is $3!(4 + 3 + 2 + 1) = 60$.

$\therefore P(\text{correct}) = \dfrac{1}{60}$.

(ii) What is the probability of being correct if he knows that A occurred sometime before D and that D occurred sometime before B?

Solution: Method 1—
3! ways of arranging the three letters A, B, and D.

$$\begin{aligned}\therefore P(\text{correct}) &= \frac{1}{5! \times {}^{1}\!/_{6}},\\ &= \frac{1}{20}.\end{aligned}$$

Solution: Method 2— $\square$ A $\square$ D $\square$ B $\square$
If C and E are together, 2×4 ways.
If C and E are separated, 4×3 ways.
Total ways $= 20$.
$\therefore P(\text{correct}) = \dfrac{1}{20}$.

Solution: Method 3—
Choose three positions for A to be before D before B,
then 2! permutations of C and E.
So the number of ways is $\binom{5}{3} \times 2$.
$\therefore P(\text{correct}) = \dfrac{1}{20}$.

Solution: Method 4— $\square\square\square\square\square$
If D is in the second position, there are three possible positions for B. Hence $3 \times 2!$
If D is in the fourth position, there are three possible positions for A. Thus $3 \times 2!$
If D is in the middle position, there are two positions for A and for B. So $2 \times 2 \times 2!$
The total of ways is $6 + 6 + 8 = 20$.
$\therefore P(\text{correct}) = \dfrac{1}{20}$.

(c) Prove the identity $2\sin^{-1} x = \cos^{-1}(1 - 2x^2)$ where $x \leqslant 0$. $\boxed{3}$

Solution:

$$\begin{aligned} \text{Put } y &= \sin^{-1} x, \\ \textit{i.e.}, \ x &= \sin y. \\ \text{R.H.S.} &= \cos^{-1}\left(1 - 2\sin^2 y\right), \\ &= \cos^{-1}(\cos 2y), \\ &= 2y, \\ &= 2\sin^{-1} x, \\ &= \text{L.H.S.} \end{aligned}$$

Q5

(a) $C(x) = \frac{x^5}{5!} + \frac{x^4}{4!} + \frac{x^3}{3!} + \frac{x^2}{2!} + \frac{x}{1!} + \frac{1}{0!}$

$= \frac{x^5}{120} + \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1$

$c'(x) = \frac{x^4}{24} + \frac{x^3}{6} + \frac{x^2}{2} + x + 1$

$\therefore c(x) = \frac{x^5}{120} + c'(x)$ —— (1)

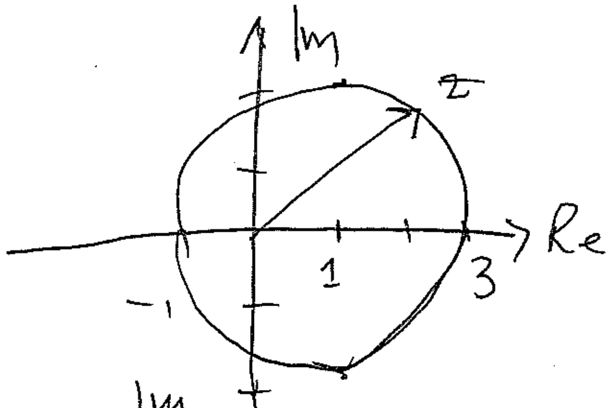
Assume α is a double root.
Clearly $\alpha \neq 0$.
Now $C(\alpha) = 0$ and $c'(\alpha) = 0$

$\therefore$ In (1): $0 = \frac{\alpha^5}{120} + 0$

This is a contradiction. [3]

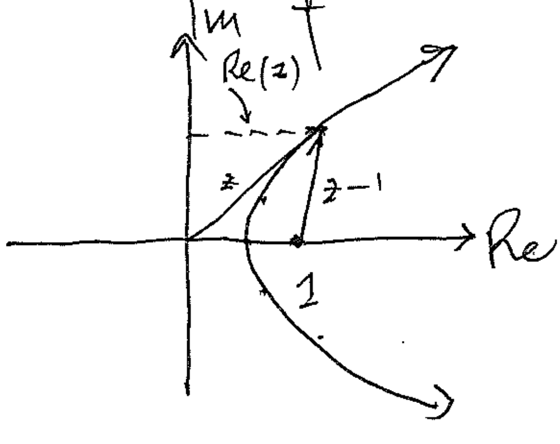
$\therefore$ There is no double root.

(b) (i) $|z - 1| = 2$



[2]

(ii) $|z - 1| = Re(z)$



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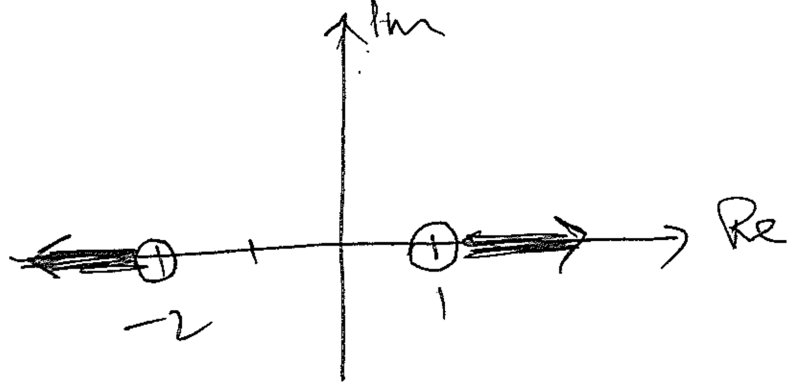
Q5 (contd)
(b) (contd) (iii) $\arg(z-1) = \arg(z+2)$



[2]

Q6

(a) (i) $\int e^x \cos x\,dx = \int \frac{d}{dx}(e^x).\cos x\,dx$

$= e^x \cos x - \int e^x \frac{d}{dx}(\cos x)dx$

$= e^x \cos x + \int e^x \sin x\,dx$

$= e^x \cos x + \int \frac{d}{dx}(e^x) \sin x\,dx$

$= e^x \cos x + e^x \sin x - \int e^x \frac{d}{dx}(\sin x)dx$

$= e^x(\cos x + \sin x) - \int e^x \cos x\,dx$

$\therefore\ 2\int e^x \cos x\,dx = e^x(\cos x + \sin x)$

$\therefore\ \int e^x \cos x\,dx = \frac{1}{2}e^x(\cos x + \sin x) + C$

[2]

Q6 (Contd)
(a) (Contd)

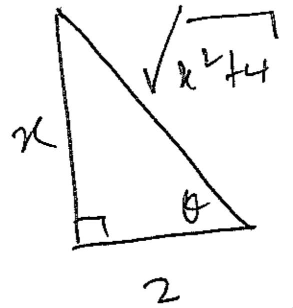
(ii) $I = \int \frac{1}{x^2\sqrt{x^2+4}}\,dx$ Let $x = 2\tan\theta$
$dx = 2\sec^2\theta\,d\theta$

$$= \int \frac{2\sec^2\theta\,d\theta}{4\tan^2\theta\sqrt{4\tan^2\theta+4}}$$

$$= \int \frac{\sec^2\theta\,d\theta}{2\tan^2\theta \times 2\sec\theta}$$

$$= \frac{1}{4}\int \frac{\sec\theta\,d\theta}{\tan^2\theta}$$

$$= \frac{1}{4}\int \operatorname{cosec}\theta\cot\theta\,d\theta$$



$$= -\frac{1}{4}\operatorname{cosec}\theta + C$$

$$= -\frac{\sqrt{x^2+4}}{4x} + C$$

[3]

(iii) $\int \frac{\sqrt{1+x}}{\sqrt{1-x}}\,dx = \int \frac{1+x}{\sqrt{1-x^2}}\,dx$ Rationalising Numerator ~~Numerator~~

$$= \int\left(\frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}\right)dx$$

$$= \sin^{-1}x - \sqrt{1-x^2}\ \cancel{dx} + C$$

[2]

Also $= -2\sin^{-1}\frac{\sqrt{1+x}}{\sqrt{2}} - \sqrt{1-x^2} + D$

Also $= -\cos^{-1}x - \sqrt{1-x^2} + E$

(4)

(b) (i) ${}^{2}C_1 \times {}^{6}C_3 \times {}^{9}C_4 = 5040$ [1]

(ii)

Green	Labour	Liberal		
1	1	6	${}^{2}C_1 \times {}^{6}C_1 \times {}^{9}C_6$	$= 1008$
1	2	5	${}^{2}C_1 \times {}^{6}C_2 \times {}^{9}C_5$	$= 3780$
1	3	4	${}^{2}C_1 \times {}^{6}C_3 \times {}^{9}C_4$	$= 5040$
2	1	5	${}^{2}C_2 \times {}^{6}C_1 \times {}^{9}C_5$	$= 756$
2	2	4	${}^{2}C_2 \times {}^{6}C_2 \times {}^{9}C_4$	$= 1890$

Total 12474

[2]

(iii) Labour Spokesman not selected

1 3 of 5 4 ${}^{2}C_1 \times {}^{5}C_3 \times {}^{9}C_4 = 2520$

Labour Spokesman selected

1 1+2 of 5 1+3 of 8 ${}^{2}C_1 \times {}^{5}C_2 \times {}^{8}C_3 = 1120$

Total 3640

[2]