CANDIDATE NUMBER

SYDNEY GRAMMAR SCHOOL

2015 Half-Yearly Examination

FORM VI

MATHEMATICS EXTENSION 2

Friday 20th February 2015

General Instructions

- Writing time — 1 hour 30 minutes
- Write using black or blue pen.
- Board-approved calculators and templates may be used.
- A list of standard integrals is provided at the end of the examination paper.

Total — 55 Marks

- All questions may be attempted.

Section I – 7 Marks

- Questions 1–7 are of equal value.
- Record your solutions to the multiple choice on the sheet provided.

Section II – 48 Marks

- Questions 8–11 are of equal value.
- All necessary working should be shown.
- Start each question in a new booklet.

Collection

- Write your candidate number on each booklet and on your multiple choice answer sheet.
- Hand in the booklets in a single well-ordered pile.
- Hand in a booklet for each question in Section II, even if it has not been attempted.
- If you use a second booklet for a question, place it inside the first.
- Place your multiple choice answer sheet inside the answer booklet for Question Eight.
- Write your candidate number on this question paper and submit it with your answers.

Checklist

- SGS booklets — 4 per boy
- Multiple choice answer sheet
- Candidature — 73 boys

Examiner

MLS

SECTION I - Multiple Choice

Answers for this section should be recorded on the separate answer sheet handed out with this examination paper.

QUESTION ONE

Which of the following represents the complex number $\sqrt{3} - i$ expressed in modulus–argument form? **1**

(A) $2\,\text{cis}\,\frac{\pi}{6}$

(B) $2\,\text{cis}\left(-\frac{\pi}{6}\right)$

(C) $2\,\text{cis}\,\frac{2\pi}{3}$

(D) $2\,\text{cis}\left(-\frac{2\pi}{3}\right)$

QUESTION TWO

What are the solutions to the quadratic equation $z^2 - 4iz + 12 = 0$? **1**

(A) $-6i$ or $2i$

(B) $6i$ or $-2i$

(C) $2 + 2\sqrt{2}\,i$ or $2 - 2\sqrt{2}\,i$

(D) $2i + 2\sqrt{2}$ or $2i - 2\sqrt{2}$

QUESTION THREE

Which of the following represents the complex number $-iz$ where $|z| = 2\sqrt{2}$ and $\arg(z) = \frac{\pi}{4}$? **1**

(A) $2(1 - i)$

(B) $2(1 + i)$

(C) $2(-1 + i)$

(D) $2(-1 - i)$

Exam continues next page ...

QUESTION FOUR

The expression $\dfrac{9x}{(x+2)(x-1)^2}$ can be expressed in partial fractions as: **1**

(A) $\dfrac{2}{x-1} - \dfrac{2}{x+2} + \dfrac{3}{(x-1)^2}$

(B) $\dfrac{2}{x+2} - \dfrac{3}{(x-1)^2}$

(C) $\dfrac{2}{x+2} + \dfrac{3}{(x-1)^2}$

(D) $\dfrac{2}{x-1} + \dfrac{2}{x+2} + \dfrac{3}{(x-1)^2}$

QUESTION FIVE

Suppose z and w are the roots of the quadratic equation $3x^2 + (2-i)x + (4+i) = 0$. Without solving the quadratic find the value of $\overline{z} + \overline{w}$. **1**

(A) $-\dfrac{2+i}{3}$

(B) $\dfrac{2+i}{3}$

(C) $-\dfrac{4+i}{3}$

(D) $\dfrac{4+i}{3}$

QUESTION SIX

Which of the following is a primitive of $\dfrac{x-1}{x^2+4}$? **1**

(A) $2\log_e(x^2+4) - 2\tan^{-1}x + C$

(B) $\frac{1}{2}\log_e(2x) - \tan^{-1}x + C$

(C) $2\log_e(x^2+4) - \tan^{-1}\dfrac{x}{2} + C$

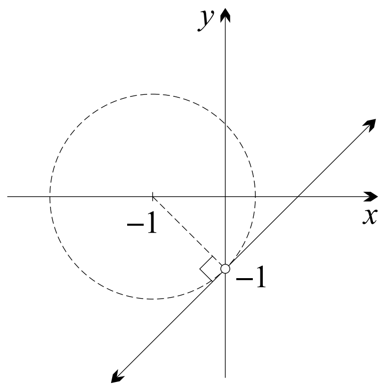
(D) $\frac{1}{2}\log_e(x^2+4) - \dfrac{1}{2}\tan^{-1}\dfrac{x}{2} + C$

Exam continues overleaf ...

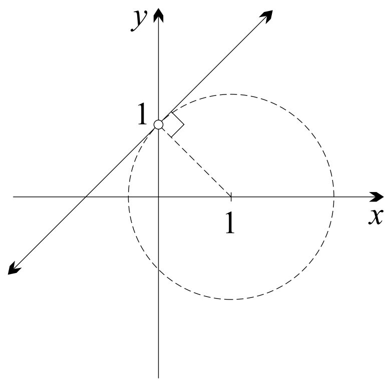
QUESTION SEVEN

If $w = \dfrac{z+1}{z+i}$ and w is imaginary, what is the locus of z? $\boxed{\mathbf{1}}$

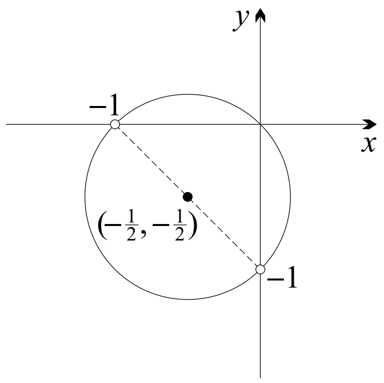
(A)



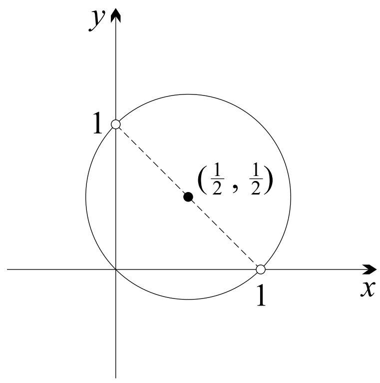
(B)



(C)



(D)



End of Section I

Exam continues next page ...

SECTION II - Written Response

Answers for this section should be recorded in the booklets provided.

Show all necessary working.

Start a new booklet for each question.

QUESTION EIGHT (12 marks) Use a separate writing booklet. **Marks**

(a) Let $u = 8 - 2i$ and $w = 4 + 3i$.

(i) Find $\text{Im}(uw)$. **1**

(ii) Find $u + \overline{w}$. **1**

(iii) Express $\dfrac{u}{w}$ in the form $a + ib$, where a and b are real numbers. **2**

(b) (i) Describe the locus of those points z such that $|z - i| = |z + i|$. **1**

(ii) Describe and sketch the locus of those points z such that $|z - i| = \sqrt{2}\,|z + i|$. Do not find any intercepts with the axes. **3**

(c) (i) Find all pairs of integers x and y such that $(x + iy)^2 = -3 - 4i$. **2**

(ii) Using part (i), or otherwise, solve the quadratic equation $z^2 - 3z + (3 + i) = 0$. **2**

QUESTION NINE (12 marks) Use a separate writing booklet. **Marks**

(a) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx$. **2**

(b) Evaluate $\displaystyle\int_3^8 \frac{x}{(x+1)-\sqrt{x+1}}\,dx$ by using the substitution $x+1=u^2$. **3**

(c) (i) Find A and B such that $\dfrac{6x+23}{2x^2+11x-6} = \dfrac{A}{2x-1} + \dfrac{B}{x+6}$. **2**

(ii) Find $\displaystyle\int \frac{6x+23}{2x^2+11x-6}\,dx$. **1**

(d) Using the substitution $t=\tan\frac{x}{2}$, or otherwise, evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{5+3\sin x+4\cos x}$. **4**

QUESTION TEN (12 marks) Use a separate writing booklet. **Marks**

(a) A curve has parametric equations **3**

$$x = \sin\theta$$
$$y = \tan\theta$$

where $0 < \theta \le \pi$ and $\theta \ne \frac{\pi}{2}$.

Show that $\dfrac{dy}{dx} = \sec^3\theta$ and $\dfrac{d^2y}{dx^2} = 3\tan\theta\sec^4\theta$.

(b) Using the substitution $x = 6\cos u$, determine $\displaystyle\int \frac{1}{x^2\sqrt{36 - x^2}}\,dx$. **3**

(c) Suppose the locus of a point P in the Argand diagram has equation $|z - 2i| = 1$.

(i) Sketch the locus of P as z varies. **1**

(ii) Find the maximum and minimum values of $\arg z$, if $0 < \arg z < \pi$. **3**

(iii) Find the value of z when $\arg z$ takes the minimum value found in part (ii). Give z in the form $a + ib$. **2**

Exam continues overleaf ...

QUESTION ELEVEN (12 marks) Use a separate writing booklet. **Marks**

(a) Use integration by parts to determine $\int x^2 \cos x \, dx$. **2**

(b) You are given that

$$2\cos A \sin B = \sin(A+B) - \sin(A-B).$$

Let $S = 1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta$.

(i) Show that $S \times \sin\dfrac{\theta}{2} = \sin\dfrac{7\theta}{2}$. **1**

(ii) Hence show that $1 + 2\cos\dfrac{2\pi}{7} + 2\cos\dfrac{4\pi}{7} + 2\cos\dfrac{6\pi}{7} = 0$. **1**

(iii) By writing S as a sum of powers of $\cos\theta$, show that $\cos\dfrac{2\pi}{7}$ is a solution of the polynomial equation $8x^3 + 4x^2 - 4x - 1 = 0$. **2**

(c) Suppose z is a complex number such that $|z-1| = 1$ and $\text{Im}(z) > 0$. Let the points O, P and Q represent $0 + 0i$, z and $z - 1$ respectively.

(i) Draw a diagram showing O, P and Q. **1**

(ii) Let $\arg(z) = \theta$. Write expressions for $\arg(z-1)$ and $\arg(z-2)$ in terms of θ. **1**

(iii) Show that $\arg\left(\dfrac{z^2 - 2z}{z-1}\right) = \dfrac{\pi}{2}$. **1**

(iv) What can we deduce about the locus of the point representing $\dfrac{z^2-2z}{z-1}$ from the result in part (iii)? **1**

(v) Find the maximum value of $\left|\dfrac{z^2-2z}{z-1}\right|$. **2**

End of Section II

END OF EXAMINATION

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The following list of standard integrals may be used:

$$\int x^n \, dx = \frac{1}{n+1} \, x^{n+1}, \;\; n \neq -1; \;\; x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \;\; x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} \, e^{ax}, \;\; a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \;\; a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \;\; a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \;\; a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \;\; a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \;\; a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a}, \;\; a > 0, \;\; -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \;\; x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} \, dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

$$\text{NOTE : } \; \ln x = \log_e x, \;\; x > 0$$

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- Record your multiple choice answers by filling in the circle corresponding to your choice for each question.
- Fill in the circle completely.
- Each question has only one correct answer.

Question One

A ◯ B ◯ C ◯ D ◯

Question Two

A ◯ B ◯ C ◯ D ◯

Question Three

A ◯ B ◯ C ◯ D ◯

Question Four

A ◯ B ◯ C ◯ D ◯

Question Five

A ◯ B ◯ C ◯ D ◯

Question Six

A ◯ B ◯ C ◯ D ◯

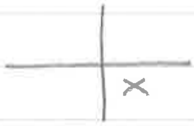
Question Seven

A ◯ B ◯ C ◯ D ◯

HY Ext II Soln 2015

MC

Q1.



B.

Q2.

$$z^2 - 4iz + 12 = 0$$
$$(z - 6i)(z + 2i) = 0$$
$$z = 6i \text{ or } -2i$$

B

Q3.

$$z = 2\sqrt{2} \operatorname{cis} \tfrac{\pi}{4}$$
$$= 2\sqrt{2}\left(\tfrac{1}{\sqrt{2}} + \tfrac{1}{\sqrt{2}} i\right)$$
$$= 2 + 2i$$

$$-iz = -i(2+2i)$$
$$= 2 - 2i$$
$$= 2(1-i)$$

A.

Q4.

$$\frac{9x}{(x+2)(x-1)^2} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$$

$$9x = A(x-1)^2 + B(x+2)(x-1) + C(x+2).$$

$x = 1$ $\qquad 9 = 3C, \qquad C = 3$

$x = -2, \qquad -18 = 9A, \qquad A = -2$

A.

Q5.

$$3x^2 + (2-i)x + (4+i) = 0$$
$$z + w = -\frac{(2-i)}{3}$$

$$\bar{z} + \bar{w} = \overline{z+w}$$
$$= -\frac{(2+i)}{3}$$

A.

6.

$$\int \frac{x-1}{x^2+4}\,dx = \frac{1}{2}\int \frac{2x}{x^2+4}\,dx - \int \frac{1}{x^2+4}\,dx \quad D$$

$$= \frac{1}{2}\ln|x^2+4| - \frac{1}{2}\tan^{-1}\frac{x}{2} + C$$

7. let $z = x+iy$

$$\omega = \frac{x+1+iy}{x+i(y+1)} \times \frac{x-i(y+1)}{x-i(y+1)}$$

$$= \frac{x(x+1) + iyx - i(y+1)(x+1) + y(y+1)}{x^2+(y+1)^2}$$

imag if $re(\omega) = 0$

ie $\quad x(x+1) + y(y+1) = 0$

$$x^2+x + y^2+y = 0 \qquad C$$

$$x^2+x+\tfrac{1}{4} + y^2+y+\tfrac{1}{4} = \tfrac{1}{2}.$$

$$\left(x+\tfrac{1}{2}\right)^2 + \left(y+\tfrac{1}{2}\right)^2 = \tfrac{1}{2}$$

circle centre $\left(-\frac{1}{2}, -\frac{1}{2}\right)$, radius $\frac{1}{\sqrt{2}}$.

Q8.

a)

(i) $uw = (8-2i)(4+3i)$

$= 32 - 8i + 24i + 6$

$\text{Im}(uw) = 16$

(ii) $u + \overline{w} = 8 - 2i + 4 - 3i$

$= 12 - 5i$

(iii) $\frac{u}{w} = \frac{8-2i}{4+3i} \times \frac{4-3i}{4-3i}$

$= \frac{32 - 8i - 24i - 6}{16+9}$

$= \frac{26 - 32i}{25}$

$= \frac{1}{25}(26 - 32i)$

(b) (i) $|z - i| = |z + i|$

We want the locus of all points equidistant from i and $-i$

ie the x axis or real axis or $y=0$.

ii) $|z-i| = \sqrt{2}\,|z+i|$

let $z = x + iy$

$|x + i(y-1)| = \sqrt{2}\,|x + i(y+1)|$ ✓

$x^2 + (y-1)^2 = 2\left(x^2 + (y+1)^2\right)$

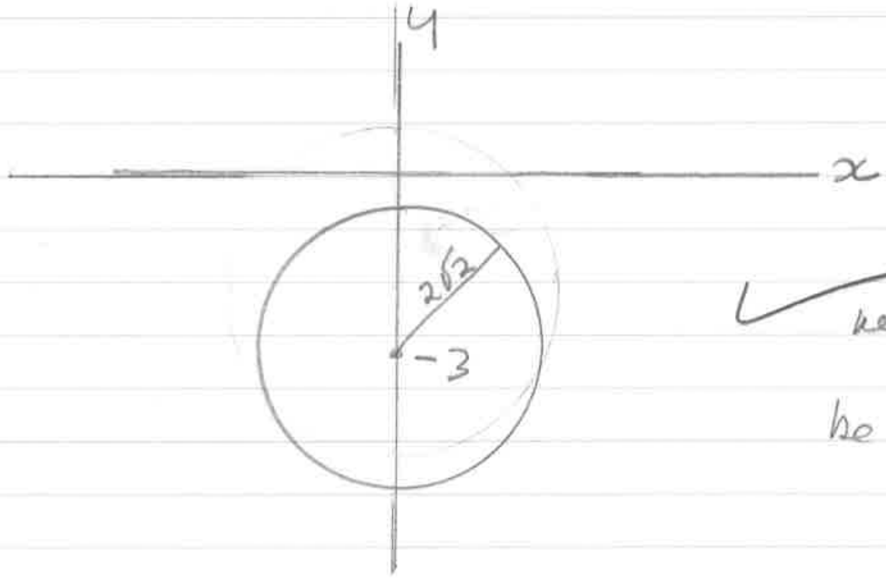
$x^2 + y^2 - 2y + 1 = 2x^2 + 2y^2 + 4y + 2$

$x^2 + y^2 + 6y + 1 = 0$

$x^2 + (y^2 + 6y + 9) = -1 + 9$

$x + (y+3)^2 = 8$ ✓

circle centre $(0,-3)$, radius $2\sqrt{2}$.



✓ need centre & radius to be clear.

c. (i) $(x+iy)^2 = -3-4i$

$$x^2 - y^2 + 2xyi = -3-4i$$

$$x^2 - y^2 = -3$$
$$xy = -2$$
$$y = \frac{-2}{x}$$

$$x^2 - \frac{4}{x^2} = -3$$

$$x^4 + 3x^2 - 4 = 0$$
$$(x^2+4)(x^2-1) = 0$$
$$x = 1 \text{ or } -1$$
$$y = -2 \text{ or } 2$$

$(1-2i)$ or $(-1+2i)$

(ii) $z^2 - 3z + (3+i) = 0$

$$z = \frac{3 \pm \sqrt{9-4(3+i)}}{2}$$

$$= \frac{3 \pm \sqrt{-3-4i}}{2}$$

$$= \frac{3 \pm (1-2i)}{2}$$

$$= \frac{4-2i}{2} \text{ or } \frac{2+2i}{2}$$

$$= (2-i) \text{ or } (1+i)$$

Q9.

(a) $$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x}\,dx = \int_0^{\frac{\pi}{4}} \sec^2 x\, e^{\tan x}\,dx$$ ✓

$$= \left[e^{\tan x}\right]_0^{\frac{\pi}{4}}$$

$$= e - 1$$ ✓

(b) $$\int_3^8 \frac{x}{(x+1)-\sqrt{x+1}}\,dx$$

$x+1 = u^2$
$dx = 2u\,du$

x	8	3
u	3	2

$$= \int_2^3 \frac{u^2-1}{u^2-u}\,2u\,du$$ ✓

$$= 2\int_2^3 \frac{(u+1)(u-1)}{u-1}\,du$$

$$= 2\int_2^3 (u+1)\,du$$ ✓

$$= \Big[u^2 + 2u\Big]_2^3$$

$$= (9+6)-(4+4)$$

$$= 7.$$ ✓

c.

(i) $\dfrac{6x+23}{2x^2+11x-6} = \dfrac{A}{2x-1} + \dfrac{B}{x+6}.$

$6x+23 = A(x+6) + B(2x-1).$ ✓

$x=-6, \quad -13 = -13B \implies B = 1$

$x=\frac{1}{2}, \quad 26 = \frac{13}{2}A \implies A = 4$ ✓

(ii) $\displaystyle\int \frac{6x+23}{2x^2+11x-6}\,dx = \int \frac{4}{2x-1} + \frac{1}{x+6}\,dx$ ✓

$= 2\ln(2x-1) + \ln(x+6) + C$

(d) $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{5+3\sin x+4\cos x}$

$= \displaystyle\int_0^1 \frac{2\,dt}{5+\frac{6t}{1+t^2}+\frac{4-4t^2}{1+t^2}} \times \frac{1}{(1+t^2)}$ ✓

$= \displaystyle\int_0^1 \frac{2\,dt}{5+5t^2+6t+4-4t^2}$

$= \displaystyle\int_0^1 \frac{2\,dt}{t^2+6t+9}$ ✓

$= 2\displaystyle\int_0^1 (t+3)^{-2}\,dt$

let $t = \tan\frac{x}{2}$

$\frac{dt}{dx} = \frac{1}{2}\sec^2\frac{x}{2}$

$= \frac{1}{2}\left(\tan^2\frac{x}{2}+1\right)$

$\frac{dt}{dx} = \frac{1}{2}(t^2+1)$

$dx = \dfrac{2\,dt}{1+t^2}$

x	$\frac{\pi}{2}$	0
t	1	0

$$= -2\left[(t+3)^{-1}\right]_0^1$$

$$= -2\left[\frac{1}{4} - \frac{1}{3}\right]$$

$$= -2 \times -\frac{1}{12}$$

$$= \frac{1}{6}.$$

Q10.

a).

$$x = \sin\theta$$

$$\frac{dx}{d\theta} = \cos\theta$$

$$y = \tan\theta$$

$$\frac{dy}{d\theta} = \sec^2\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta}\,\frac{d\theta}{dx}$$

use of chain rule

$$= \frac{\sec^2\theta}{\cos\theta}$$

$$= \sec^3\theta$$

$$\frac{d^2y}{dx^2} = \frac{d(\sec^3\theta)}{dx}$$

$$= \frac{d(\sec^3\theta)}{d\theta} \times \frac{d\theta}{dx}$$

$$= 3\sec^2\theta \,.\, \sec\theta\tan\theta\,\frac{d\theta}{dx}$$

$$= 3\sec^3\theta\tan\theta\,\frac{1}{\cos\theta}.$$

$$= 3\tan\theta\sec^4\theta$$

b)

$$\int \frac{1}{x^2\sqrt{36-x^2}}\,dx$$

$x = 6\cos u$

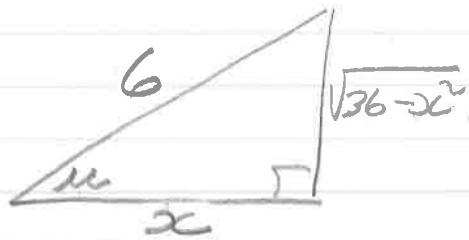
$dx = -6\sin u\, du$

$\sqrt{36-x^2} = \sqrt{36-36\cos^2 u}$
$= 6\sin u.$

$$= \int \frac{1}{36\cos^2 u \; 6\sin u}\left(-6\sin u\, du\right).$$

$$= -\int \frac{1}{36\cos^2 u}\, du$$

$$= -\frac{1}{36}\int \sec^2 u\, du$$

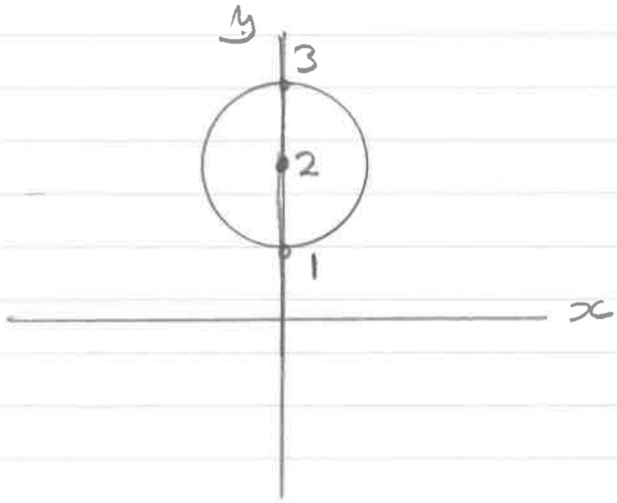


$$= -\frac{1}{36}\tan u + C.$$

$$= -\frac{1}{36}\,\frac{\sqrt{36-x^2}}{x} + C$$

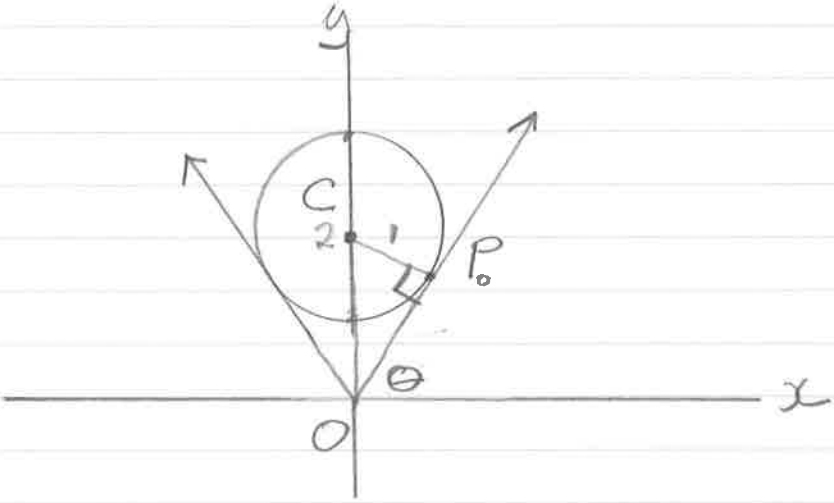
c)

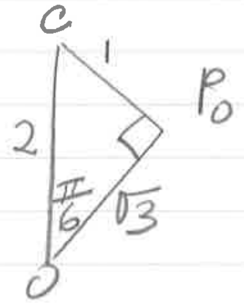
(i) $|z-2i| = 1$



need centre & radius .

ii)





$\angle COP_0 = \frac{\pi}{6}$ ✓

so $\theta = \frac{\pi}{2} - \frac{\pi}{6}$

$= \frac{\pi}{3}.$

minimum value of arg z is $\frac{\pi}{3}$ ✓

So maximum value of arg z is $2\pi - \frac{\pi}{3}$

$= \frac{2\pi}{3}$ ✓

iii) At P_0, $z = \sqrt{3}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ ✓

$= \sqrt{3}\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{2} + \frac{i3}{2}$ ✓

Q11.

(a) $\int x^2 \cos x \, d = x^2 \sin x - \int 2x \sin x \, dx$ ✓

$= x^2 \sin x - 2\left[-x\cos x - \int -\cos x \, dx\right]$

$= x^2 \sin x + 2x\cos x - 2\sin x + c$ ✓

(b)

i) LHS $= S \times \sin\frac{\theta}{2}$

$= \sin\frac{\theta}{2}(1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta)$

$= \sin\frac{\theta}{2} + 2\cos\theta\sin\frac{\theta}{2} + 2\cos 2\theta \sin\frac{\theta}{2} +$

$2\cos 3\theta \sin\frac{\theta}{2}$

$= \sin\frac{\theta}{2} + \left(\sin\frac{3\theta}{2} - \sin\frac{\theta}{2}\right) + \left(\sin\frac{5\theta}{2} - \sin\frac{3\theta}{2}\right)$

$+ \left(\sin\frac{7\theta}{2} - \sin\frac{5\theta}{2}\right)$ ✓

$= \sin\frac{7\theta}{2}$

$=$ RHS

ii) Let $\theta = \frac{2\pi}{7}$

then $\sin\frac{\pi}{7}\left(1 + 2\cos\frac{2\pi}{7} + 2\cos\frac{4\pi}{7} + 2\cos\frac{6\pi}{7}\right)$

$= \sin\frac{7}{2}\left(\frac{2\pi}{7}\right)$

$= \sin\pi$ ✓

$= 0$

Now $\sin\frac{\pi}{7} \neq 0$ so $1 + 2\cos\frac{2\pi}{7} + 2\cos\frac{4\pi}{7} + 2\cos\frac{6\pi}{7}$

$= 0$

iii)

$$S = 1 + 2\cos\theta + 2\cos 2\theta + 2\cos 3\theta$$
$$= 1 + 2\cos\theta + 2(2\cos^2\theta - 1) + 2(4\cos^3\theta - 3\cos\theta).$$

[aside $\cos 3\theta = \cos\theta\cos 2\theta - \sin\theta\sin 2\theta$
$= \cos\theta(2\cos^2\theta - 1) - 2\cos\theta(1 - \cos^2\theta)$
$= 4\cos^3\theta - 3\cos\theta$]

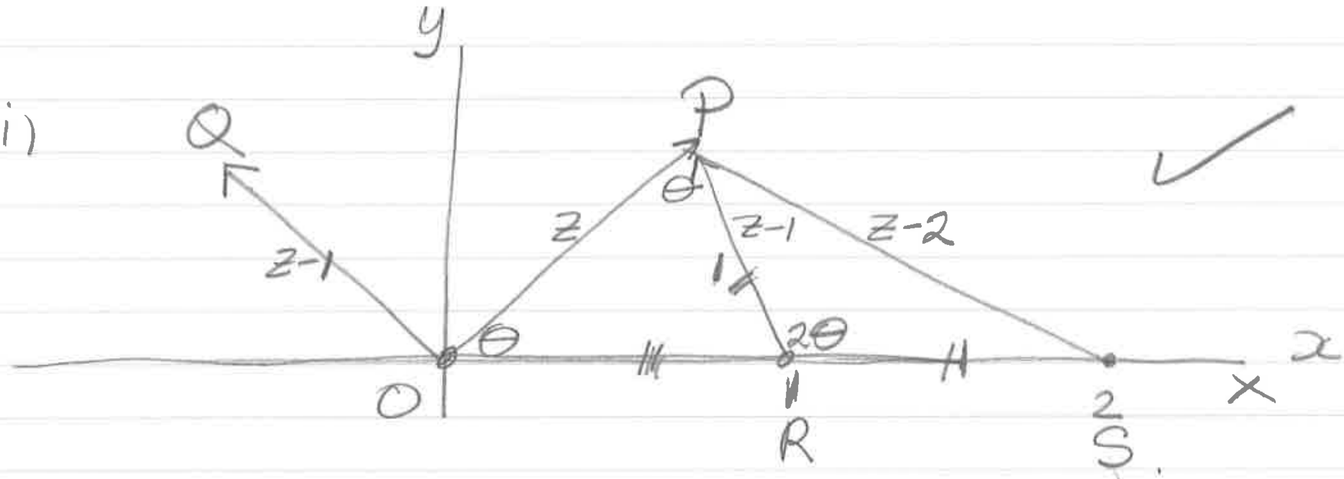
So $S = 8\cos^3\theta + 4\cos^2\theta - 4\cos\theta - 1.$
$= 0$ for $\theta = \frac{2\pi}{7}.$

So $x = \cos\frac{2\pi}{7}$ is a solution of $8x^3 + 4x^2 - 4x - 1 = 0$

✓✓

C.

(i)



(ii) Call (1,0) R, (2,0) S.

Then OR = RP = 1
So △OPR is isosceles
So ∠ROP = ∠OPR = θ
The exterior angle of a triangle equals
the sum of the interior opposite angles
So ∠PRS = 2θ
ie $\arg(z-1) = 2\theta$

OR = RP = RS = 1, So ∠OPS is an angle
in a semi circle
So ∠OPS = $\frac{\pi}{2}$.

So ∠PSX = $\theta + \frac{\pi}{2}$, ext angle of △OPS
ie $\arg(z-2) = \theta + \frac{\pi}{2}$

iii) $\arg\left(\frac{z^2-2z}{z-1}\right) = \arg \frac{z(z-2)}{z-1}$

$= \arg z + \arg(z-2) - \arg(z-1)$

$= \theta + \theta + \frac{\pi}{2} - 2\theta$

$= \frac{\pi}{2}$

iv) It lies on the upper half of the imaginary axis

v) Let $z = 1 + \text{cis}\,\alpha$

$$\frac{z(z-2)}{z-1} = \frac{(\text{cis}\,\alpha + 1)(\text{cis}\,\alpha - 1)}{\text{cis}\,\alpha}$$

$$= \frac{\text{cis}\,2\alpha - 1}{\text{cis}\,\alpha} \times \frac{\text{cis}(-\alpha)}{\text{cis}(-\alpha)}.$$

$$= \text{cis}\,\alpha - \text{cis}(-\alpha)$$

$= 2\sin\alpha$. which has a maximum of 2

So maximum value of $\left|\frac{z^2-2z}{z-1}\right|$ is 2.