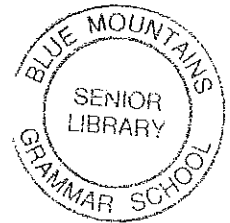**2010**
SEMESTER ONE
EXAMINATION

General Mathematics

General Instructions
- Reading time – 5 minutes
- Working time – $2\frac{1}{2}$ hours
- Write using blue or black pen
- Calculators may be used
- A formulae sheet is provided at the back of this paper

Total Marks – 100

Section I:
22 marks
- Attempt questions 1-22
- Allow about 30 minutes for this section

Section 2:
78 marks
- Attempt questions 23-28
- Allow about 2 hours for this section

Section 1 (22 marks)

Attempt Questions 1–22
Allow about 30 minutes for this section

1. Use the formula $b = a \sqrt{1-e^2}$ to find the value of b, correct to 3 significant figures if $a = 4$ and $e = 0.5$

   A.   1·50   B.   3·46

   C.   3·75   D.   12·0

2. Which of the following events does **NOT** have a probability of 0.5?

   A.   Selecting black card from a standard deck of cards

   B.   Selecting an even number from the numbers 1 to 25.

   C.   Rolling a die and obtaining an odd number.

   D.   Tossing two coins and having one land heads and the other tails.

3. Thirty students sat for tests in four different subjects. Each test was marked out of four. A histogram of the results for each subject is shown below.

   Which subject had marks with the highest standard deviation?

   A.   History   B.   Mathematics

   C.   English   D.   Hebrew

4. If $a^{\Delta} \times a^{\nabla} = a^{12}$ and $\Delta = 4$, then which one of the following is true:

   A. $\nabla = 3$ B. $\nabla = 4$ C. $\nabla = 8$ D. $\nabla = 12$

5. The side of a square microchip is 0.000 000 6 mm. In scientific notation, its area is:

A. $6 \times 10^{-7} \, mm^2$

B. $36 \times 10^{-7} \, mm^2$

C. $3.6 \times 10^{-14} \, mm^2$

D. $3.6 \times 10^{-13} \, mm^2$

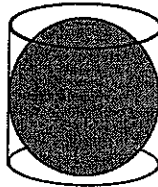**6.** Simplify $8(x-1) - 2(3x-2)$

A. $14x - 12$

B. $2x - 2$

C. $2x - 20$

D. $2x - 4$

7. A sphere fits exactly into a cylindrical container as shown below.

If the diameter of the sphere is 20cm, the volume of the cylinder to the nearest cubic centimetre is:

A. $3\ 142 \ cm^3$

B. $6\ 283 \ cm^3$

C. $12\ 566 \ cm^3$

D. $25\ 132 \ cm^3$

8. The collection of data which has a mean of 4, a median of 3 and a mode of 2 is:

A. 2, 3, 2, 3, 3

B. 2, 2, 3, 2, 11

C. 2, 2, 4, 2, 10

D. 2, 3, 7, 6, 2

9. A car with a cash price of $12 750 is bought on the following terms. A deposit of 20% to be paid with the balance plus interest to be paid in 36 equal monthly payments of $385.33. The annual rate of simple interest charged is:
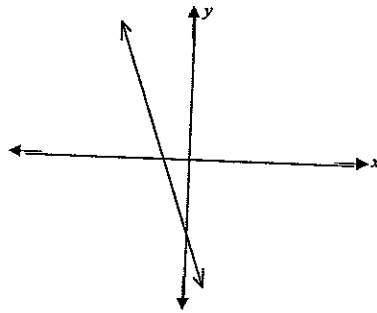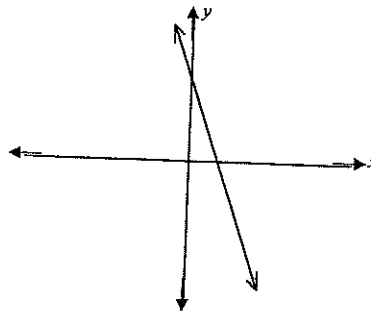
A. 2.9%

B. 3.7%

C. 9.6%

D. 12%

10. The diagram below shows the cross section of a river. Bart used one application of Simpson's Rule to approximate the area of the cross section. All units are in metres.

NOT TO SCALE

The area of the cross section of the river would be closest to:

A. 24 m^2

B. 48 m^2

C. 72 m^2

D. 143 m^2

11. The ratio of blue balls to green balls in a hat is $3:4$.

The number of balls in the hat could be:

(A) 9   (B) 16   (C) 34   (D) 49

12. The box and whisker plot below shows the scores obtained by a team during a Rugby League season:

16 18 20 22 24 26 28 30 32 34 36

The inter-quartile range for the rugby team is:

A. 16

B. 8

C. 22

D. 18

13. Which of the following graphs could be the graph of $y = 4x - 3$?

A.

B.

C.
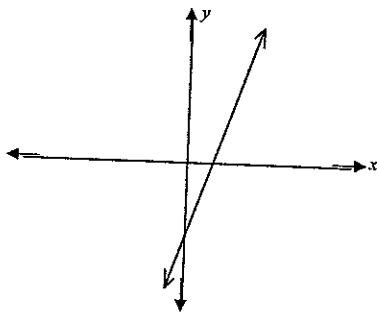
D.
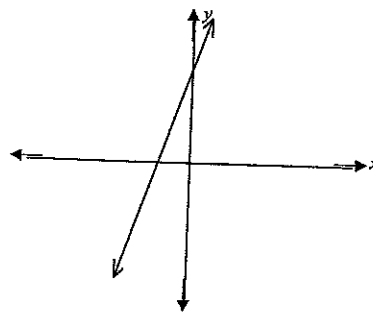
14. The area of the shaded region is approximately:

Not to scale
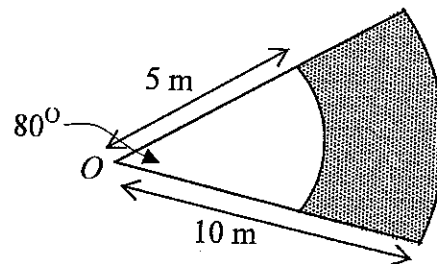
(A) 236 m^2

(B) 53 m^2

(C) 52 m^2

(D) 70 m^2

15. The daily interest rate on a credit card is 0.04796%. How much interest will Ashley pay on a purchase of $699, if he is charged interest from 3rd March to 14th March inclusive?
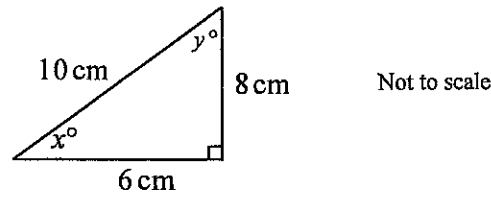
A. $3.69

B. $4.02

C. $368.76

D. $402.29

16.

10 cm 8 cm Not to scale

6 cm

In the triangle above, which of the following ratios is the largest?

(A) sin x (B) sin y (C) tan x (D) tan y

17. 25 students sat for a General Mathematics test. The ordered Stem and Leaf plot displays their marks.

1	2	2	5	6	8		
2	6	7	8	8	9	9	
3	0	0	0	2	5	6	7	8
4	1	1	4	5			
5	0	2					

The percentage of students who scored more than 30% is:

A. 11%   B. 14%   C. 44%   D. 56%

18. In NSW postcodes consist of four digits, with the first number always a '2'. The number of distinct possible postcodes in NSW is given by:

A. 2^4 B.   2 x 10 x 9 x 8

C. 10^3 D. 10^4
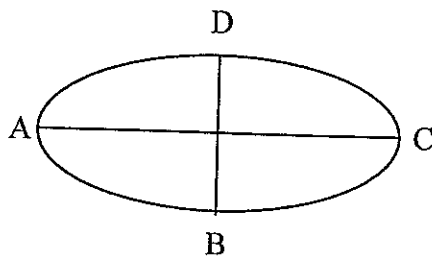
19. The advertised price of a plasma TV is $1397, which includes 10% GST.
What is the amount of GST included in the price?

A. $125.73   B. $127.00

C. $139.70   D. $397.00

20. Shannon walks at 4 km/h. If he walks for 40 minutes how far will he walk?

A.   2·3 km

B.   2·67 km

C.   10 km

D.   160 km

21. Find the area of the ellipse below given that AC = 15cm and DB = 8cm.

A.   28 cm^2

B.   55 cm^2

C.   94 cm^2

D.   377 cm^2

22. Nikki wishes to survey the students at BMGS to suggest changes to the Canteen menu. She decides to give the survey to a proportion of the students in each year group so that her sample reflects of the composition of the school population. What type of sampling is this?

A.   Random sampling

B.   Systematic sampling

C.   Census

D.   Stratified sampling

END OF SECTION I

START A NEW BOOKLET FOR THIS QUESTION.

Question 23   (13 marks)

Marks

Use the table below to answer the following questions:

Annual Tax Rates

Taxable income	Tax on this income
$0 – $6,000	Nil
$6,001 – $34,000	15c for each $1 over $6,000
$34,001 – $80,000	$4,200 plus 30c for each $1 over $34,000
$80,001 – $180,000	$18,000 plus 40c for each $1 over $80,000
$180,001 and over	$58,000 plus 45c for each $1 over $180,000

The above rates **do not** include the Medicare Levy of 1.5% on taxable income.

These are Daniella's financial details for the last tax year:
She received a gross weekly wage of $1 222
She earned $162 for the year in interest from her Supersaver account.
She had a profit of $2 600 from her investment in Newtown.
Her total allowable tax deductions were $2 425.
During the year, she **paid monthly** PAYG tax instalments of $1568.35.

Find:

(a)   her total income   **2**

(b)   her taxable income   **1**

(c)   her Medicare Levy to be paid   **1**

(d)   the total tax payable on her taxable income (incl. Medicare levy).   **3**

(e)   her refund due or balance payable.   **2**

(f)   the percentage of her income that she pays in tax.   **2**

(g)   how much more Daniella would receive each week if she is given a wage rise of $28 per week.   **2**

End of Question 23

Question 24 (13 marks)

Marks

(a)   A car is advertised for sale as follows:

Cash: $25 070
Terms: $5 070 deposit and $1033.33 per calendar month for 2 years.

(i)   What is the total amount paid for the car on these terms?

1

(ii)   Calculate the **annual** simple interest rate charged by the car dealer when the car is purchased on these terms.

2

(b)   The frequency distribution below shows the results of a student survey about the number of movies watched in the last week.

Number of Movies	Number of students
1	5
2	7
3	10
4	13
5	5

What is the relative frequency of watching four movies? (express as a percentage)

1

(c)   Georgia bought a new machine costing $121 000 for her confectionary factory.

(i)   What would be the value of the machine after 3 years if the depreciation is calculated using the declining balance method and the annual depreciation rate is 9%?

1

(ii)   What depreciation rate would be needed for the machine to depreciate to half its value after only 2 years?
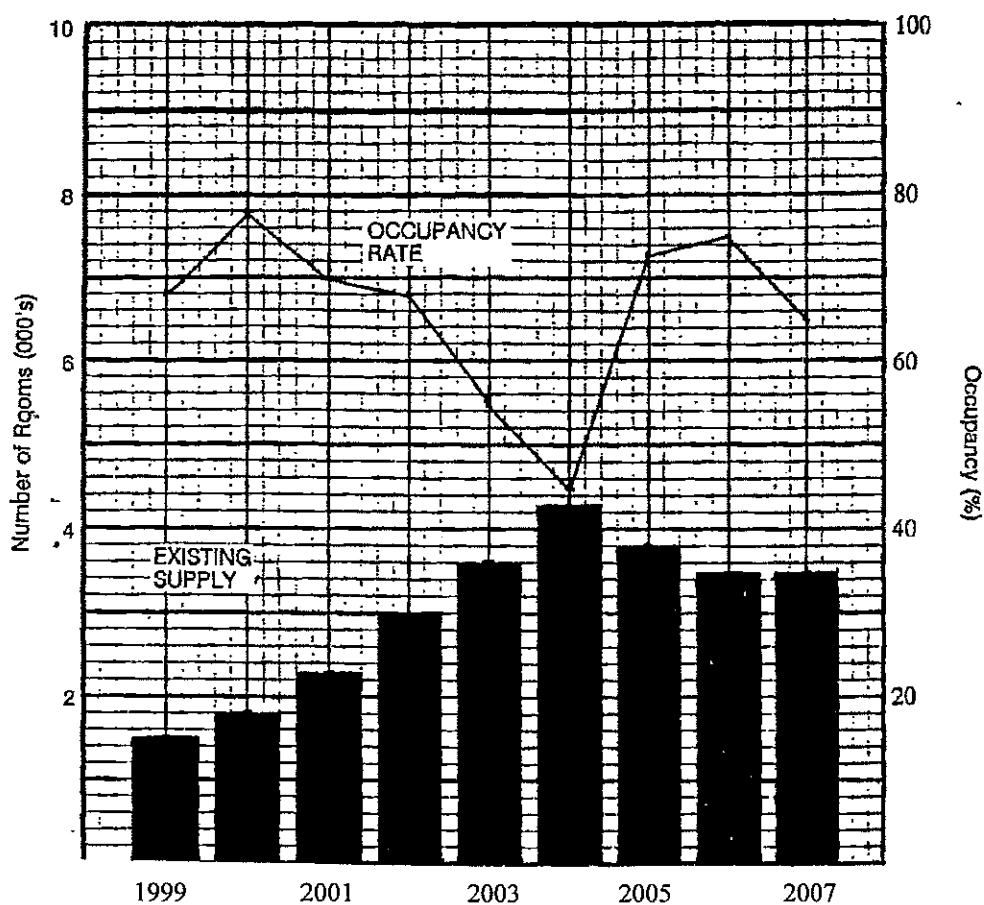
2
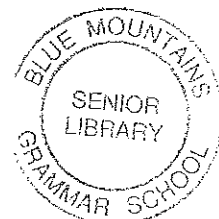
Question 23 continued

(d) The height of a Year 12 student was measured to be 172 cm correct to the nearest centimetre. What is the percentage error of this measurement correct to 2 decimal places?　　　　**2**

(e) The information below refers to the supply and demand for hotel

HOTEL ROOMS — SUPPLY AND DEMAND (RESORT TOWN)

(i) How many rooms were available in 2003?　　　　**1**

(ii) What percentage of rooms were **unoccupied** in 2000?　　　　**1**

(iii) When did the greatest increase in occupancy rate occur? Justify your answer.　　　　**2**
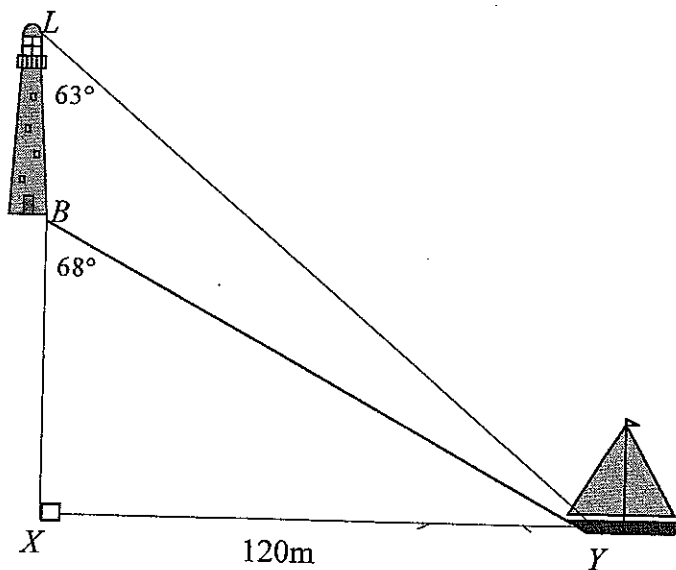
End of Question 24

Question 25 (13 marks)

Marks

(a) A lighthouse master stands at the top of the lighthouse and sights a yacht which is 120m from the base of the cliff. She determines that the angle of depression of the yacht is 27°. She runs down the stairs to the bottom of the lighthouse and determines that the new angle of depression when looking at the yacht is 22°.

(i) Explain why $\angle YLX = 63°$. You must show working out.   **1**

(ii) What is the distance from the base of the cliff to the bottom of the lighthouse XB?
Give your answer correct to 1 decimal place.   **2**

(iii) Using your answer to (ii) or otherwise, find the height of the lighthouse BL.
Give your answer correct to 1 decimal place.   **2**

(b) Adam is a marine biologist. He spends the day on a boat and catches 350 fish. Adam notes the type of fish caught. There are 126 tuna, 185 whitehead and 39 mullet.

(i) Is Adam's data categorical or quantitative?   **1**

(ii) The fish are tagged and released back into the school from which they were caught. Another 140 are then caught and it is noted that 70 have tags.
Estimate the population in the school.   **2**
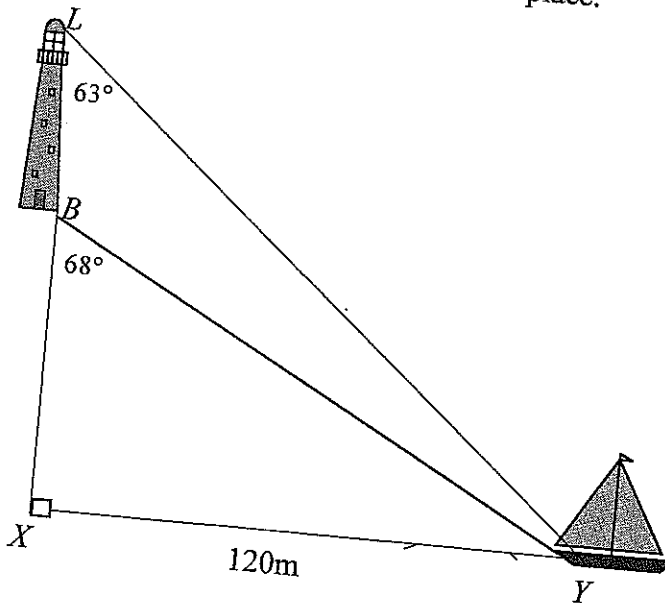
Question 25 (13 marks)

Marks

(a) A lighthouse master stands at the top of the lighthouse and sights a yacht which is 120m from the base of the cliff. She determines that the angle of depression of the yacht is 27°. She runs down the stairs to the bottom of the lighthouse and determines that the new angle of depression when looking at the yacht is 22°.

(i) Explain why $\angle YLX = 63°$. You must show working out.

1

(ii) What is the distance from the base of the cliff to the bottom of the lighthouse XB?

Give your answer correct to 1 decimal place.

2

(iii) Using your answer to (ii) or otherwise, find the height of the lighthouse BL.

Give your answer correct to 1 decimal place.

2

(b) Adam is a marine biologist. He spends the day on a boat and catches 350 fish. Adam notes the type of fish caught. There are 126 tuna, 185 whitehead and 39 mullet.

(i) Is Adam's data categorical or quantitative?

(ii) The fish are tagged and released back into the school from which they were caught. Another 140 are then caught and it is noted that 70 have tags. Estimate the population in the school.

1

2

Question 25 continued

(c) Below is a field book entry and a field sketch of an offset survey of a block of land

D

14
m Y   45
              m   C

(i)

X
    31
20 m
m
                        B

                A
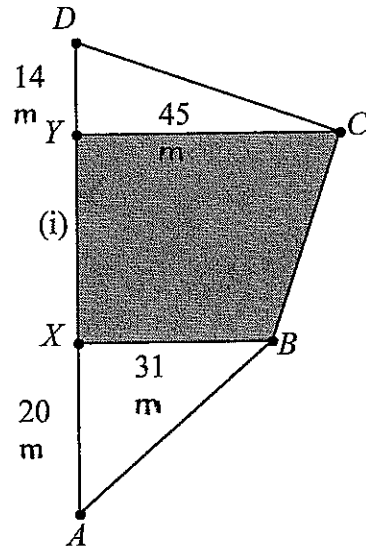
D	
80	
66	45
20	31
0	
A	

(i)   Determine the missing information (i) on the field sketch.   1

(ii)   Find the area of the shaded region *XBCY*.   2

(d) The average temperature in June for the past *30* years in a city has been18.4°C .
    This year the new June temperature average was 22.1°C .

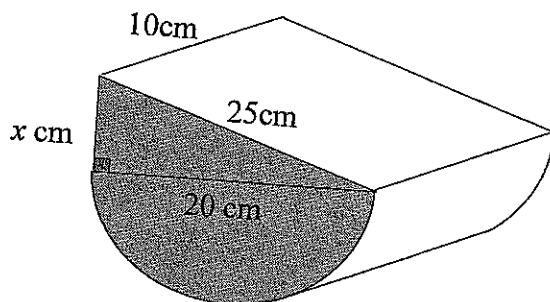   What is the new 31 year average? *(Answer correct to one decimal place)*   2

End of Question 25

Question 26 (13 marks)

Marks

(a) The cross section of this solid prism is a semi-circle and a right-angled triangle.

10cm

25cm

x cm

20 cm

(i) Find the length of x.

2

(ii) What is the total surface area of this solid?

4

(b) Measurements were taken every 2 km apart across the area of a rural community, as shown below:

1.9k

8km   1.3k

3.4k
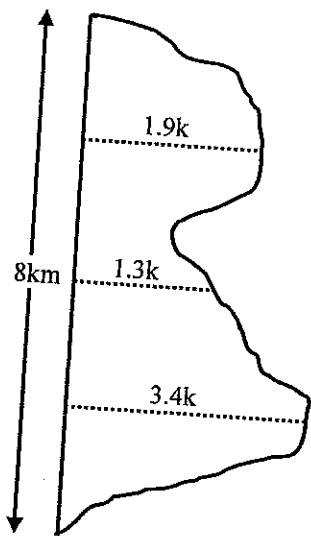
(i) Using two applications of Simpson's Rule, approximate the area of this community to the nearest km²

3

(ii) Find the area in hectares.

2

(c) Nasya's steelworks manufactures special steel washers used on large bridges according to the dimensions shown in the diagram below. Calculate the volume of metal used in each washer.

2

110mm

15mm

255mm
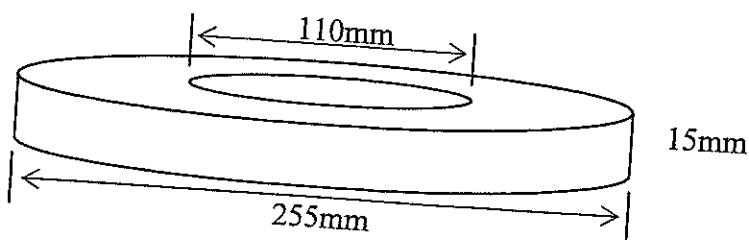
End of Question 26

START A NEW BOOKLET FOR THIS QUESTION.

Question 27 (13 marks)

(a) Hassan borrowed $75 000 to buy a home unit which he is going to use as a rental investment.
This table shows some of the figures involved in the repayment of his loan.

	Home Loan Table			
	Amount =	$75 000	This table assumes the same number of days in each month. Interest = rate/12 × principal	
	Annual interest rate	10%		
	Monthly repayment R =	$900		
N	Principal (P)	Interest (I)	P + I	P + I − R
1	$75 000.00	$625.00	$75 625.00	$74 725.00
2	$74 725.00	$622.71	$75 347.71	$74 447.71
3	$74 447.71	$620.40	$75 068.11	$74 168.11
4	$74 168.11	A	B	C
185	$1219.77	$10.16	$1229.93	$329.93
186	$329.93			

(i) Find the values of A, B and C   **3**

(ii) After Hassan's 186th repayment, the loan will be repaid. Calculate the value of his last repayment.   **2**

(b) Telstra shares have a face value of $2 and a current market value of $4.55. Sophie bought 500 shares when the stock was first floated. In addition she had to pay a 2.5% brokerage fee on the total cost of the shares.

(i) What was the total cost of the shares, including brokerage fee, when Sophie bought the shares?   **2**

The Government charges stamp duty on the purchase of shares of 15cents for every $100 worth of shares or part thereof.

(ii) How much stamp duty did Sophie have to pay?   **1**

(c) When Andrea was born, her grandparents deposited $500 into an account that pays 6%p.a. interest, compounded annually. They added $500 to the account each birthday, making the last deposit on Andrea's 21st birthday. The investment was given to Andrea as a 21st birthday present.

(i) What was the total value of the investment on Andrea's 21st birthday?

2

(ii) What was the total amount of interest that the investment had earned by Andrea's 21st birthday?

1

Andrea decided to leave this 21st birthday investment in the account to earn interest. She was hoping that it would help to support her during University by providing her with a monthly allowance (withdrawal) of $600 for the next 3 years.

(iii) Will Andrea's investment provide a monthly allowance of $600 over the next 3 years?

2

Assume that interest is 6%p.a. compounded monthly. Show all working out.

End of Question 27

Question 28 (13 marks)   **Marks**

(a)   Solve   (i) $15 - 2y = 10$ **2**

                (ii) $\dfrac{(2y+3)}{5} - \dfrac{(y-4)}{4} = 2$ **3**

(b)   A gardening expert recommends 15 litres of water per week for each 1
metre height of gum tree.
A garden hose fills a 10 litre bucket in 30 seconds.

Calculate the time that should be spent each week watering a 9 metre high
gum tree with a hose.   **2**

(c)   Mr Peabody bought 500 Restac $2 shares at $1.50 each

Six months later the shares paid a dividend of 8% and the market price was
$1.65 per share.

    (i)   How much dividend did Mr Peabody receive?   **1**

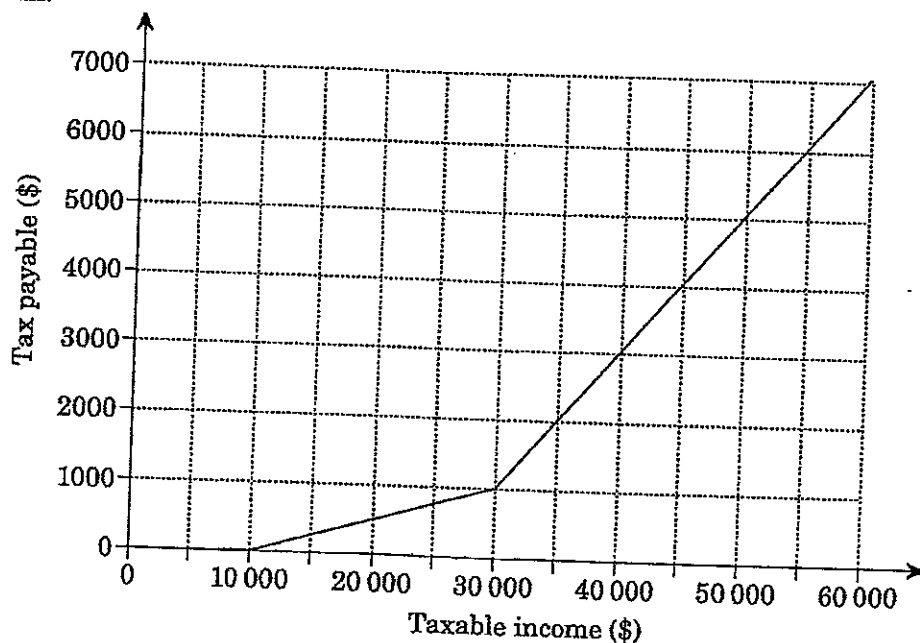   (ii) Calculate the percentage yield of the shares.   **2**

(d)   The table of values below gives four points which lie on a straight line.

x	5	7	8	11
y	3	7	9	15

What is the equation of the straight line.   **1**

QUESTION 28 CONTINUED ON NEXT PAGE

(e) The graph shows the tax payable for taxable incomes up to $60 000 in a proposed tax system.

Calculate the tax rate as a percentage for incomes of $30 000 and above.　　2

END OF QUESTION 28

END OF EXAMINATION

GENERAL MATHEMATICS
SEMESTER 1 EXAMINATION

MULTIPLE CHOICE ANSWER SHEET

STUDENT NUMBER:_____

1 A◯ B◯ C◯ D◯	12 A◯ B◯ C◯ D◯
2 A◯ B◯ C◯ D◯	13 A◯ B◯ C◯ D◯
3 A◯ B◯ C◯ D◯	14 A◯ B◯ C◯ D◯
4 A◯ B◯ C◯ D◯	15 A◯ B◯ C◯ D◯
5 A◯ B◯ C◯ D◯	16 A◯ B◯ C◯ D◯
6 A◯ B◯ C◯ D◯	17 A◯ B◯ C◯ D◯
7 A◯ B◯ C◯ D◯	18 A◯ B◯ C◯ D◯
8 A◯ B◯ C◯ D◯	19 A◯ B◯ C◯ D◯
9 A◯ B◯ C◯ D◯	20 A◯ B◯ C◯ D◯
10 A◯ B◯ C◯ D◯	21 A◯ B◯ C◯ D◯
11 A◯ B◯ C◯ D◯	22 A◯ B◯ C◯ D◯

FORMULAE SHEET

Area of an annulus

$$A = \pi\left(R^2 - r^2\right)$$

R = radius of outer circle

r = radius of inner circle

Area of an ellipse

$$A = \pi ab$$

a = length of semi-major axis

b = length of semi-minor axis

Area of a sector

$$A = \frac{\theta}{360}\pi r^2$$

θ = number of degrees in central angle

Arc length of a circle

$$l = \frac{\theta}{360}2\pi r$$

θ = number of degrees in central angle

Simpson's rule for area approximation

$$A \approx \frac{h}{3}\left(d_f + 4d_m + d_l\right)$$

h = distance between successive measurements

d_f = first measurement

d_m = middle measurement

d_l = last measurement

Surface area

Sphere $\qquad A = 4\pi r^2$

Closed cylinder $\qquad A = 2\pi rh + 2\pi r^2$

r = radius

h = perpendicular height

Volume

Cone $\qquad V = \frac{1}{3}\pi r^2 h$

Cylinder $\qquad V = \pi r^2 h$

Pyramid $\qquad V = \frac{1}{3}Ah$

Sphere $\qquad V = \frac{4}{3}\pi r^3$

r = radius

h = perpendicular height

A = area of base

Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Area of a triangle

$$A = \frac{1}{2}ab\sin C$$

Cosine rule

$$c^2 = a^2 + b^2 - 2ab\cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

FORMULAE SHEET

Simple interest

$$I = Prn$$

P = initial quantity

r = percentage interest rate per period, expressed as a decimal

n = number of periods

Compound interest

$$A = P(1+r)^n$$

A = final balance

P = initial quantity

n = number of compounding periods

r = percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$A = M\left\{\frac{(1+r)^n - 1}{r}\right\}$$

M = contribution per period, paid at the end of the period

Present value (N) of an annuity

$$N = M\left\{\frac{(1+r)^n - 1}{r(1+r)^n}\right\}$$

or

$$N = \frac{A}{(1+r)^n}$$

Straight-line formula for depreciation

$$S = V_0 - Dn$$

S = salvage value of asset after n periods

V_0 = purchase price of the asset

D = amount of depreciation apportioned per period

n = number of periods

Declining balance formula for depreciation

$$S = V_0(1-r)^n$$

S = salvage value of asset after n periods

r = percentage interest rate per period, expressed as a decimal
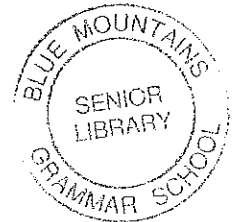
Mean of a sample

$$\bar{x} = \frac{\sum x}{n}$$

$$\bar{x} = \frac{\sum fx}{\sum f}$$

$\bar{x}$ = mean

x = individual score

n = number of scores

f = frequency

Formula for a z-score

$$z = \frac{x - \bar{x}}{s}$$

s = standard deviation

Gradient of a straight line

$$m = \frac{\text{vertical change in position}}{\text{horizontal change in position}}$$

Gradient–intercept form of a straight line

$$y = mx + b$$

m = gradient

b = y-intercept

Probability of an event

The probability of an event where outcomes are equally likely is given by:

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$