MOUNT ST JOSEPH MILPERRA

.

HIGHER SCHOOL CERTIFICATE

GENERAL MATHEMATICS

HALF YEARLY 2001

Section I

Total marks **(20)**
Attempt all questions 1- 20
Allow about 30 minutes for this section

Section II

Total marks (60)
Attempt all questions 21 - 25
Allow about $1\frac{1}{2}$ hours for this section.

General Instructions

- Reading time - 5 minutes
- Working time - 2 hours
- Write using blue or black pen
- Calculators may be used
- Write your name on each page
- A Formula sheet is provided

SECTION I (20 Marks)

1. The diameter of a human hair is 0.0000456mm . In scientific notation this is equal to

A. 4.56×10^{-4} B. 4.56×10^{-5} C. 4.56×10^{4} D. 4.56×10^{5}

2. A cube has a side of length 4cm.
   I. The surface area of the cube is $64cm^2$
   II. The volume of the cube is $96cm^3$
Which of the above statements is correct ?

A. I only   B. II only   C. Both I and II   D. Neither I nor II.

3. Which of the following is an example of a random sample ?

A. The first 50 students to arrive at school take the survey.
B. Fifty student's names are drawn from a hat and those students take the survey.
C. Ten students from each year of the school are asked to complete the survey.
D. One class in the school is asked to complete the survey.

4. A spinner is to be designed so that the probability of the arrowhead stopping in the blue region is twice that of stopping in the green region.

Which of the following designs meets this requirement ?
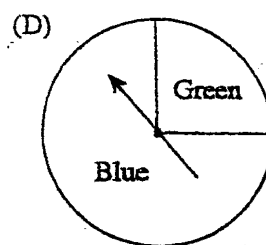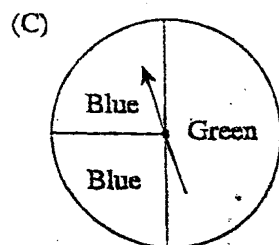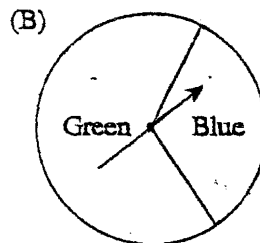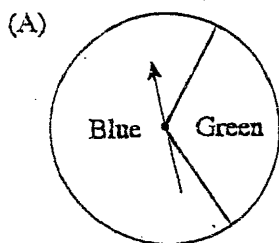
5. For the formula $v = u + at$, find the value of v if $u = 8$, $a = 10$ and $t = 5$.
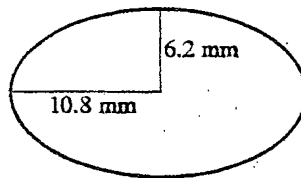
A. $v = 58$ B. $v = 42$ C. $v = 90$ D. $v = 113$

6. $3x(2x - 4y) + 2y(4y - 6x) =$

     A. $6x^2 - 8y^2$ B. $6x^2 + 8y^2$

     C. $6x^2 - 4xy - 8y^2$ D. $6x^2 - 24xy + 8y^2$

7. What is the area of the ellipse below ?

A. $\pi \times 6.2^2$ B. $\pi \times 8.5^2$ C. $\pi \times 10.8^2$ D. $\pi \times 10.8 \times 6.2$

6.2 mm
10.8 mm

8. Fifty tickets are sold in a raffle. There are two prizes and Michael buys five tickets.

    Which expression gives the probability that Michael wins both prizes?

       A. $^5/_{50} + {}^4/_{50}$ B. $^5/_{50} + {}^4/_{49}$

       C. $^5/_{50} \times {}^4/_{50}$ D. $^5/_{50} \times {}^4/_{49}$

9. Ali buys a television costing $1494 on interest free terms over 2 years. If he pays one third deposit, how much will he be required to pay each month ?

A. $20.75   B. $41.50   C. $43.58   D. $83.00

10. A $115 000 loan is repaid over a 25 year term at the rate of
$1221.21 per month. What is the total amount of interest that is paid on
this loan ?

A. $30280.25   B. $145 280.25   C. $251 363   D. $366 336

11. Mr & Mrs Yousef research the typical price of a family car. At one car
yard they find six family cars. Five are priced between $30 000 and
$40 000, while the sixth car is priced at $80 000. What is the best
measure of the price of a typical family car ?

A. Median   B. Mean   C. Mode   D. All are equally valid

12. Noel sells computer software and receives a retainer of $250 per week,
plus a commission of 5% for all sales over $10 000. In one week
Noel's sales reach $13460, he is paid a total of :

A. $17   B. $423   C. $673   D. $923

13. On a set of house plans a room which measures 4.5m x 3.2m is
shown as 9cm x 6.4cm. The scale of the house plan is :

A. 1:2   B. 1:50   C. 1:100   D. 1:200

14. The total surface of a square based pyramid with side of base b and
height of the triangular face h, is given by $A = b^2 + 2bh$. The total
surface area of the pyramid is 64cm^2 and the length of the side of the base
is 4cm, find the height of the triangular face.

A. 6cm   B. 10cm   C. 20cm   D. 24cm

15. The solution to $5^x = 300$ is closest to

A. 30   B. 3.7   C. 3.5   D. 3

16. Madeline invests $1000 per year for twenty years into an annuity. The interest rate is 6.5% p.a. and interest compounds annually. What is the present value of the annuity ?

A. $11 018.65   B. $3523.65   C. $18 825.31   D. $38 825.31

17. The solution to the equation $3(2x + 4) = 4(2x - 7)$ is :

A. $x = -\frac{1}{2}$ B. $x = 24$ C. $x = 8$ D. $x = 20$

18. A group of six people consists of Darren, Shinavo, Jim, Damien ,John and Allan. From these six people a group of three is chosen. The probability of choosing Darren, Jim and John is ?

A. $^3/_6$ B. $^1/_{20}$ C. $^1/_{12}$ D. $^1/_{120}$

19. A bag contains four black and six white marbles. Two marbles are drawn from the bag one after the other without replacement. If the first marble drawn is black, the probability that the second marble drawn is white is ?

A. $^4/_9$ B. $^3/_5$ C. $^2/_3$ D. $^2/_5$

20. The box-and-whisker plot drawn below shows Emma's performance in her physics and chemistry exams.
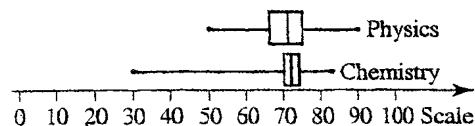
Which of the following statements is correct ?

A. The median of Emma's mark in physics is greater than for chemistry.
B. The range of Emma's marks in physics is greater than for chemistry.
C. The interquartile range of Emma's marks in physics is greater than in chemistry.
D. All of the above.

SECTION II (60 marks)

**Question 21** (12 marks) *Start a new sheet of paper.*

(a) An offset survey of a park is conducted and the notebook entries are shown below.

```
                          D
                          80
            (C) 42       65
                          40   28  (E)
            (B) 35       10
                          0
                          A
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(i) Use the notebook entries to draw a neat sketch of the park. (2 marks)
(ii) Calculate CD to the nearest metre.   (2marks)
(iii) Find the area of the park.   (3 marks).

(b)A water storage tank is the shape of a cube side length 1.8m, surmounted by a cylinder of diameter 1m, with height of 0.5m .
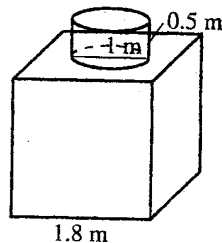
(i) Find the volume of the water storage tank .   (2 marks)

(ii) Calculate the capacity of the water tank, correct to nearest 100L.   (1 mark)

(c) A drop of water contains about 1.7×10^{21} molecules. A typical waterfall may have a flow of 7.5×10^{11} drops of water per minute. How many molecules of water flow over the waterfall in one minute. Give answer in scientific notation correct to three significant figures.   (2 marks)

Question 22 (12 marks)   *(Start a new page.)*

(a) Along a road there are three sets of traffic lights. The probability of catching a green light is 0.35 .

(i) What is the probability that any given light is not green ?   (1 mark)

By drawing a probability tree or otherwise, find the probability of :

(ii) Catching all three green lights   (1 mark)

(iii) Missing all three green lights   (1 mark)

(iv) Catching at least one green light   (1 mark)

(b) The table shows the result of random breathtesting by Sydney Police over one weekend. A driver is charged if they record a reading over 0.05% prescribed concentration of alcohol (PCA).

	MALES	FEMALES	TOTALS
Over 0.05 PCA	26	7	33
Below 0.05 PCA	962	743	1705
TOTALS	988	750	1738

(i) What percentage of those tested were female ?   (1 mark)
(ii) What percentage of the females tested had a PCA over 0.05 ? (1 mark)
(iii) What Percentage of male drivers had a PCA over 0.05 ?   (1 mark)
(iv) Based on the above results, can any conclusion be drawn, concerning the prevalence of drink driving among men and women ? Explain your answer.

(1 mark)

(c) The payment on a loan of $150 000 over a 20 year term at 9.6% p.a. is $1408.01 per month.

(i) Use this information to find values A, B and C in the table below.
(3 marks)

Month	Principal (P)	Interest (I)	P+I	Balance owing
1	$150 000	$1200	$151 200	$149 791.99
2	$149 791.99	$1198.34	A	$149 582.32
3	$149 582.32	B	$150 778.98	$149 370.97
4	$149370.97	$1194.98	$150 565.94	C

(ii) How much is paid off the loan at the end of four months ?   (1 mark)

Question 23 (12 marks)   (*Start a new page)*

(a) Theresa borrows $12 000 to buy a car. This is to be repaid over five years at $320 per month.

(i) Calculate the total amount to be repaid.   (1 mark)

(ii) How much interest does she pay ?   (1 mark)

(iii) Calculate the flat interest that Theresa has been charged.   (2 marks)

(b) The surface area of a closed cylinder is given by $A = 2\pi r(r+h)$, where r is the radius of the cylinder and h is the height in cm.

(i) Rearrange the formula of the surface area to make **h** the subject.

(2 marks)

(ii) If the surface area of a closed cylinder with a diameter of 6cm is $678.58cm^2$, find the height of the cylinder.   (1 mark)

(iii) What is the volume of the cylinder ?   (1 mark)

(c) Solve the following equation, showing all working.   (2 marks)

$$\sqrt{5x} - 3 = 4$$

(d) Zita is doing an exam and realises that she is almost out of time. There are still 10 questions that she has to answer, 6 are true/false questions and 4 are multichoice questions with answers A,B,C or D. If Zita guesses the answers to the last 10 questions, calculate the probability that she :

(i) Answers all the true/false questions correctly.   (1 mark)

(ii)Answers all the multichoice questions incorrectly.   (1 mark)

Question 24 (12 marks)   *Start a new page.*

(a) The diagram shows a vertical cross-section of a river.
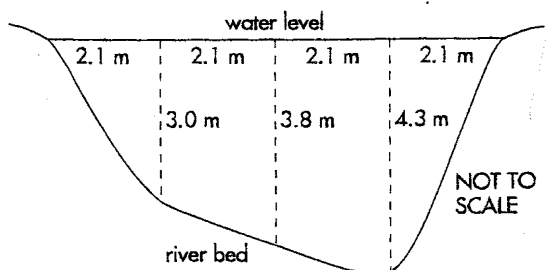
(i) Use two applications of Simpson's rule to find the approximate area of the river's cross-section. **(3 marks)**

(ii) Estimate the volume of water, in cubic metres, in a 50-metre length of this river, assuming the cross-section is the same as above and uniform along the 50-metre length. Give your answer to the nearest cubic metre.

(1 mark)

(b) The heights (to the nearest cm) of 17-year old boys and girls in a school year are shown in the stem and leaf plot below.

Boys		Girls
	15	4
	15*	5 6 8 9
4 3 0 0	16	2 3 3 3 4
9 9 8 8 8 8 7 7 7 6 5	16*	5 5 5 5 6 6 6 8 8 8 9 9
4 4 3 2 2 1 1 1 0	17	0 0 0 0 0 1 1 2
9 7 7 5 5	17*	5 6 7
0 1	18	
5	18*	

(i) Write down the range (in cm) of the heights of the girls in the survey.

(1 mark)

(ii) This is the five figure summary for the girls' heights :
minimum value = 154cm; maximum value = 177cm; medium = 166cm; lower quartile = 163cm; upper quartile = 170cm.
Determine the five figure summary for the boys' heights. **(3 marks)**

(iii) Consider the statement : " Generally, the boys are taller than the girls and the spread of the boy's heights is less than the girls,"
Do you agree with this statement ? Give reasons for your decision.
(Your answer to part (ii) should be useful in your explanation.) **(2 marks)**

(c) The area of a particular skin cancer is increasing at the rate of 1% per month. The initial area of the skin cancer was 0.8cm^2. Using an estimate or refinement technique or an alternative method, find when the area will be greater than 1cm^2.   (2 marks)

Question 25 (12 marks)   *Start a new page.*

(a)Two cars leave from the same place, X, at the same time. Car A travels due east at a speed of 60 km/hour. Car B travels due south at a speed of 80 km/hour.

Not drawn to scale
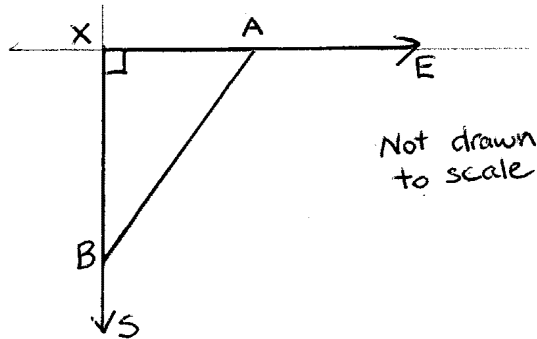
(i) Find the shortest distance between the two cars after 45 minutes.

(2 marks)

(ii) Find the size of $\angle$ XBA to the nearest degree.   (2 marks)

(b) Marcus and Simon both started working at the age of 20 years.

Marcus invested $1200 at the end of each year for the first 8 years of his working life. He then stopped these regular investments, but left his money to earn interest and grow for the next 32 years. The investment earned interest of 9% p.a. compounded yearly, over the entire 40 years.

(i) How much did Marcus invest over the first 8 years of his working life ?   (1 mark)
(ii) Show that the total value of Marcus' investment after 8 years was **$13 234.17**.   (2 marks)
(iii) Calculate the value of the investment after a further 32 years.

(1 mark)

Simon did not make any investments in the first 8 years of his working life. He then started investing $1200, at the end of each year, for the next 32 years, again earning 9% p.a. compounded yearly. After making contributions of $1200 p.a. for 4 times as long as Marcus, Simon thought that the value of his investment would be far more than the value of Marcus'.

(iv) Explain why the value of Simon's investment could never be greater than the value of Marcus' . (Your answer to (b) (ii) above may be useful in your explanation.)   (2 marks)

(c) A rectangular piece of metal 70cm by 45cm is cut to make the circular base and the side of an open cylinder, as shown.

Cut along line

15 cm

30 cm

20 cm

70 cm

15 cm

NOT TO SCALE
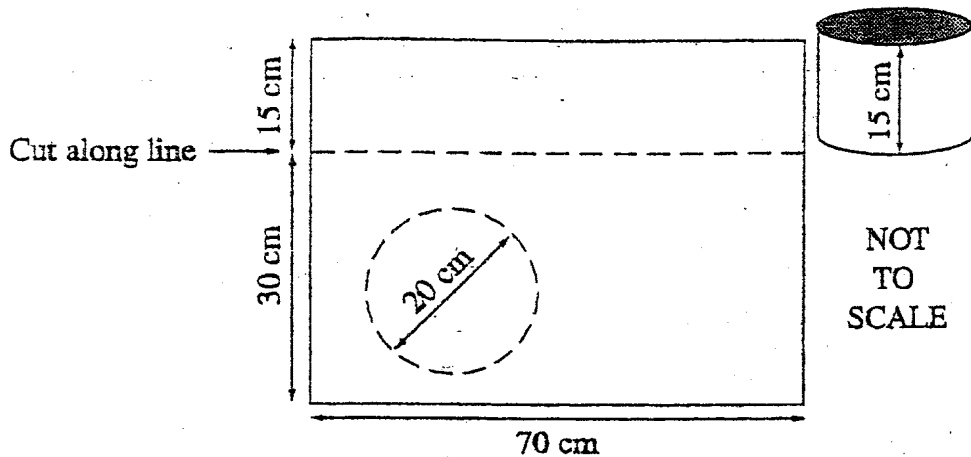
(i) The strip of width 15cm must be trimmed to fit around the base of diameter 20 cm, with no overlap.

Calculate the length of the strip required, to the nearest millimetre.
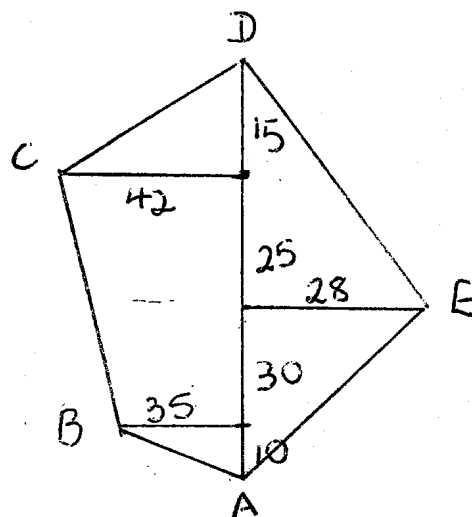   (2 marks)

21

```
        D
        80
 C 42 | 65
        40    28 E
 B 35 | 10
        O
        A
```

(i)

(2 marks)

(ii) $CD = \sqrt{42^2 + 15^2}$ (A)

$= 44 \cdot 6 m$ (1)

CD is 45m to nearest metre (1)

(iii) Area park $= \underset{315}{\underbrace{\dfrac{42 \times 15}{2}}} + \underset{2117\cdot5}{\underbrace{(42+35) \times \dfrac{55}{2}}} + \underset{175}{\underbrace{\dfrac{35 \cdot 40}{2}}} + \underset{1120}{\underbrace{\dfrac{80 \times 28}{2}}}$

$= 3727 \cdot 5 \, m^2$

(b) (i) Cylinder $= \pi x \times 0.5^2$ Cube $= 1.8^3$ (1)
       Total Volume $= 6 \cdots 2 \, m^3$ (2dp)

(ii) $6 \cdots 2 \times 1000 = 6 \cdot 20$ Litres. (A)
         Capacity $= 6 \cdot 20L = 6 \cdot 00 L$ (nearest 100L)

c) No molecules water / minute $= 1.7 \times 10^{21} \times 7.5 \times 10^{11}$ (1)
                    $= 1.28 \times 10^{33}$ (1)

a) (i) 0.65   (ii)

(ii)
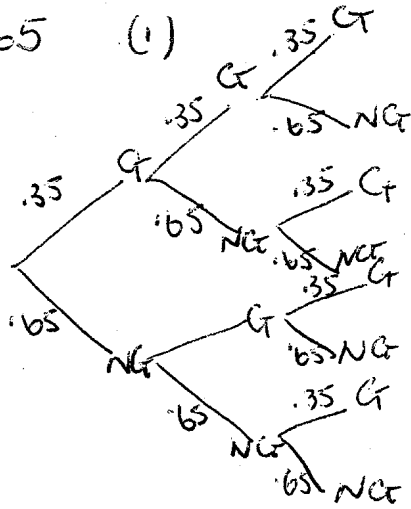
$$P(3\ G) = (0.35)^3 = 0.042875 \qquad (1)$$

(iii) $P(3\ NG) = (0.65)^3 = 0.274625 \qquad (1)$

(iv) $P(\text{at least one green}) = 1 - P(3NG) \qquad (1)$
$$= 0.725375.$$

b) i) 43.2%

   ii) 0.93%

   (iii) 2.63%

   (iv) The % of males driving above 0.05% was 3 times that of the females

c) i) A = $150 990.33 \qquad (1)$
   B = $1196.66 \qquad (1)$
   C = $149 157.93 \qquad (1)$
   ii) 150 000 − C = $842.07 \qquad (1)$

a) (i) $\$320 \times 60 = \$19200$ (1)

(ii) Interest $= \$19200 - \$12000 = \$7200$ (1)

(iii) Flat rate interest $= \dfrac{7200}{12000} \times 100 \div 5 = 12\%$ (1)

(1)

(b) $A = 2\pi r (r+h)$

i) $\dfrac{A}{2\pi r} = r+h$ (1)

$h = \dfrac{A}{2\pi r} - r$ (1)

(ii) $h = \dfrac{678.58}{2 \times \pi \times 3} - 3$

$= 33\,cm.$ (1)

(iii) $V = \pi r^2 h$

$= \pi \times 9 \times 33$

$= 933.05\,cm^3$ (1)

c) $\sqrt{5x} - 3 = 4$

$\sqrt{5x} = 7$ (1)

$5x = 49$

$x = 9.8$ (1)

d) (i) $\left(\dfrac{1}{2}\right)^6 = 0.015625.$ (1)

(ii) $\left(\dfrac{3}{4}\right)^4 = 0.316\ (3\,dp)$ (1)

Q24

a) Simpsons Rule (two applications)

i) Area $\doteq \frac{21}{3}(0 + 4\times 3.0 + 3.8) + \frac{21}{3}(3.8 + 4\times 4.3 + 0)$ (1)

$= 7(15.8) + 7(21)$

$= 257.6 m^2$ (1)

ii) $257.6 \times 50 = 12880 m^3$ (1)

(b) i) $154 \rightarrow 177$ - Range 23cm

ii) More For Boys   Minimum 160
(iii),   LQ   167
                    Med   170.5   (1) move max + min
                    UQ   174.5   correct.
                    Maximum 185   (1) med, LQ UQ
                                         correct.

(iii) Agree ~~better ly~~ as the boys generally are taller
as the median of boys height is 4.5cm taller than girls
+ the UQ + LQ one around 4cm taller. ~~~~
The range for both groups is similar but the IQR
for boys is around half that of the girls which
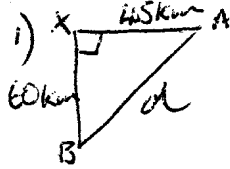means the boys heights are less spread out than the
girls.
   1 - looking at median for heights
   1 - IQR for spread.

c) $0.8(1.1)^n = 1$ Let n=1

n	value
1	0.88
2	0.968
3	1.064

The area becomes larger than
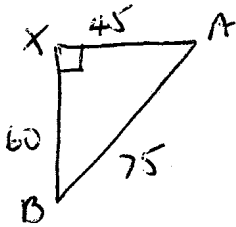1cm during the 3rd month.
    n = 2.34 mths.

Q 25

a)

i)

$d = \sqrt{45^2 + 60^2}$

$= 75 \text{km}$

(i) correct distances for car A + B

(i) correct use of Pyth.

ii)

$\tan B = \dfrac{45}{60} = .75 \text{ (or equivalent trig ratio)}$ (i)

$B = 36.9°$

$\therefore B = 37°$ to nearest degree (i)

b)

i) $9600 (i)

ii) Use future value $N = \dfrac{1200\left(1.09^8 - 1\right)}{0.09}$

M = 1200, r = 0.09

n = 8

$= \$13234.17$

(i) correct formula

(i) correct values for m, n, r.

iii) $13234.17 $\times \left(1.09\right)^{32} = \$208\,614.57$ (i)

iv) Simon's invest will amount to $196844.38 which is $11770.19 less than Marcus'. Simon starts with $13234.17 at beginning of the 32 years. Interest earned on this for one year is $14425 — which is more than Simon contributes each year, Simons interest will increase as the years go on, but so will Marcus', so Simon will not catch up.

— NOT sure about allocation — need to discuss! (2 marks)

c) length = circumference = $2\pi r$

r = 10 cm (i)

length = $2 \times \pi \times 10$

$= 62.8 \text{cm}$. (1 nearest mm)