## General Test Instructions

- Total marks: 100
- Reading time - 5 minutes
- Working time -2 hours 30 minutes
- Write using black or blue pen
- Calculators may be used
- A formulae sheet is provided at the back of this paper
- A multiple choice answer sheet is provided at the back of this paper


## GENERAL MATHEMATICS



## Section I Pages 5-11

## 22 marks

- Attempt Questions 1-22
- Allow about 30 minutes for this section.
- Use the answer sheet provided.


## Section II Pages 12-22

## 78 marks

- Attempt Questions 23-28
- Allow about 2 hours for this section.
- Answer each question in a separate booklet.


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## Section I

22 marks
Attempt Questions 1-22
Allow about 30 minutes for this section

Use the multiple-choice answer sheet.
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.
Sample:
$2+4=$
(A) 2
(B) 6
(C) 8
(D) 9
$\mathrm{A} \bigcirc$
$B$
C $\bigcirc$
D $\bigcirc$

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.
A
B

C

D


If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.


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## Section I

## 22 marks

Attempt Questions 1-22
Allow about 30 minutes for this section
Use the multiple-choice answer sheet for Questions 1-22

1. Expand and simplify $3 x(x-1)-x^{2}+4 x$
A. $4 x^{2}-7 x$
B. $2 x^{2}-7 x$
C. $4 x^{2}+x$
D. $2 x^{2}+x$
2. A survey was taken asking people about the number of songs on their I phones. This data is
A. Discrete
B. Continuous
C. Qualitative
D. Ordered
3. The total repayment for a loan of $\$ 45000$ on a flat rate of interest at $7.5 \%$ p.a. over four years is:
A. $\$ 13500$
B. $\$ 58500$
C. $\$ 59000$
D. $\$ 57000$
4. Three partners, Joe, Zelsa, and Jadon, invest in a business in the ratio $3: 4: 1$. The total amount invested is $\$ 125000$.

How much did Joe invest?
A. $\$ 15625$
B. $\$ 41667$
C. $\$ 46875$
D. $\$ 75000$
5. At the end of 2009 the price of a house in Dunedoo was $\$ 267500$. This price had fallen by $11 \%$ since the beginning of 2009.

What would have been the price of the house at the beginning of 2009 ?
A. $\$ 238075$
B. $\$ 240991$
C. $\$ 296925$
D. $\$ 300562$
6. Find the value of $v=\sqrt{u^{2}+2 a s}$

Find the value of $v$, given that $u=5, a=2 \cdot 2$, and $s=10$.
A. 6.63
B. 7
C. 8.31
D. 31.6

7 Which expression below represents the length of AB

A. $\frac{15 \sin 80^{\circ}}{\sin 40^{\circ}}$
B. $\frac{15 \sin 40^{\circ}}{\sin 60^{\circ}}$
C. $\frac{15 \sin 80^{\circ}}{\sin 60^{\circ}}$
D. $\frac{\sin 40^{\circ}}{15 \sin 80^{\circ}}$
8. The time taken for a journey varies inversely with a car's average speed. The journey takes 4 hours when the car travels at an average speed of $88 \mathrm{~km} / \mathrm{h}$.

How long would the same journey take at an average speed of $66 \mathrm{~km} / \mathrm{h}$ ?
A. 3 h
B. 5 h 20 min
C. 5 h 33 min
D. 22 h
9. Which one of the following groups of scores has a mean of 60 and a median of 50 ?
A. $10,50,60,70,80,90$
B. $40,40,45,55,70,90$
C. $40,45,45,55,85,90$
D. $30,40,50,50,70,80$
10. Which of the graphs below represents the equation $y=2^{x}$

(B)

(C)

(D)

11. A microchip is square, with side length $0 \cdot 0000003 \mathrm{~mm}$.

What is its area, expressed in scientific notation?
A. $3 \times 10^{-7} \mathrm{~mm}^{2}$
B. $3 \times 10^{7} \mathrm{~mm}^{2}$
C. $9 \times 10^{-14} \mathrm{~mm}^{2}$
D. $9 \times 10^{14} \mathrm{~mm}^{2}$
12. The perimeter of a rectangle is 84 cm . The length of the rectangle is twice the breadth. What is the length of the rectangle?
A. 7 cm
B. 14 cm
C. 21 cm
D. 28 cm
13. Solve for $p$ :
$\frac{p-3}{3}-\frac{p-2}{4}=1$
A. $\quad p=7$
B. $p=13$
C. $p=18$
D. $p=30$
14. Find the value of $h$

A. 2.25 m
B. $\quad 3.75 \mathrm{~m}$
C. 4.5 m
D. $\quad 7.5 \mathrm{~m}$
15.


The paddock enclosed by the fence shown in the diagram consists of a rectangle and a semicircle.

What is the area of the paddock to the nearest square metre?
A. 119
B. 159
C. 237
D. 394
16. Which of the following expressions is equivalent to $\left(6 t^{3}\right)^{2}$
A. $12 t^{6}$
B. $36 t^{6}$
C. $12 t^{5}$
D. $36 t^{5}$
17. Brett and Lance are very competitive in class tests. Lance claims that Brett overestimates what he will get after an exam by $25 \%$.

After his Trial HSC Brett estimates that he got $80 \%$. What does Lance think Brett got?
A. $20 \%$
B. $60 \%$
C. $64 \%$
D. $100 \%$
18. The formula below gives the approximate volume of a barrel:

$$
V=\frac{\pi}{12} h\left(2 D^{2}+d^{2}\right)
$$

where $h$ is the height of the barrel
$D$ is the diameter at the centre of the barrel $d$ is the diameter at the base of the barrel.


Find the approximate volume, in litres, of a barrel with dimensions $h=1 \cdot 1$ metres, $D=0.85$ metres and $d=0.70$ metres. $\left(1 \mathrm{~m}^{3}=1000 \mathrm{~L}\right)$
A. 0.490
B. 0.557
C. 490
D. 557
19. Bart made two errors in his solution to the following equation.

$$
\begin{aligned}
& 2(x+5)-5(x-5)=27 \\
& 2 x+10-5 x-25=27 \text {.......................... Line } 1
\end{aligned}
$$

$$
\begin{aligned}
& -3 x=42 \text {.......................... Line } 3 \\
& x=14 \quad \text {.......................... Line } 4
\end{aligned}
$$

Which lines DO NOT follow correctly from the previous line?
A. Line 1 and Line 4
B. Line 1 and Line 3
C. Line 2 and Line 4
D. Line 2 and Line 3
20.

Which of the following graphs represents the equation $y=2 x+4$ ?
(A)

(B)

(C)

(D)


21 It was found that in a test one of the questions had a printing error which made the question invalid. As no students were awarded any marks for answering that question it was decided that 5 marks would be added to every student's mark. Which of the following will be true?
(A) The mean and the standard deviation will remain the same.
(B) The mean will increase by 5 and the standard deviation will remain the same.
(C) The mean will increase by 5 and the standard deviation will increase by 25.
(D) The mean will increase by 5 and the standard deviation will increase by 5 .
22. William wanted to treat himself to an overseas trip after his HSC. He started a part time job when he was back in Year 9 to finance this holiday. He opened a special Holiday Investment account which paid him $6 \%$ p.a. monthly compound interest into which he deposited $\$ 800$ every month. Which calculation would give the future value of this annuity after 36 months?
(A)

$$
\$ 800\left\{\frac{(1.06)^{36}-1}{.06}\right\}
$$

$$
\begin{equation*}
\$ 800\left\{\frac{(1.06)^{6}-1}{.06}\right\} \tag{B}
\end{equation*}
$$

(C)

$$
\$ 800\left\{\frac{(1.005)^{6}-1}{.005}\right\}
$$

$$
\begin{equation*}
\$ 800\left\{\frac{(1.005)^{86}-1}{0.005}\right\} \tag{D}
\end{equation*}
$$

## END OF SECTION 1

## Section II

## 78 Marks

Attempt Questions 23-28
Allow about $\mathbf{1 2 0}$ minutes for this section
Answer each question in the appropriate writing booklet. Extra writing booklets are available.
All necessary working should be shown in every question.

## Marks

Question 23 (13 marks) Use a Separate Writing Booklet
(a) Substitute $a=2.21, b=-1.50$ and $c=\frac{3}{4}$ into $a^{2}-b^{2}+c^{2}$, giving your answer correct

2 to two decimal places.
(b) PURA-WATER comes in a large cylindrical container with diameter 30 cm and height 40 cm . PURA disposable cups are conical. They have a diameter of 5 cm and can be filled to a depth of 6 cm .

i) Find the volume of water in a full PURA - WATER container, answer correct to the nearest $\mathrm{cm}^{3}$.
ii) If a cup is filled to a depth of 6 cm , how many cups can be filled from a full container.
iii) A company uses two containers of PURA-WATER per week. Cups can only be bought in boxes of 1000, at a cost of $\$ 18$ per box. Find the cost of cups for one year.
(Assume that only PURA cups are used and that they are filled and used only once)
(c) Bill Gates bought the new iPad 3 for $\$ 699$ on hire purchase. Terms included a $10 \%$ deposit with the balance being paid the balance at $\$ 60$ per month for 1 year.
i) What was the total cost of Bill Gates' iPad 3?
ii) How much interest did Bill Gates pay?
iii) What rate of interest was Bill Gates charged?
d) Convert $30 \mathrm{~km} / \mathrm{h}$ to $\mathrm{m} / \mathrm{s}$

## END OF QUESTION 23

Question 24 (13 marks) Use a Separate Writing Booklet
a) The location of three towns, Ubiri $(U)$, Vanati $(V)$ and Wallarah $(W)$ is shown in the diagram. Vanati is due east of Ubiri.


NOT TO SCALE
i) What is the bearing of $W$ from $V$.
ii) How far south of $V$ is $W$. Answer correct to the nearest km .
iii) Find the distance between $U$ and $W$.

Answer correct to the nearest metre.
b) Given the formula $e^{2}=1-\frac{b^{2}}{a^{2}}$,
i) make $a$ the subject of the formula.
ii) If $e=0.64$ and $b=7.5$, find the value of $a$ correct to 3 significant figures.
c) A farmer wants to build a small rectangular enclosure for a vegetable garden. She has 18 metres of fencing available.
i) Let the dimensions of the rectangle be $x$ and $y$. Find an expression for $y$ in terms of $x$.
ii) Show that the area of the rectangle is given by the formula $A=9 x-x^{2}$.
iii) Complete the table of values below for the equation $A=9 x-x^{2}$

| $\boldsymbol{x}$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Area |  |  |  |  |  |  |  |  |  |

Draw a neat graph of the function in your answer booklet.
Hence state the maximum area of the vegetable garden and the dimensions that give the maximum area.

## END OF QUESTION 24

Question 25 (13 marks) Use a Separate Writing Booklet
a) Aaron decides to borrow $\$ 150000$ over a period of 20 years at a rate of $7.0 \%$ per annum.

| Monthly Repayment Table <br> Principal and interest per \$1000 |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Interest rate | Terms of loan - years |  |  |  |  |  |
| $\mathbf{p . a .}$ |  |  |  |  |  |  |

i) Using the Monthly Repayment Table, calculate Aaron's monthly repayment.
ii) How much interest does he pay over the 20 years?
iii) Aaron calculates that if he repays the loan over 15 years, his total repayments would be $\$ 242730$. How much interest would he save by repaying the loan over 15 years instead of 20 years?
b) Sydney (Australia) is located at $\left(34^{\circ} \mathrm{S}, 151^{\circ} \mathrm{E}\right)$. Los Angeles is located at $\left(34^{\circ} \mathrm{N}, 119^{\circ} \mathrm{W}\right)$.
i) What is the difference in longitude between the two cities?
ii) It is 11.30 pm in Los Angeles on a Saturday. What is the day and time in Sydney?
iii) Find the distance between Sydney and the Equator. Give you answer to the nearest kilometre. (You may assume the radius of the Earth is 6400 km ).
c) Solve $\frac{(x-1)}{3}=\frac{2 x}{7}$

## END OF QUESTION 25

## Question 26 (13 marks) Use a Separate Writing Booklet

a) Luke invests $\$ 520$ at the end of each quarter into an annuity which compounds quarterly with an annual interest rate of $4 \%$.
i) Show that after five years the value of Luke's annuity is $\$ 11450$ (to the nearest dollar).
ii) Calculate how much Luke's sister, Jane, needs to invest today to have a total amount of $\$ 11450$ after five years at $4 \%$ interest per annum compounding quarterly.
iii) Explain who invests more to achieve a return of $\$ 11450$ after 5 years and state how much more is invested.
iv) Write down the main advantage of Luke's investment over Jane's.
v) Explain how Luke could have increased the return on his investment without changing the amount he invested or the time over which he invested it. Give reasons for your answer.
b) A radial survey of a disused tip was taken and the following diagram was produced.

i) Find the size of $\angle A O D$.
ii) Calculate the area of $\triangle A O D$ to the nearest $\mathrm{m}^{2}$
c) The diagram shows a vertical cross-section of a creek.

i) By using Simpson's rule twice, find an approximation for the area of this cross-section of the creek.
ii) Assume that an 85 metre length of this creek has approximately the same cross-section as above. Estimate the volume of water in this section of the creek.
iii) Convert the volume of the 85 metre section of creek to capacity.

Answer to the nearest kL.

## END OF QUESTION 26

Question 27 (13 marks) Use a separate writing booklet
(a) A standard die (faces numbered 1 to 6 ) is rolled twice and the two numbers on the uppermost faces are multiplied together.


Copy the table below into your booklet and complete it to show all of the possible outcomes.

| $\mathbf{X}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{1}$ | 1 |  |  |  |  |  |
| $\mathbf{2}$ |  |  |  | 8 |  |  |
| $\mathbf{3}$ |  |  |  |  |  |  |
| $\mathbf{4}$ |  |  |  |  |  |  |
| $\mathbf{5}$ |  |  |  |  |  |  |
| $\mathbf{6}$ |  | 12 |  |  |  |  |

(i) What is the probability of getting an odd product? $\mathbf{1}$
(ii) What is the probability of getting a perfect square?

Grant plays a game where it costs $\$ 3$ to play. If he throws an odd product he wins $\$ 5$. If he throws a perfect square he wins $\$ 10$ and if he throws anything else he loses.
(iii) What is the financial expectation of this game?

## (b)

John East Cannery farm salmon in Tasmania. Ideally they want the fish to be normally distributed and to have a mean length of 30 cm with a standard deviation of 2.5 cm .
(i) What percentage of fish would have a length between 27.5 cm and 32.5 cm ?
(ii) Fish are rejected if they are less than 25 cm in length.

What percentage of fish would be rejected?
(iii) A fish is chosen at random. What is the probability that it is longer than 37.5 cm ? 1
(iv) The owners of the cannery want to estimate the number of fish that they have in the farm. Describe how this could be done and create an example.
(c) Seoul in South Korea has position coordinates ( $37^{\circ} \mathrm{N}, 127^{\circ} \mathrm{E}$ ). What are the position coordinates of a town that is 2400 nautical miles due south of Seoul?

END OF QUESTION 27

Question 28 (13 marks) Use a separate writing booklet
(a) The graph illustrates the repayment schedules for a loan of $\$ 200000$ taken out by Aaron and Abel at the same time.
The interest rate remains the same for the period of the loan.


Use the above graph to answer the following questions.
(i) Who pays their loan off earlier and by how many months?
(ii) How much does Aaron owe when Abel has finally paid off his loan?
(iii) After 170 months, how much more does Aaron owe on his loan than Abel? $\mathbf{1}$
(iv) Write down the gradient of the line representing Abel's repayments and describe what it represents.

## Question 28 continues over page

(b) (i) Calculate the $z$-score for a score of 57 when the mean is 45 and the standard deviation is 4 .
(ii) Calculate the mean of a distribution which has a standard deviation of 8 and a score of 32 that corresponds to $z$-score of 1.25 .
(c) The swing period of a pendulum varies directly as the square root of its length.
(i) For a pendulum of length 9 cm with a swing period of 6 seconds, show that the constant of variation is 2 .
(ii) Hence find the length of the pendulum with a swing period of 98 seconds.

## END OF EXAMINATION

## General Mathematics

## FORMULAE SHEET

## Area of an annulus

$A=\pi\left(R^{2}-r^{2}\right)$
$R=$ radius of the outer circle
$R=$ radius of the inner circle

## Surface area

Sphere $\quad A=4 \pi r^{2}$
Closed cylinder $\quad A=2 \pi r h+2 \pi r^{2}$
$r=$ radius
$h=$ perpendicular height

Volume
Cone
$V=\frac{1}{3} \pi r^{2} h$
Cylinder

$$
V=\pi r^{2} h
$$

Pyramid $V=\frac{1}{3} A h$

Sphere
$v=\frac{4}{3} \pi r^{3}$
$r=$ radius
$h=$ perpendicular height
$A=$ area of base

Sine rule
$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

## Area of a triangle

$A=\frac{1}{2} a b \sin C$

## Cosine rule

$$
c^{2}=a^{2}+b^{2}-2 a b \cos C
$$

or

$$
\cos C=\frac{a^{2}+b^{2}-c^{2}}{2 a b}
$$

## Simple interest

$I=P r n$

## Declining balance formula for depreciation

$P=$ Initial quantity
$R=$ percentage interest rate per period,
expressed as a decimal
$n=$ number of periods

## Compound interest

$A=P(1+r)^{n}$
$A=$ final balance
$P=$ initial balance
$n=$ number of compounding periods
$r=$ percentage interest rate per compounding period, expressed as a decimal

Future value (A) of an annuity

$$
A=M\left\{\frac{(1+r)^{n}-1}{r}\right\}
$$

$M=$ contributions pr period, paid at the end of the period

Present value ( $N$ ) of an annuity

$$
N=M\left\{\frac{(1+r)^{n}-1}{r(1+r)^{n}}\right\} \quad \text { or }
$$

$$
N=\frac{A}{(1+r)^{n}}
$$

## Straight -line formula for depreciation

$S=V_{0}-D n$
$S=$ Salvage value of assets after $n$ periods
$V_{0}=$ purchase price of the asset
$D=$ amount of depreciation apportioned per
Period
$n=$ number of periods
$S=V_{0}(1-r)^{n}$
$S=$ Salvage value of assets after $n$ periods
$r=$ percentage interest rate per period, expressed as a decimal.

## Mean of a sample

$\bar{x}=\frac{\sum x}{n}$
$\bar{x}=\frac{\sum f x}{\sum f}$
$\bar{x}=$ mean
$x=$ individual score
$n=$ number of scores
$f=$ frequency

## Formula for a $\boldsymbol{z}$-score

$z=\frac{x-\bar{x}}{s}$
$s=$ standard deviation

## Gradient of a straight line

$m=\frac{\text { vertical change in position }}{\text { horizontal change in position }}$

## Gradient-intercept form of a straight line

$y=m x+b$
$m=$ gradient
$b=y$-intercept

## Probability of an event

The probability of an event where outcomes are equally likely is given by:
$P($ event $)=\frac{\text { number of favourable outcomes }}{\text { total number of outcomes }}$
ПП|

## 2012 GENERAL MATHEMATICS

 HIGHER SCHOOL CERTIFICATE Half Yearly Examination
## SECTION I

Multiple Choice Questions Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.
Start
Here $\rightarrow$ 1. $\mathrm{A} \circ \mathrm{B} \circ \mathrm{C} \circ \mathrm{D} \circ$
2. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
3. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
4. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
5. $\mathrm{A} \bigcirc \mathrm{B} \bigcirc \mathrm{C} \bigcirc \mathrm{D} \bigcirc$
6. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
7. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
8. $\mathrm{A} O$
B $O$
$C \bigcirc D O$
9. $\mathrm{A} O$
B $O$
C
D
10. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
11. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
12. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
13. $A \bigcirc B \bigcirc C \bigcirc D O$
14. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
15. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
16. $A \bigcirc B \bigcirc C \bigcirc D O$
17. $A \bigcirc B \bigcirc C \bigcirc D O$
18. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
19. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
20. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
21. $A \bigcirc B \bigcirc C \bigcirc D \bigcirc$
22. $\mathrm{A} \bigcirc \mathrm{B} \bigcirc \mathrm{C} \bigcirc \mathrm{D} \bigcirc$

