

# TRINITY GRAMMAR SCHOOL MATHEMATICS DEPARTMENT



## YEAR 12 2009 HALF YEARLY EXAMINATION

## GENERAL MATHEMATICS

MONDAY, 4<sup>TH</sup> MAY 2009 '8:30am-11:00am

Weighting: 30% towards final grade.

### **INSTRUCTIONS:**

- 1. Reading time 5 minutes
- 2. Working time TWO AND A HALF Hours
- 3. Write using black or blue pen
- 4. Board approved calculators may be used
- 5. A formulae sheet is provided on pages 27 and 28 of the examination booklet.
- 6. Write your candidate number and teacher's name on the multiple choice sheet and the 6 answer booklets.
- 7. Attempt ALL questions.
- 8. Show all necessary working.
- 9. Begin each question in Section II in a new booklet.
- 10. Mark values are shown at the side of each question.
- 11. Hand in all sections, multiple choice sheet and Question paper.

#### Section I 22 marks

- Attempt Questions 1-22
- Allow about 30 minutes for this section

#### Section II 78 marks

**TOTAL MARKS - 100** 

- Attempt Questions 23-28
- Allow about 2 hours for this section

Section I: Multiple choice	/22
Section II:	
Q23 Equations and Functions	/13
Q24: Area and Volume	/13
Q25: Credit and Loans	/13
Q26: Statistical Distributions	/13
Q27: Trigonometry	/13
Q28: Probability	/13

Total = /100

## Section I

22 ma ks
Attempt Questions 1–22
Allow about 30 minutes for this section

Use the multiple-choice answer sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:

2 + 4 =

(A) 2

(B) 6

(C) 8

 $c \bigcirc$ 

(D) 9

A ()

D 🔾

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A  $\blacksquare$ 

в 💓

c O

 $D \bigcirc$ 

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

corr

A 💓

В

c (

D 🔾

1 The results of a geography test are displayed in a stem-and-leaf plot.

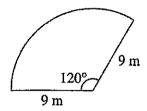
What is the range of the data?

- (A) 15
- (B) 27
- (C) 29
- (D) · 31
- 2 Which of the following is the correct simplification of  $8x^3 5x^3$ ?
  - (A)  $3x^6$
  - (B)  $3x^3$
  - (C) 3x
  - (D) 3
- The Great Pyramid of Egypt has a square base of side 230 m. Its perpendicular height is 135 m.

What is the volume of the pyramid?

- (A)  $10\,350\,\mathrm{m}^3$
- (B) 1 397 250 m<sup>3</sup>
- (C) 2380500 m<sup>3</sup>
- (D)  $7141500 \,\mathrm{m}^3$

4 This is a sketch of a sector of a circle.

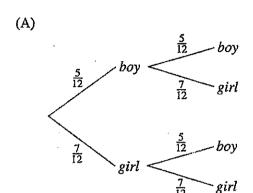


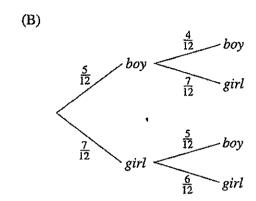
Casculate the area of this sector (correct to one decimal place).

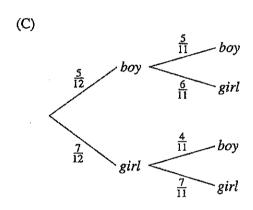
- (A)  $9.4 \text{ m}^2$
- (B) 18.8 m<sup>2</sup>
- (C)  $36.8 \text{ m}^2$
- (D) 84.8 m<sup>2</sup>
- 5 Which graph best represents  $y = 3^x$ ?
  - (A) y
- (B) y
- (C) y
- (D) y

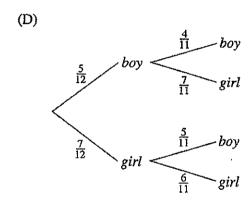
From 5 boys and 7 girls, two children will be chosen at random to work together on a project.

Which of the following probability trees could be used to determine the probability of choosing a boy and a girl?

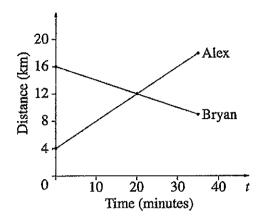








At the same time, Alex and Bryan start riding towards each other along a road. The graph shows their distances (in kilometres) from town after t minutes.

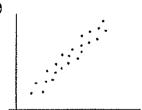


How many kilometres has Alex travelled when they meet?

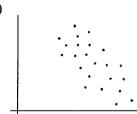
- (A) 4
- (B) 8
- (C) 12
- (D) 20

8 Which scatterplot shows a low (weak) positive correlation?

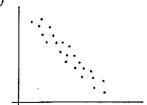
(A)



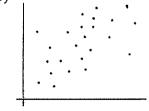
(B)



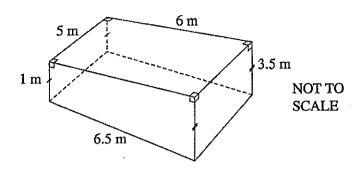
(C)



(D)



9 A swimming pool has a length of 6 m and a width of 5 m. The depth of the pool is 1 m at one end and 3.5 m at the other end, as shown in the diagram.

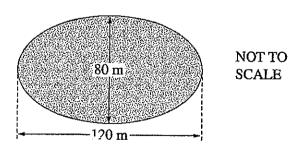


What is the volume of this pool in cubic metres?

- (A) 67.5
- (B) 105
- (C) 109.375
- (D) 113.75
- Joyce measures the length of a piece of wood as 250 mm, correct to the nearest mm.

What is the percentage error in her measurement?

- (A) ±0.002%
- (B) ±0.004%
- (C) ±0.2%
- (D) ±0.4%
- The council wants to put new grass on a park which is in the shape of an ellipse. If grass costs \$7.50 per square metre, what is the total cost to the nearest dollar?



- (A) \$7 540
- (B) \$30 159
- (C) \$56 549
- (D) \$226 195

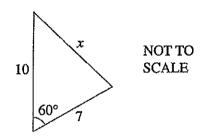
Use the following information to answer Questions 12 and 13.

Joy asked the students in her class how many brothers they had. The answers were recorded in a frequency table as follows.

Number of brothers	Frequency
6	5
1	10
2	3
3	1
4	1

- 12 What is the mean number of brothers?
  - (A) 1.15
  - (B) 2
  - (C) 2.3
  - (D) 4
- One of the students is chosen at random. What is the probability that this student has at least two brothers?
  - (A) 0.10
  - (B) 0.15
  - (C) 0.25
  - (D) 0.75

14



Which equation should be used to obtain the value of x in this triangle?

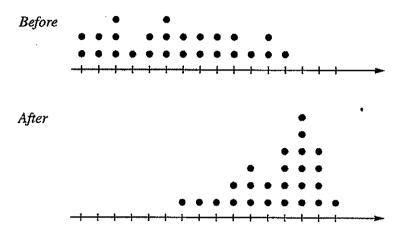
$$(A) \quad \frac{x}{\sin 60^{\circ}} = \frac{7}{\sin 10^{\circ}}$$

(B) 
$$x^2 = 10^2 + 7^2 - 2 \times 10 \times 7\cos 60^\circ$$

(C) 
$$\cos 60^\circ = \frac{x^2 + 10^2 - 7^2}{2 \times 10 \times 7}$$

(D) 
$$x^2 = 10^2 - 7^2$$

The dot plots below are drawn on the same scale. They show the class scores in tests taken before and after a unit of work was completed.



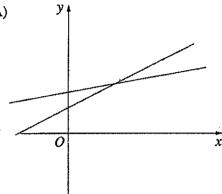
Which statement about the change in scores is correct?

- (A) The mean increased and the standard deviation decreased.
- (B) The mean increased and the standard deviation increased.
- (C) The mean decreased and the standard deviation decreased.
- (D) The mean decreased and the standard deviation increased.

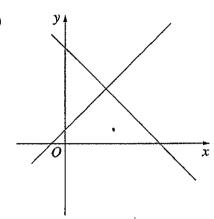
George drew a correct diagram that gave the solution to the simultaneous equations y=2x-5 and y=x+6.

Which diagram did he draw?

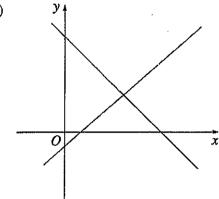
(A)



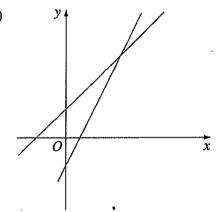
(B)



(C)



(D)



17 Rita purchased a camera for \$880 while on holidays in Australia. This price included 10% GST. When she left Australia she received a refund of the GST.

What was Rita's refund?

- (A) \$80
- (B) \$88
- (C) \$792
- (D) \$800

- 18 Which of the following correctly expresses T as the subject of  $B = 2\pi \left(R + \frac{T}{2}\right)$ ?
  - $(A) \quad T = \frac{B}{\pi} 2R$
  - (B)  $T = \frac{B}{\pi} R$
  - (C)  $T = 2R \frac{B}{\pi}$
  - (D)  $T = \frac{B}{4\pi} \frac{R}{2}$
  - 19  $I_{at}$  one year, the population of a city increased by 20%. The next year, it decreased by 10%.

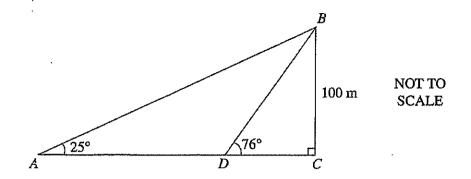
What was the percentage increase in the population over the two years?

- (A) 8%
- (B) 10%
- (C) 15%
- (D) 30%
- Dave's school has computer security codes made up of four digits (eg 0773). Juanita's school has computer security codes made up of five digits (eg 30568).

How many more codes are available at Juanita's school than at Dave's school?

- (*f.*) 10
- (B) 50
- (C) 90 000
- (D) 100 000

21 Calculate the length of AD (to the nearest metre).



- (A) 25 m
- (B) 134 m
- (C) 190 m
- (D) 214 m

#### 22 Join knows that

- one Australian dollar is worth 0.62 euros
- one Vistabella dollar (\$V) is worth 1.44 euros.

John changes 25 Australian dollars to Vistabella dollars.

How many Vistabella dollars will he get?

- (A) \$V10.76
- (B) \$V22.32
- (C) \$V28.00
- (D) \$V58.06

(c) Two unbiased dice, A and B, with faces numbered 1, 2, 3, 4, 5 and 6 are rolled. The numbers on the uppermost faces are noted. This table shows all the possible outcomes.

		Die A					
,		1	2	3	4	5	6
	1	1,1	1,2	1,3	1,4	1,5	1,6
	2	2,1	2,2	2,3	2,4	2,5	2,6
Die B	3	3,1	3,2	3,3	3,4	3,5	3,6
DIC D	4	4,1	4,2	4,3	4,4	4,5	4,6
	5	5,1	5,2	5,3	5,4	5,5	5,6
	6	6,1	6,2	6,3	6,4	6,5	6,6

A game is played where the difference between the highest number showing and the lowest number showing on the uppermost faces is calculated.

(i) What is the probability that the difference between the numbers showing on the uppermost faces of the two dice is one?

In the game, the following applies.

Difference	Result
0	Win \$3.50
1	Lose \$5
2, 3, 4 or 5	Win \$2.80

- (ii) What is the financial expectation of the game?
- (iii) If Jack pays \$1 to play the game, does he expect a gain or a loss, and how much will it be?

End of paper

3

#### Section II

78 marks Attempt Questions 23–28 Allow about 2 hours for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

All necessary working should be shown in every question.

Marks

1

1

Question 23 (13 marks) Use a SEPARATE writing booklet.

- (a) Calculate  $(3.21 \times 10^{23}) \times (7.34 \times 10^{-21})$ . Give your answer in scientific notation. 1
- (b) A clubhouse uses four long-life light globes for five hours every night of the year. The purchase price of each light globe is \$6.00 and they each cost \$d\$ per hour to run.
  - (i) Write an equation for the total cost (\$c) of purchasing and running these four light globes for one year in terms of d.
  - (ii) Find the value of d (correct to three decimal places) if the total cost of running these four light globes for one year is \$250.
  - (iii) If the use of the light globes increases to ten hours per night every night of the year, does the total cost double? Justify your answer with appropriate calculations.
  - (iv) The manufacturer's specifications state that the expected life of the light globes is normally distributed with a standard deviation of 170 hours.

What is the mean life, in hours, of these light globes if 97.5% will last up to 5000 hours?

Question 23 continues on page 14

- (c) An alcoholic drink has 5.5% alcohol by volume. The label on a 375 mL bettle says it contains 1.6 standard drinks.
  - (i) How many millilitres of alcohol are in a 375 mL bottle?

2

(ii) It is recommended that a fully-licensed male driver should have a maximum of one standard drink every hour.

2

Express this as a rate in millilitres per minute, correct to one decimal place.

(d) Solve  $\frac{5x+1}{2} = 4x-7$ .

3

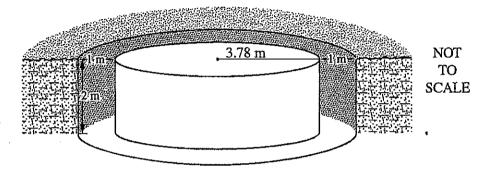
**End of Question 23** 

3

2

(a) A cylindrical water tank, of height 2 m, is placed in the ground at a school.

The radius of the tank is 3.78 metres. The hole is 2 metres deep. When the tank is placed in the hole there is a gap of 1 metre all the way around the side of the tank.



- (i) When digging the hole for the water tank, what volume of soil was removed? Give your answer to the nearest cubic metre.
- (ii) Sprinklers are used to water the school oval at a rate of 7500 litres per hour.

The water tank holds 90 000 litres when full.

For how many hours can the sprinklers be used before a full tank is emptied?

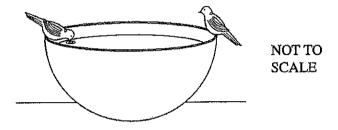
(iii) Water is to be collected in the tank from the roof of the school hall, which has an area of 400 m<sup>2</sup>. During a storm, 20 mm of rain falls on the roof and is collected in the tank.

How many litres of water were collected?

Question 24 continues on page 16

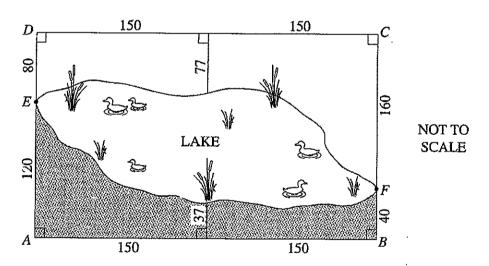
(b) In her garden, Keryn has a birdbath in the shape of a hemisphere (half a sphere). The internal diameter is 45 cm.

2



What is the internal surface area of this birdbath? (Give your answer to the nearest square centimetre.)

(c) In order to find the area of a lake, Bob took some measurements (in metres) and drew the following diagram.



(i) Use Simpson's Rule to find the shaded area ABFE.

2

(ii) Calculate the area of the lake.

3

## **End of Question 24**

(a) Aaron decides to borrow \$150 000 over a period of 20 years at a rate of 7.0% per annum.

MONTHLY REPAYMENT TABLE						
Principal and interest per \$1000 borrowed						
Interest		Term of loan – years				
rate (pa)						30
6.5%	19.57	11.35	8.71	7.46	6.75	6.32
7.0%	19.80	11.61	8.99	7.75	7.07	6.65
7.5%	20.04	11.87	9.27	8.06	7.39	6.99
8.0%	20.28	12.13	9.56	8.36	7.72	7.34

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- (i) Using the Monthly Repayment Table, calculate Aaron's monthly repayment.
- (ii) How much interest does he pay over the 20 years?
- (iii) Aaron calculates that if he repays the loan over 15 years, his total repayments would be \$242 730.

How much interest would he save by repaying the loan over 15 years instead of 20 years?

Question 25 continues on page 18

(b) David is paid at these rates:

Weekday rate	\$18.00 per hour
Saturday rate	Time-and-a-half
Sunday rate	Double time

His time sheet for last week is:

	Start	Finish	Unpaid break
Friday	9.00 am	1.30 pm	30 minutes
Saturday	9.00 am	4.00 pm	1 hour
Sunday	8.00 am	2.00 pm	I hour

(i) Calculate David's gross pay for last week.

3

1

(ii) David decides not to work on Saturdays. He wants to keep his weekly gross pay the same. How many extra hours at the weekday rate must he work? ,

- (c) Vicki earns a taxable income of \$58 624 from her job with an insurance company. She pays \$14 410.80 tax on this income.
  - (i) Vicki has a second job which pays \$900 gross income per month.

1

What is Vicki's total annual taxable income from both jobs, assuming that she has no allowable tax deductions?

(ii) Use the tax table below to calculate the total tax payable on her income from both jobs.

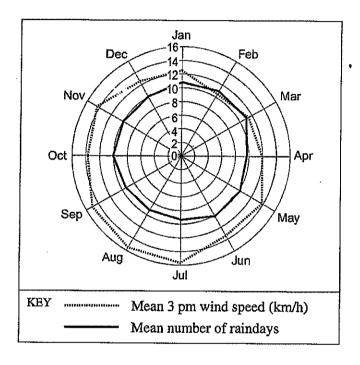
2

Taxable income	Tax payable
\$0-\$6 000	NIL
\$6 001–\$22 000	18 cents for each \$1 over \$6 000
\$22 001-\$55 000	\$2 880 plus 30 cents for each \$1 over \$22 000
\$55 001–\$66 000	\$12 780 plus 45 cents for each \$1 over \$55 000
\$66 001 and over	\$17 730 plus 48 cents for each \$1 over \$66 000

**End of Question 25** 

(a) The following graphs have been constructed from data taken from the Bureau of Meteorology website. The information relates to a town in New South Wales.

The graphs show the mean 3 pm wind speed (in kilometres per hour) for each month of the year and the mean number of days of rain for each month (raindays).



(i) What is the mean 3 pm wind speed for September?
(ii) Which month has the lowest mean 3 pm wind speed?
(iii) In which three-month period does the town have the highest number of raindays?
(iv) Briefly describe the pattern relating wind speed with the number of raindays for this town. Refer to specific months.

Question 26 continues on page 20

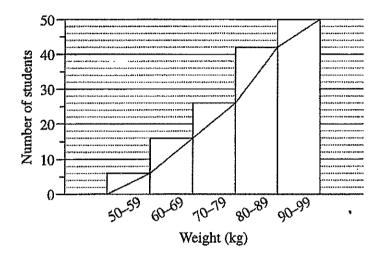
1

1

1

2

(b) Armand recorded the weights of a random sample of male students in his Year. The cumulative frequency graph displays the results.

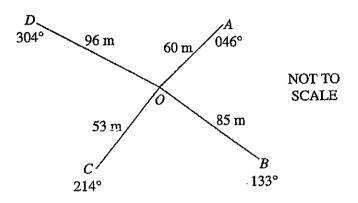


- (i) How many of the students surveyed were in the 80-89 kg class?
- (ii) Estimate the median weight of the students surveyed.
- (iii) Of the 300 male students in Armand's Year, how many would you expect to weigh less than 70 kg?
- (iv) 1 In order to select a sample, Armand's friend suggested selecting the first 50 male students in his Year to arrive at school on Monday morning. Explain why this would NOT be a random sample.
  - 2 Describe a method that could have been used to select a random sample of the male students.
- (c) Christina has completed three Mathematics tests. Her mean mark is 72%.

What mark (out of 100) does she have to get in her next test to increase her mean mark to 73%?

**End of Question 26** 

(a) The following notebook entry was made during a radial survey of a field.



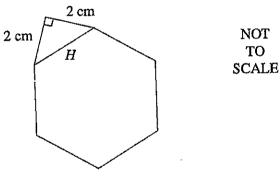
(i) What is the size of  $\angle AOB$ ?

- 2
- (ii) Calculate the area of triangle AOB. Round your answer to the nearest square metre.
- 2

(iii) Find the distance from A to B.

- 2
- (b) This shape is made up of a right-angled triangle and a regular hexagon.



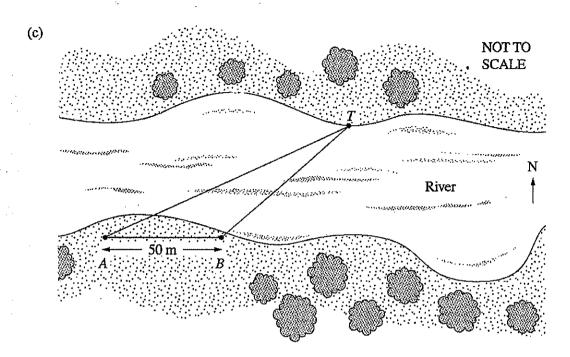


The area of a regular hexagon can be estimated using the formula  $A = 2.598H^2$  where H is the side-length.

Calculate the total area of the shape using this formula.

Question 27 continues on page 22

## Question 27 (continued)



In the diagram above, the following measurements are given:

 $\angle TAB = 30^{\circ}$ .

B is 50 m due east of A.

The bearing of T from B is 020°.

Copy or trace  $\triangle ABT$  into your answer booklet.

(i) Explain why  $\angle ABT$  is 110°.

1

(ii) Calculate the distance BT (to the nearest metre).

3

End of Question 27

2

2

1

## Question 28 (13 marks) Use a SEPARATE writing booklet.

- (a) Five men and three women are living on an island, but not all will be able to stay.
  - (i) If one person is selected at random, what is the probability that this person is female?
  - (ii) Two people are to be randomly selected to leave the island. .
    - 1 Copy the tree diagram into your writing booklet, and complete the diagram by writing the probabilities on all the branches.

1st selection 2nd selection

Female

Male

Male

Male

Male

- 2 Calculate the probability that the selection includes exactly one female.
- (iii) Antoinette is one of the women on the island. Before the two people are randomly selected to leave, Antoinette calculates her chance of remaining on the island. She concludes that she has a good chance of remaining.

Do you agree? Justify your answer.

(b) Give an example of an event that has a probability of exactly  $\frac{3}{4}$ .

Question 28 continues on page 24

Year 12 General

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Solutions

2009,

## Multiple Choice.

- 1 D
- 2. B
- 3. C
- 4. **9**
- 5. D
- 6. D
- 2 B
- 8. 0
- 9. A
- 10. D
- 11. C

- 12. A
- 13. B
- 14. B.
- 15. A
- IL. D
- 12. A
- N. A.
- 19. A
- 20 C
- 21. C
- 12. A

# Year 12 Carol 1/2 yrly Enon. 2009

multiple choice sowthers

3. 
$$V = \frac{1}{3} A \times L_1$$
  
=  $\frac{1}{3} (230^2) \times 135$   
= 2 380 500 m<sup>3</sup> (C)

5. 
$$y=3^{2}$$
 Exponential curve investing as a gets larger. (D)

9. 
$$V = Ah$$

$$= \left\{ \left( \frac{6}{2} \left( 1+3.5 \right) \right) \times S \right\}$$

$$= \left\{ \left( \frac{6}{2} \left( 1+3.5 \right) \right) \times S \right\}$$

$$= 67.5 \text{ m}^{3}$$
A

$$\frac{B}{2\pi} = R + \frac{T}{2}$$

$$\frac{B}{2\Pi} - R = \frac{1}{2}$$

$$T = \frac{B}{T} - 2R$$
 (A)

$$100 \times \frac{120}{100} \times \frac{90}{100} = 108$$
 A

(a) 2.35614 × 103



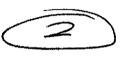
(for I light globe) (b) (i) c = \$6 + 5 × 365 × \$d

> not: (No of days in you could be given as 36514).

For \$ light globes

C = \$24 + 7300x 4d.

C = 24 + 7200d.



IF C = 250

250 = 24 + 73eOd

226 = 7,000d

d = 30 per hour or equivalent

Total cost will clouble!!

C= \$24 + 3600d where of = 0:03 C = \$24+ 7386×2×093

=\$462.



: , 5000 - 2(170) = 4660.

## Question 23 (cont.)

- (c) (i)  $\frac{5.5}{100} \times 375 = 20.625 \text{ mLs}$ 
  - $\frac{1}{100} \times 375 = 20.625 \text{ mLs}$
  - (ii) 1.6 stendard of his = 20.625 mls

    1 stendard drink = 20.625

    1 12.89 mls (2dp) / hour.
    - = 0.21 mLs (2017) / mihute.
- (d) 5x+1 = 2(4x-7)(1) 5x+1 = 8x-14(1)
  - 3x = 15
  - x = S.
- 3

Quenton 24

(iii) 
$$Vol = 400n^2 \times 0.02n$$
 () [20m = 0.02n]  
=  $8n^3$   
=  $8 \times 1000L$ 

$$= 60000 - 42800$$
$$= 17200 n^{2}$$

(a) i) 150 × 7.75 = \$1162-50/ north.

ii) \$1162.50x 12x20 = \$279 000 ()

Interest = 1279000 - 9150000

= 1129 000 (

iii) Interest bred = \$129000 - \$ 92730 (1)

= \$36 270

 $(5) \quad (1) \quad (4 \times 318 = 172) \\ (6 \times 1.5 \times 318 = 1160)$ 

Total = \$414 (1)

ii) +162 + 18 = 9 hours entra. 1) De 6x = 9

c) \$58624 + (900 x12) = \$69 424 ()

ii) Tor Payable = \$17 730 + [(69424-66000) × 0.48] ()

= \$ 19373.52

Q,	se_tho	., 2.	6
_			
	* 4		

(ء	·)	12	km/hr	(1)

- ii) Feb. (1)
  iii) Jan, Feb. Mar (1)
- iv) when wind speed increases, rainfall decreases () Relation an average. i.e. Jul - Oct what speed was ( marty at its now, but rantall was at a minimum.
- 5) i) 16 students
  - ii) 77 kg [accept 76-78].
  - iii) Out of 50 in the souple 16 were & 70kg
    - would expect 96 students approx < 70 kg ()
  - iv) 1. Radon selection is not an ordered selection It is a sample where each stoded would have equally likely clance of being selected. Not () of this cax. 2. Systematic or stratified. & Reference to ായിഗ്. Different enemples
- Total Scores 3×72 = 216 1 292-216 = 76%. 4×73: 292 .

## · Overtien 27

- a) i) LAOB = 133-46 ()
  - ii) Area = \frac{1}{2}abSm (
    = \frac{1}{2}(60)(85) \times Sm 87° (1)
    - = 2546.50 ---
    - = 2547m2.(1)
  - iii) Using come whe: a2 = 62 + c2 260 col A.
    - AB2 = 602 + 852 2(60)(65) (65) (65)
      - 2 10291.17
      - = 10! 45 m (2dx). ()
- b) H2= 22+2° (Pyt. Th)
  - H= 18 cm.
    - A = 2.598 × 18 + 2 (6×4)
    - = 2.598×8 + 2(2×2)
      - = 20.784 + 2
      - = 22.784 cm²

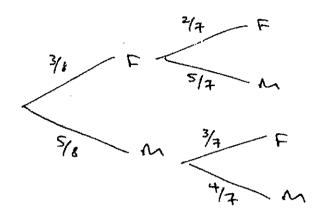
(1)

- 1) LAST = 90+ 20° (Bearing) (
  - BT = 50 Sh 30° = Sh 40°
  - BT = 50 × Sm 30
    - >14 40°
      - 38.89m (24) = 39m (never) a

Question 28

(a) i) 
$$P(fende) = \frac{3}{8} (0.375)$$
 (1)

, (ii



2. 
$$I(\text{Excet Ifende}) = P(\text{FM}) \text{ or } P(\text{MF})$$

$$= \left(\frac{3}{8} \times \frac{5}{7}\right) + \left(\frac{5}{8} \times \frac{3}{7}\right)$$

$$= \frac{15}{56} + \frac{15}{56}$$

$$= \frac{30}{56}$$

$$= \frac{30}{56}$$

$$= \frac{30}{56}$$

ii) Yes. (1)
Probability that Antomette is selected
$$= \frac{1}{8} \times \frac{1}{7}$$

$$= \frac{1}{56}$$
 which is a low prob.

(c) i) 
$$P(aiff, b) = \frac{10}{36}$$

$$= \frac{5}{18}$$

ii) Finacial empectation
$$= \left(3.50 \times \left(\frac{6}{36}\right)^{1} - 45\left(\frac{5}{18}\right) + 42.80\left(\frac{20}{36}\right)^{1}\right)$$

$$= 40.75 \qquad \boxed{}$$