TRINITY GRAMMAR SCHOOL

Mathematics Department

2012

HALF YEARLY EXAMINATION

Year 12

General

Mathematics

Total marks – 100

(**Section 1**)   Pages 1 – 8

25 marks

- Attempt Questions 1 – 25
- Allow about 30 minutes for this section

(**Section 2**)   Pages 9 – 20

75 marks

- Attempt Questions 26 – 30
- Allow about 2 hours for this section

Assessment Task: Weighting 30%

General Instructions

- Reading time – 5 minutes
- Working time – 2½ hours
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may
  be used
- A formula sheet is provided
- Show all necessary working in
  Questions 26 – 30
- Write your Board of Studies Student
  Number **and** Class Teacher on the
  answer sheet **and** writing booklet(s) **or**
 sheet(s) submitted

Attempt Questions 1-25

Select the alternative A, B, C or D that best answers the question and fill in the response circle completely. Answer on the separate multiple choice sheet.

1. The cash price for a computer is $1 990. The computer could be bought on terms for 20% deposit and $36 per week for one year. What is the saving if it is bought for cash?

    (A) $118   (B) $280   (C) 398   (D) $1 872

2. A sphere fits exactly inside a cylindrical container as shown.
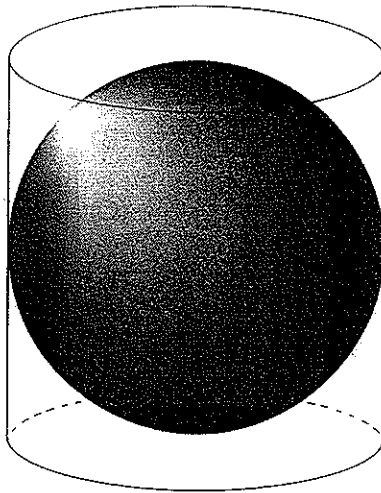
    The diameter of the sphere is 20cm. Calculate the volume of the cylindrical container to the nearest cubic centimetre.

    (A) 3 142cm^2 (B) 6 283cm^3 (C) 12 566cm^3 (D) 25 132cm^3

3. A restaurant menu offers a choice of 3 entrees, 5 mains and 2 desserts. How many combinations of meal are possible?
    (A) 3   (B) 10   (C) 15   (D) 30

4. Jacob, Tom, Nick and Julian are nominated for School Captain and Vice Captain. How many combinations of School Captain and Vice Captain are possible?
    (A) 2   (B) 12   (C) 16   (D) 24

5. Chris wishes to conduct a survey using the school population. He obtains a school roll and decides to survey every 25th student on the roll. This method of sampling is best known as:
(A) Biased sampling
(B) Random sampling
(C) Stratified random sampling
(D) Systematic sampling

6. The diagram below shows a block of land that has been surveyed. All measurements are in metres. The length of AD is closest to:

C

22

23 D

11

29

B

15

A
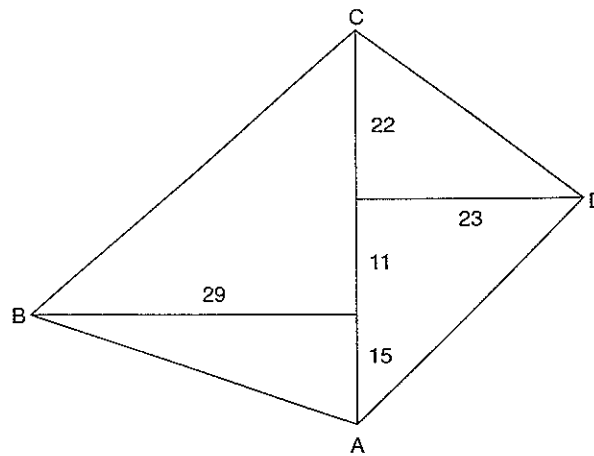
(A) 25.5m   (B) 27.5m   (C) 34.7m   (D) 49m

7. A score of 25 is added to this Frequency Distribution Table.

Score	Frequency
21	2
22	4
23	6
24	1
25	1

Which of these measures will change?

(A) Range   (B) Median   (C) Mode   (D) Mean

3

8. A bird nesting in a tree observes a worm on the ground. The angle of depression is $12°$ and the distance between the bird and the worm is 9 metres. Which expression gives the height (h) of the bird above the ground?
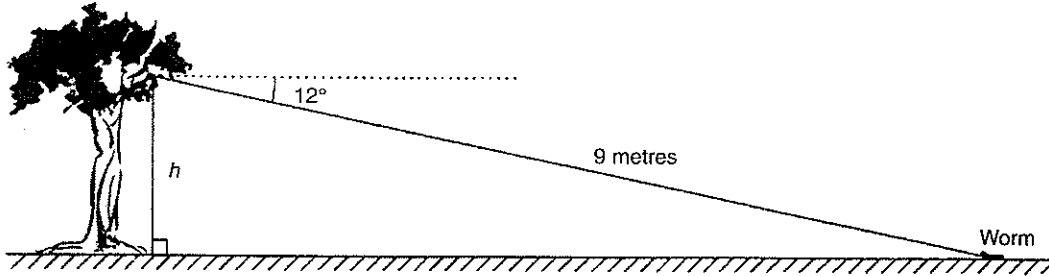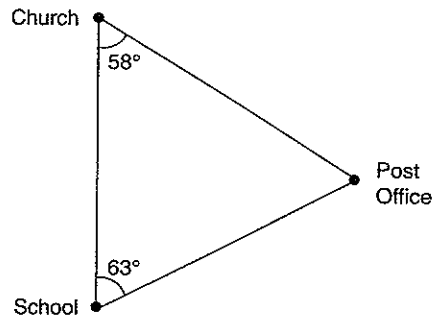

(A) $h = \dfrac{9}{sin12°}$ (B) $h = \dfrac{9}{sin78°}$ (C) $h = 9sin12°$ (D) $h = 9sin78°$

9. A woman walks 32 kilometres from B to A on a bearing of $060°$T. Which diagram best illustrates this information?

(A)

(B)

(C)

(D)
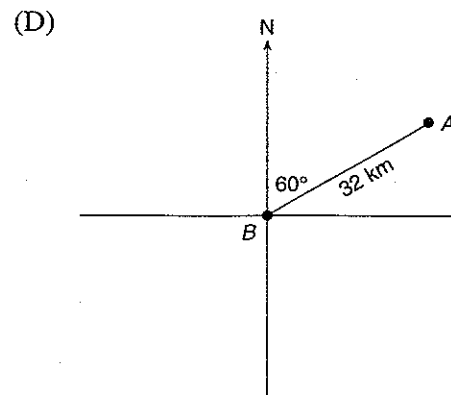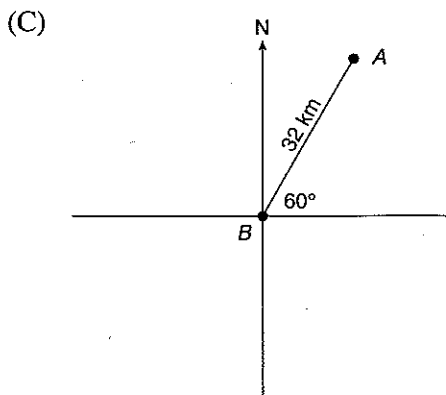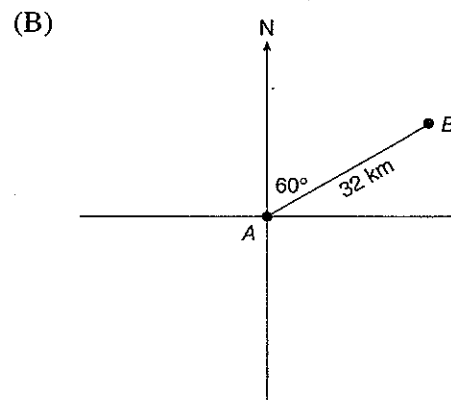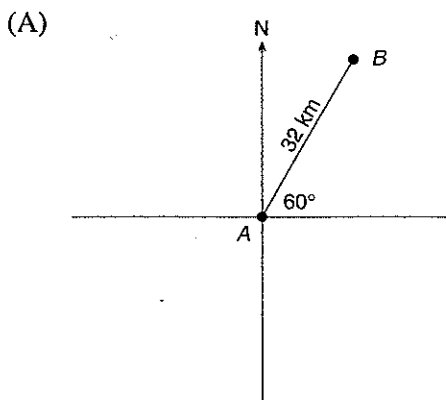
10. The diagram shows the positions of three buildings in a town.

The church is due north of the school. What is the bearing of the post office from the church?

(A) 058°   (B) 059°   (C) 122°   (D) 302°

11. After five Maths tests, Michael's mean mark was 65. In the next three Maths tests he scored 70, 75 and 80. Calculate Michael's mean mark for all of these Maths tests.

(A) 68.75   (B) 70   (C) 72.5   (D) 75

12. Simplify: $10(x + 3) - 2(4x + 2)$.

(A) $2x + 5$ (B) $2x + 6$ (C) $2x + 26$ (D) $2x + 34$

13. Solve: $\frac{2x-5}{x} = \frac{2}{3}$

(A) $x = \frac{15}{8}$ (B) $x = \frac{17}{6}$ (C) $x = \frac{15}{4}$ (D) $x = \frac{5}{4}$

14. At the beginning of the rugby league season, the Bulldogs were given a 70% chance of reaching the semi-final round, and the Rabbitohs were given a 20% chance. What is the probability that neither the Bulldogs nor the Rabbitohs reach the semi-final round?

(A) 0.14   (B) 0.24   (C) 0.86   (D) 0.90

15. There are three birds in a cage. Two are green and one is blue. If two birds escape, find the probability that one of them is blue and the other is green.

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{2}{9}$

16. Kieran borrows $35 000 and agrees to pay the loan and interest in monthly instalments over 5 years at 11% p.a. flat rate of interest. How much will Kieran pay each month?

(A) $64.17

(B) $320.83

(C) $647.50

(D) $904.17

17. The length of a rectangle is 4 cm more than its width. The width is w cm. What is the area of the rectangle?

(A) $(w^2 + 4w)cm^2$

(B) $(w^2 + 4)cm^2$

(C) $4w\ cm^2$

(D) $(4w + 8)cm^2$

18. Which of the following would yield data that would be classified as discrete?

(A) The distance a discus is thrown.
(B) The number of goals scored in a soccer game.
(C) The time taken to run 50 metres.
(D) The height jumped in a high jump.

19. Three partners, Sebastian, Ola and Robert, invest in a business in the ratio 3 : 4 : 1. The total amount invested is $125 000. How much did Sebastian invest?

(A) $15 625

(B) $41 667

(C) $46 875
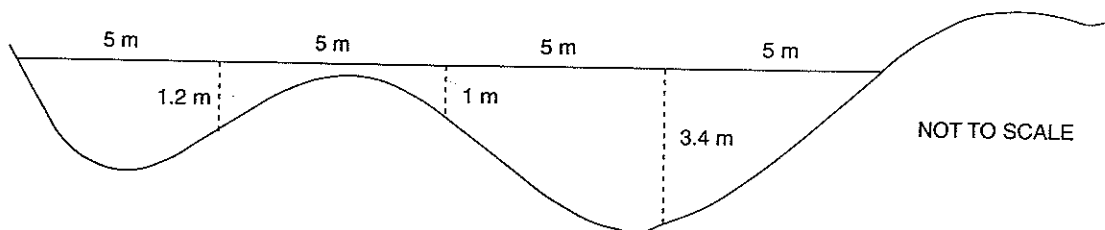
(D) $75 000

20. Simplify: $\frac{12x^6y^2}{24x^2y^8}$

(A) $2x^3y^4$

(B) $\frac{x^4}{2y^6}$

(C) $\frac{x^3}{2y^4}$

(D) $2x^4y^6$

6

21. The diagram below shows a vertical cross-section of a creek.

5 m   5 m   5 m   5 m

1.2 m   1 m   3.4 m   NOT TO SCALE

Which expression for the area of this cross-section, in square metres, would be obtained from two applications of Simpson's rule?

(A) $\frac{5}{3}(1.2 + 4 \times 1 + 3.4)$

(B) $\frac{5}{3}(4 \times 1.2 + 1) + \frac{5}{3}(1 + 4 \times 3.4)$

(C) $\frac{20}{3}(1.2 + 4 \times 1 + 3.4)$

(D) $\frac{20}{3}(4 \times 1.2 + 1) + \frac{20}{3}(1 + 4 \times 3.4)$

22. Rory was required to solve an equation for homework. This is his solution.

$$5 - 3(2x - 4) = 20$$

$$5 - 6x + 12 = 20 \ldots\ldots\ldots Line\ 1$$

$$-6x - 7 = 20 \ldots\ldots\ldots Line\ 2$$

$$-6x = 27 \ldots\ldots\ldots Line\ 3$$

$$x = -\frac{6}{27} \ldots\ldots\ldots Line\ 4$$

Which lines do not follow correctly from the previous line?

(A) Line 1 and line 2   (B) Line 1 and line 4   (C) Line 2 and line 3   (D) Line 2 and line 4

23. Use the formula $s = ut + \frac{1}{2}at^2$ to find s if $u = 8, t = 5, a = 10$.

(A) 90   (B) 165   (C) 665   (D) 1 290

7

24. If $\frac{1}{2}$ L of paint covers $3m^2$, how much paint is needed to cover $17m^2$?
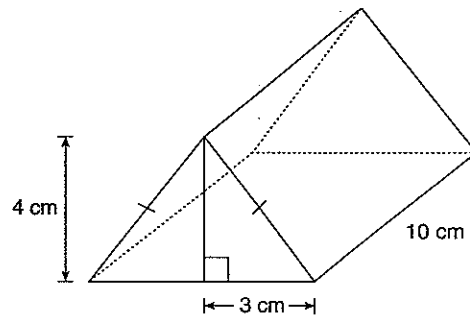
(A) $\frac{17}{6}$ L   (B) $\frac{6}{17}$ L   (C) $\frac{34}{3}$ L   (D) $\frac{3}{34}$ L

25. What is the surface area of this solid triangular prism?

(A) $120cm^2$ (B) $124cm^2$ (C) $172cm^2$ (D) $184cm^2$

End of Section 1

8

SECTION 2   75 Marks   Attempt Questions 26-30

Allow about 2 hours for this section.

Answer each question in a new writing booklet. All necessary working should be shown.

**Question 26   (15 marks)   Use the Question 26 writing booklet.**

a) The position of Boston (USA) is $42°N, 71°W$, and the position of Santiago (Chile) is $33°S, 71°W$.

i)   When it is 12 noon on Tuesday, Greenwich Mean Time, what time should it be in Boston (ignoring time zones)?   **3**

ii)   What is the difference in latitude between Boston and Santiago?   **1**

iii) Find the distance between Boston and Santiago in nautical miles. (Assume 60 nautical miles = $1°$ of latitude)   **1**

b) Jadphur is a town 75 kilometres directly south of the town Kimbala, as shown below.

N
Kimbala

75 km
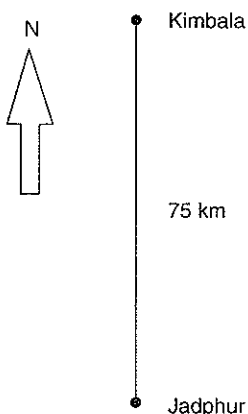
Jadphur

The town Lati is 50 kilometres from Kimbala on a bearing of $125°$.

i)   Copy the above diagram and using the information, mark the appropriate Position of the town Lati.   **1**

ii)   Use the cosine rule to find the distance from Lati to Jadphur.   **3**

(Question 26 continued)

c) The diagram below shows the surface plan of a fish pond. The pond has vertical sides and a constant depth of 45 cm.

FIGURE NOT TO SCALE
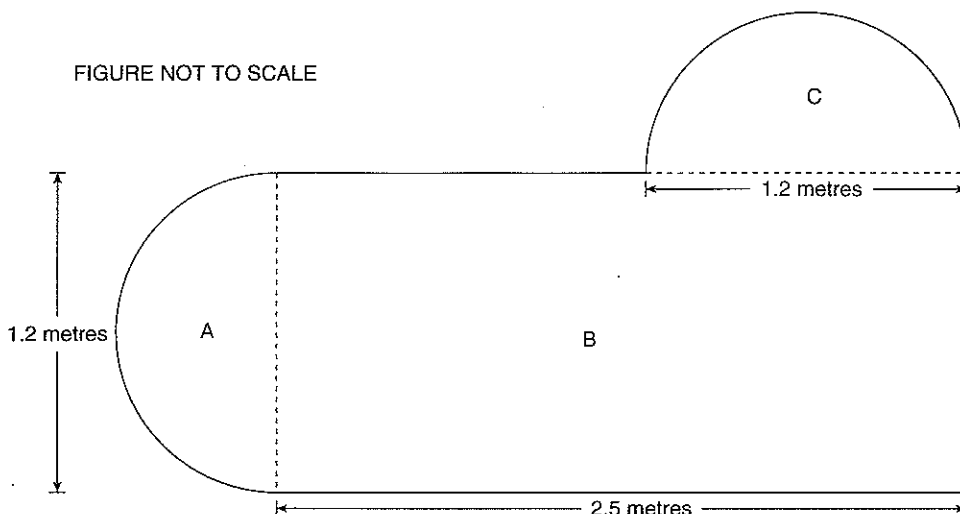
In the diagram, sections A and C are semicircular with diameters of 1.2 metres, and section B is rectangular with length 2.5 metres and width 1.2 metres.

i)   Calculate the total area of A and C to 2 decimal places.   **1**

ii) Find the total area of the surface of the pond.   **1**

iii) Calculate the maximum volume of water, in litres, that the pond can hold.   **2**

d) Given that 1.852 km = 1 M (nautical miles), find the number of metres in 5 M. **2**

End of question 26.

a) Sarah borrows $20 000 to buy a car. She can choose between several different

monthly repayment plans as shown in the table.

Number of repayments	24	36	48	60
Monthly instalments	$912	$638.60	$505	$425

How much **more** interest will she pay if she chooses to repay the loan in 60 months

rather than 36 months? **3**

b) Andy is employed as a salesman. He is offered two methods of calculating his income.

Method 1: Only 13% commission on all sales.
Method 2: $350 per week plus a commission of 4.5% on all sales.

Andy's research determines that the average sales total per employee per month is $15 670.

   i)   Based on his research, how much could Andy expect to earn in a year if he were
         to choose method 1? **2**

   ii)   If Andy were to choose a method of payment based on average sales figures,
         state which method he should choose in order to earn the greater income.
         Justify your answer with appropriate calculations. (Assume 52 weeks in 1 year) **3**

(Question 27 continued)

c) A new phone was purchased for $725 which included 10% GST. Find the price of the

phone without GST, correct to the nearest cent. **2**
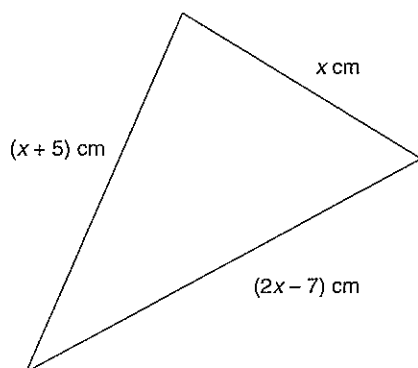
d) Convert 72 km/hour to metres per second. **2**

e) Two places on the Equator have longitude of $150°E$ and $90°W$. Taking the radius of the

Earth to be 6 400 km, find the *shortest* distance between these two places.

(Answer to the nearest km) **3**

End of question 27

a)

    i)   Show that the perimeter P cm of the above triangle is given by $P = 4x - 2$.   **1**

    ii)   If the perimeter is 82 cm, calculate the value of x.   **1**

b) Solve for x: $\dfrac{x-3}{3} - \dfrac{x-2}{4} = 1$ **3**

c) Assume that there is an equally likely chance of being born on any particular day of the week.

    i)   What is the probability that a person selected at random was <u>not</u> born on a Tuesday?   **1**

    ii)   What is the probability that, of any two people selected at random, exactly one was born on a Tuesday?   **1**

d) Fully simplify: $\dfrac{4a^2}{3b} \div \dfrac{ab}{5}$ **3**

(Question 28 continued)

e) Cathy plays a game of chance with the following outcomes.

* $\frac{1}{5}$ chance of winning $10

* $\frac{1}{2}$ chance of winning $3

* $\frac{3}{10}$ chance of losing $8

The game has a $2 entry fee.

What is the financial expectation from this game after paying the entry fee? **2**

f) Rodney invited 175 movie critics to preview his new movie. After seeing the movie,

he conducted a survey. Rodney has almost completed the two-way table below.

	Aged < 40	Aged ≥ 40	Totals
Movie critics who liked movie	65		102
Movie critics who did not like movie		31	
Totals		**A**	175

i) Determine the value of **A**. **1**

ii) A movie critic is selected at random. What is the probability that the critic

was less than 40 years old and did not like the movie? **2**

End of Question 28

a) Five people, A, B, C, D and E, turn up to compete in a fencing competition.

    i)   List all possible ways two people can be chosen to compete first.   **1**

    ii)   What is the probability that D competes in the first match?   **1**

    iii)   What is the probability that B and C compete first?   **1**

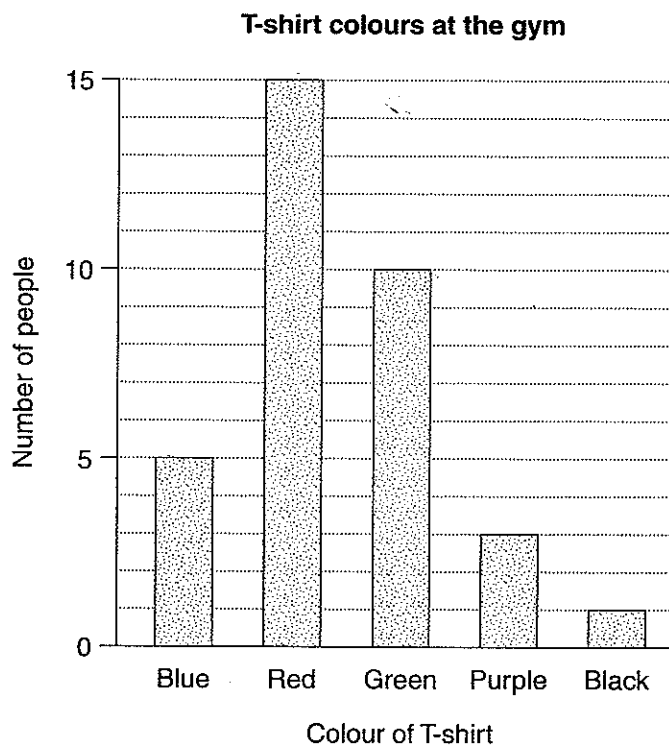b) The wheel shown is spun around until it stops on a number.
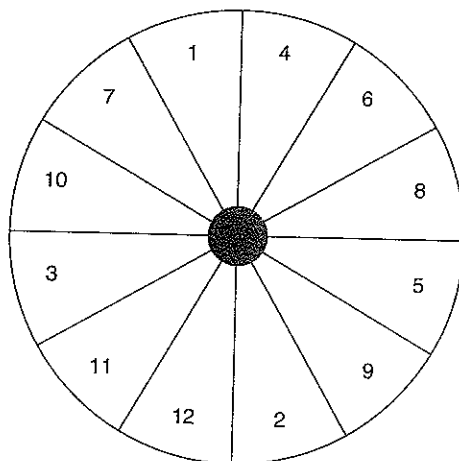
Given that the wheel is equally likely to stop on any number, find the probability that the wheel stops on:

    i)   12   **1**

    ii)   A number divisible by 5   **1**

(Question 29 continued)

c) On Saturday, Matt recorded the colour of T-shirts worn by people at his gym. The results are shown in the graph.

T-shirt colours at the gym

i) How many people were at the gym on Saturday?
(Assume everyone was wearing a T-shirt) 1

ii) What is the probability that a person selected at random at the gym on Saturday would be wearing either a blue or green T-shirt? 1

(Question 29 continued)

d) In a stack of 20 DVD's, there are 10 rated **PG**, 6 rated **G** and 4 rated **M.**

i) A DVD is selected at random. What is the probability that it is rated **G**? **1**

ii) Felix chooses two DVD's at random from the stack. Copy or trace the tree diagram into your writing booklet. Complete your tree diagram by writing the correct probability on each branch. **2**

```
          1st choice        2nd choice

                                  PG
                    PG            G
                                  M

                                  PG
                    G             G
                                  M

                                  PG
                    M             G
                                  M
```
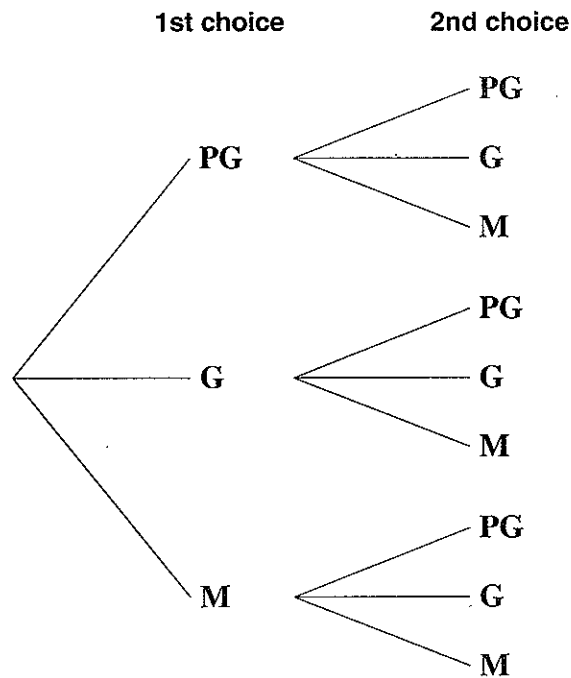
iii) Calculate the probability that Felix chooses two DVD's with the same rating. **2**

17

(Question 29 continued)

e) The angle of depression from J to M is 75°. The length of JK is 20m and the length of MK is 18m.
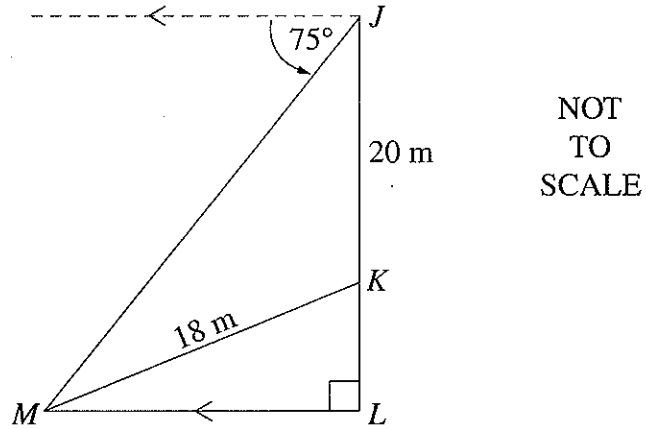
Copy or trace this diagram into your writing booklet and calculate the angle of elevation from M to K. Give your answer to the nearest degree.

3

End of Question 29

Question 30   (15 marks)   Use the Question 30 writing booklet.

a) A triangle has sides 7cm, 9cm and 10cm. Find the size of the largest angle.

(Answer to the nearest degree)   **3**

b) Calculate the area of the triangle below to one decimal place.   **2**
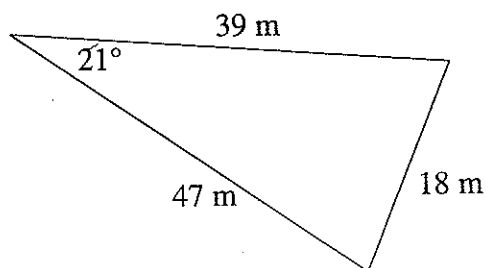
c) Five girls and five boys are attending a camp.

    i)   A group of four girls is needed to collect firewood. How many different groups
            can be chosen?   **1**

    ii)   A group of eight, consisting of four boys and four girls, are going kayaking.
            How many different groups can be chosen?   **1**

d) The number of errors on each page of a book was recorded. The results are below.

Number of errors	0	1	2	3	4
Number of pages	12	9	7	7	5

    i)   How many pages did the book have?   **1**

    ii)   What was the average number of errors on each page?   **2**

(Question 30 continued)

e) The graphs show the distribution of the ages of children in Ashville in 2000 and 2010.

Distribution of the ages of children in Ashville
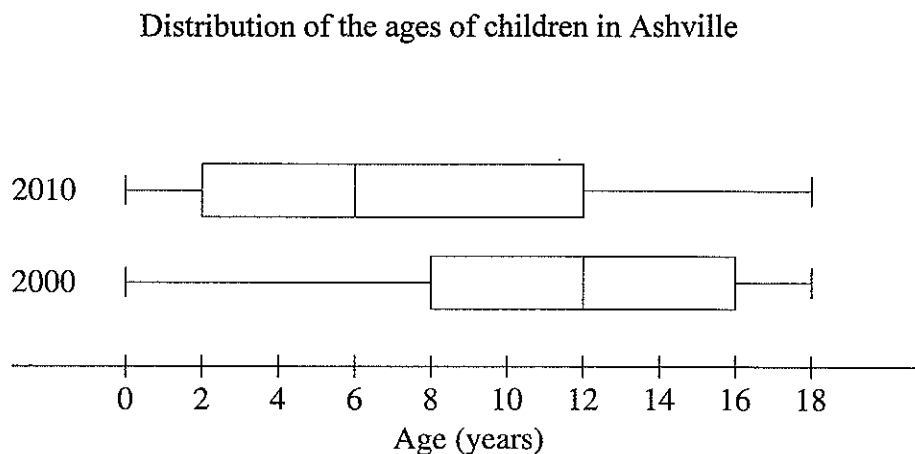
i)   In 2000 there were 1 750 children aged 0-18 years. How many children
     were aged 12-18 years in 2000?   **1**

ii) The number of children aged 12-18 years is the same in both 2000 and 2010.
     How many children aged 0-18 years are there in 2010?   **1**

iii) Identify **THREE** changes in the distribution of ages between 2000 and 2010.
     In your answer, refer to measures of <u>location</u>, <u>spread</u> and <u>shape</u> of the
     distributions.   **3**

End of Examination

20

SECTION 1   25 Marks

Sol.

Attempt Questions 1-25

Select the alternative A, B, C or D that best answers the question and fill in the response circle completely

#	A	B	C	D
1	○	●	○	○
2	○	●	○	○
3	○	○	○	●
4	○	●	○	○
5	○	○	○	●
6	○	○	●	○
7	○	○	○	●
8	○	○	●	○
9	○	○	○	●
10	○	○	●	○
11	●	○	○	○
12	○	○	●	○
13	○	○	●	○
14	○	●	○	○
15	○	○	●	○
16	○	○	○	●
17	●	○	○	○
18	○	●	○	○
19	○	○	●	○
20	○	●	○	○
21	○	●	○	○
22	○	○	○	●
23	○	●	○	○
24	●	○	○	○
25	○	○	○	●

Section ②

26

a)

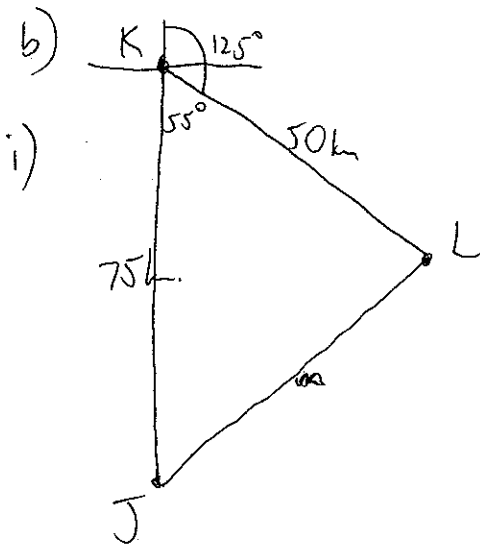

i) $1° = 4 \text{ min}$

$\therefore$ long. diff $= 71°$ — ①

$\therefore$ time diff $= 71 \times 4$

$= 284 \text{ min}$

$= 4h / 44\text{min}$ ⎤—①

$\therefore$ in Boston 7:16am Tues. — ①

ii) $75°$ — ①

iii) $75 \times 60 = 4500 \text{ n miles}$ — ①

$\left(\text{or } L = \frac{75}{360} \times 2\pi \times 6400 \doteqdot 8377.5 \text{ km} \atop \doteqdot 4523.5 \text{ nmiles} \right]$ pay I mark if shown. $\Big)$

b)

i)

① for diagram

ii) $\angle JKL = 55°$ — ①

$\therefore JL^2 = 75^2 + 50^2 - 2 \times 75 \times 50 \times \cos 55°$

$= 3823.1767..$ — ①

$\therefore JL = 61.8318...$ ⎤— ①

$\doteqdot 62 \text{ km}$

c) i) $A = \pi \times 0.6^2 \times \frac{1}{2} \times 2$

$= 1.13\, m$ — ①

ii) $A = 1.13 + (2.5 \times 1.2)$

$= 4.13\, m^2$ — ①

iii) $V = 4.13 \times 0.45$

$= 1.859\, m^3$ — ①

$= 1859\, L$ — ①

d) $1M = 1.852\, km$

$\therefore 5M = 1.852 \times 5$ — ①

$= 9.26\, km$

$= 9260\, m$ — ①

/5

(27) a) (60 month) $\Rightarrow$ 60 x 425 = 25500

$$\therefore \; Int = 5500 \qquad\qquad -\textcircled{1}$$

(36 months) $\Rightarrow$ 36 x 638·60 = 22989·6

$$\therefore \; Int = 2989·6 \qquad -\textcircled{1}$$

$$\therefore \; extra \; int = \$2510·40 \qquad -\textcircled{1}$$

b) i) 13% x 15670 $\qquad\qquad -\textcircled{1}$

$\qquad\qquad$ = 2037·10 $\quad$ (pm)

method 1.

$$\therefore \; I_{year} = \$24,445·20 \qquad -\textcircled{1}$$

ii) Method 2: 4·5% x 15670 = 705·15

$$\therefore \; 1 \; year = 8461·80 \qquad\qquad -\textcircled{1}$$

plus $\qquad$ \$350 x 52 = 18200

$$\therefore \; total = \$26,661·80 \qquad\qquad -\textcircled{1}$$

$\therefore$ choose method 2 $\qquad$ (\$2216·60 more)

$$\textcircled{1}$$

c) $110\% = 725$

$\therefore 1\% = \dfrac{725}{110}$ — ①

$\therefore 100\% = \dfrac{725}{110} \times 100$

$= \$659.09$ — ①

d) $72 \text{ km/hr} = 72000 \text{ km}/3600 \text{ sec}$ ① for conversion.

$= 20 \text{ m/sec}$ — ①

e) $\dfrac{120°}{360} \times 2 \times \pi \times 6400$ → ① shortest distance angle.

$= 13,404.128 \text{ km}$ — ①

$\doteq 13,404 \text{ km}$ — ①   * rounding ⓐ

* if find other distance lose 1 mark.
* also penalise if not rounded properly.

$/ 15$

(28) a) i) $P = x + x+5 +2x-7 \leftarrow$ ①

   <u>need this line.</u>

   $\therefore P = 4x-2$

  ii) $4x -2 = 82$

   $4x = 84$

   $x = 21$ ── ①

b) $^{12}x\left(\dfrac{x-3}{3}\right) - {}^{3}x\left(\dfrac{x-2}{4}\right) = 1^{\times 12}$

   $4(x-3) - 3(x-2) = 12$ ── ①

   $4x -12 -3x +6 = 12$ ── ①

   $x -6 = 12$

   $x = 18$ ── ①

c) i) $P(\bar{T}) = \dfrac{6}{7}$ ── ①
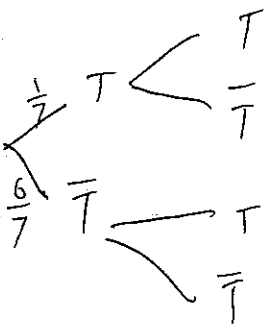
  ii) $P(\bar{T},T) = \left(\dfrac{1}{7} \times \dfrac{6}{7}\right) + \left(\dfrac{6}{7} \times \dfrac{1}{7}\right)$

   $= \dfrac{12}{49}$ ── ①

d) $\dfrac{4a^2}{3b} \div \dfrac{ab}{5} = \dfrac{4a^2}{3b} \times \dfrac{5}{ab}$ — ①

$= \dfrac{20a}{3b^2}$ — ① for correct pronumeral

          — ① for correct numeral.

e) $F \cdot E = \left(\dfrac{1}{5} \times (0)\right) + \left(\dfrac{1}{2} \times 3\right) - \left(\dfrac{3}{10} \times 4\right)$

     — ①

$= \$1 \cdot 10$

$\therefore$ with \$2 to play $\therefore$ $\boxed{\text{loss of } 90¢}$ — ①

f) i) $A = 68$ — ①

ii)   total $< 40 = 107$

$\therefore$ N° of critic < 40 did not like is ㊷ — ①

$\therefore p_4 = \dfrac{42}{175} = \dfrac{6}{25}$

     ↖ ①

15

(29)

a) i) AB   BC   CD   DE
        AC   BD   CE   — ①
        AD   BE
        AE

   ii) AD, BD, CD, DE $\therefore \dfrac{4}{10} = \dfrac{2}{5}$ — ① either.

   iii) BC only $\dfrac{1}{10}$ — ①
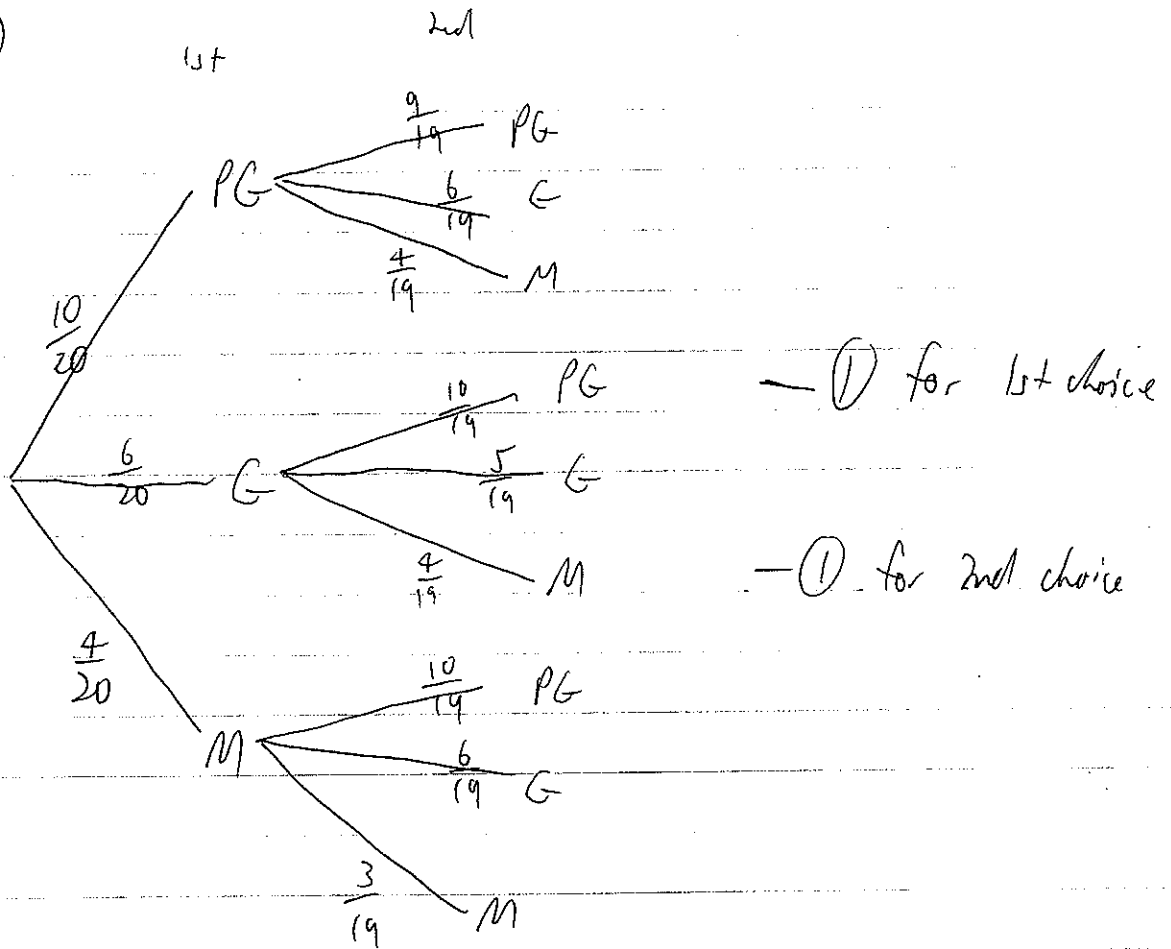
b) i) $\dfrac{1}{12}$ — ①

   ii) $\dfrac{2}{12} = \dfrac{1}{6}$ — ① either

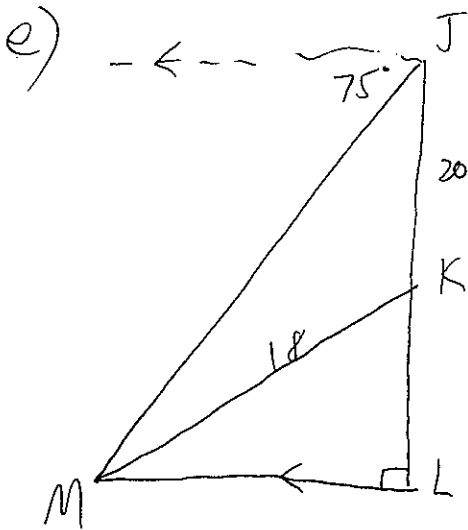c) i)   34   people   - ①

   ii) $\dfrac{15}{34}$ — ①

d)   i) $P(G) = \dfrac{6}{20} = \dfrac{3}{10}$ ——— ① title

ii)

           1st   2nd

$$P(G) = \frac{6}{20} = \frac{3}{10}$$

— ① for 1st choice

— ① for 2nd choice

iii) $P(\text{same rating}) = \left(\dfrac{10}{20} \times \dfrac{9}{19}\right) + \left(\dfrac{6}{20} \times \dfrac{5}{19}\right) + \left(\dfrac{4}{20} \times \dfrac{3}{19}\right)$ —— ①

$$= \frac{33}{95} \qquad —— ①$$

e)

$\angle MJK = 15°$ (comp) — ①

$$\frac{\text{Sin} \angle JMk}{20} = \frac{\text{Sin } 15°}{18}$$

$$\text{Sin} \angle JMk = 20 \times \frac{\text{Sin } 15°}{18}$$

$$= 0.28757...$$

$\therefore \angle JMK = 16°42'$

$\doteqdot 17°$ — ①

$\therefore \angle KML = 75 - 17$

$= 58°$ — ①

$\angle$ of elevation is $58°$

$/15$

(30)

a)

7 / θ \ 9 ... $\cos\theta = \dfrac{7^2+9^2-10^2}{2\times7\times9}$ —①

10

$= 0.23809...$ —①

$\therefore \theta = 76°13'33''$

$\left.\vphantom{\begin{array}{c}a\\b\end{array}}\right\}$ —①

$\doteqdot 76°$
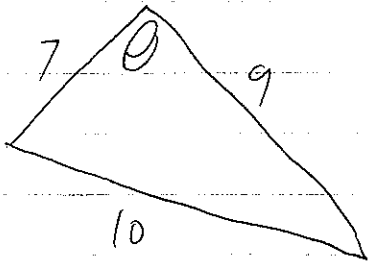
b) $\text{Area} = \dfrac{1}{2} \times 39 \times 47 \times \sin 21°$ —①

$= 328.4\,m^2$ —①

c) i) $\dfrac{5 \times 4 \times 3 \times 2}{4!} = 5$ —①

   ii) $5 \times 5 = 25$ —①

(if ① is wrong, but use it to answer ii) correctly, award mark)

d)

e	0	1	2	3	4
f	12	9	7	7	5	— 40

0   9   14 21 20   — 64

i) $\quad$ 40 $\qquad$ — ①

ii) $\quad \bar{x} = \dfrac{64}{40} = 1.6$

① for total $N°$ of errors $= 64$

① for answer.

e) $\quad$ i) $\quad \frac{1}{2} \times 1750 = 875 \qquad$ — ①

ii) $\quad 25\% = \dfrac{1}{4} = 875$

$\qquad \therefore 100\% = 875 \times 4$

$\qquad\qquad = 3500 \qquad$ — ①

iii)

	location	Shape	Spread
2000	median $= 12$	neg. skew	$IQR = 8$
2010	median $= 6$	pos. skew	$IQR = 10$
	①	①	①

15