

Total marks (120)**Attempt Questions 1 – 10****All questions are of equal value**

Answer all questions in a SEPARATE writing booklet. Extra writing booklets are available.

Question 1 (12 marks) Use a SEPARATE writing booklet.**Marks**

(a) Solve the equation $5 - 4(x - 2) = 19$.

2

(b) Simplify $\frac{2}{x-3} - \frac{1}{x+1}$.

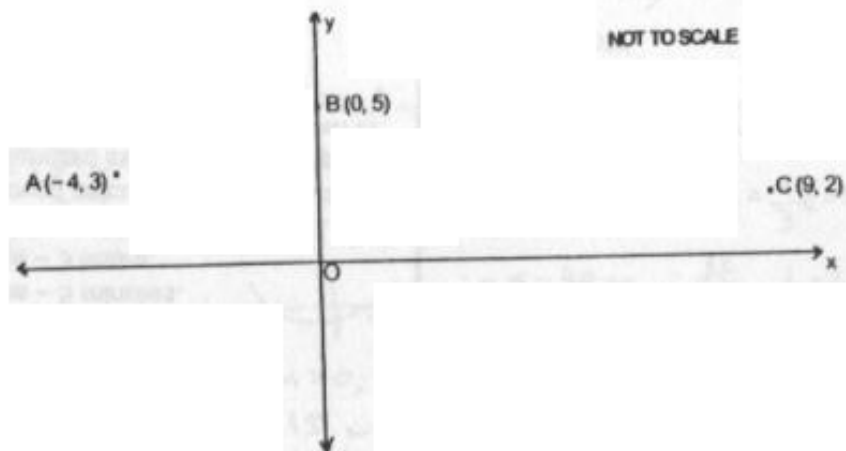
2(c) Find $\log_e 13$ correct to 3 significant figures.**2**(d) Given that p and q are rational numbers, find the values of p and q if
 $(2\sqrt{3} - 1)(3\sqrt{3} + 2) = p + q\sqrt{3}$ **2**

(e) Solve $2x^2 - 7x + 6 = 0$.

2(f) Graph the solution of $|2x - 1| < 7$ on a number line.**2**

Question 2 (12 marks) Use a SEPARATE writing booklet.

Marks



The diagram shows the origin O and the coordinates of the points $A(-4, 3)$, $B(0, 5)$ and $C(9, 2)$.

Copy or trace this diagram into your writing booklet.

- Find the exact length of the interval BC .
- Show that the equation of the line k , drawn through A and parallel to BC is $x + 3y - 5 = 0$.
Clearly indicate the line k on your diagram.
- Find the coordinates of D , the point where the line k meets the x axis.
- Prove $ABCD$ is a parallelogram.
- Find the perpendicular distance from the point B to the line k .
- Hence, or otherwise, find the area of $ABCD$.

2

3

1

2

2

2

Question 3 (12 marks) Use a SEPARATE writing booklet.

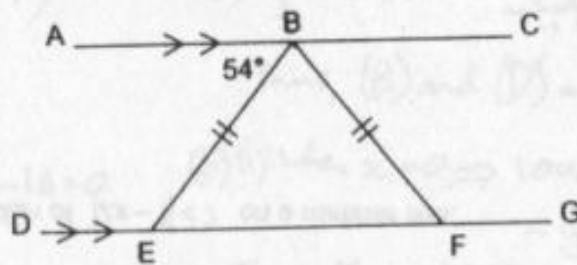
Marks

(a) Differentiate the following:

(i) $\frac{4x+5}{3x-2}$ 2

(ii) $x^2 \sin x$ 2

(b) In the diagram $AC \parallel DG$, $\angle ABE = 54^\circ$, $BE = BF$.



Copy the diagram into your writing booklet.

Find the value of $\angle BFG$, giving all reasons. 2

(c) Find:

(i) $\int_0^2 \frac{1}{x+1} dx$ 2

(ii) $\int \frac{e^x - 1}{e^x} dx$ 2

(d) A curve has a gradient function $\frac{dy}{dx} = 2x - 3$. Find the equation of the curve if it passes through the point (1, 4). 2

Question 4 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) An arithmetic series has a seventh term of 8 and the difference between the twelfth and eleventh terms is -4 .
- (i) What is the common difference? 1
 - (ii) Find the first term of the series. 1
 - (iii) Find the sum of the first 17 terms. 2
- (b) Consider the curve given by $y = x^3 - 3x^2$.
- (i) Find the coordinates of the two stationary points and determine their nature. 3
 - (ii) Find any point of inflexion. 2
 - (iii) Sketch the curve, showing the main features, for $-1 \leq x \leq 4$. 2
 - (iv) What is the maximum value of the function over the given domain? 1

Please turn over

Question 5 (12 marks) Use a SEPARATE writing booklet.

Marks

(a) A red die and a green die are thrown at the same time. What is the probability that the pair of dice show:

(i) at least one six

1

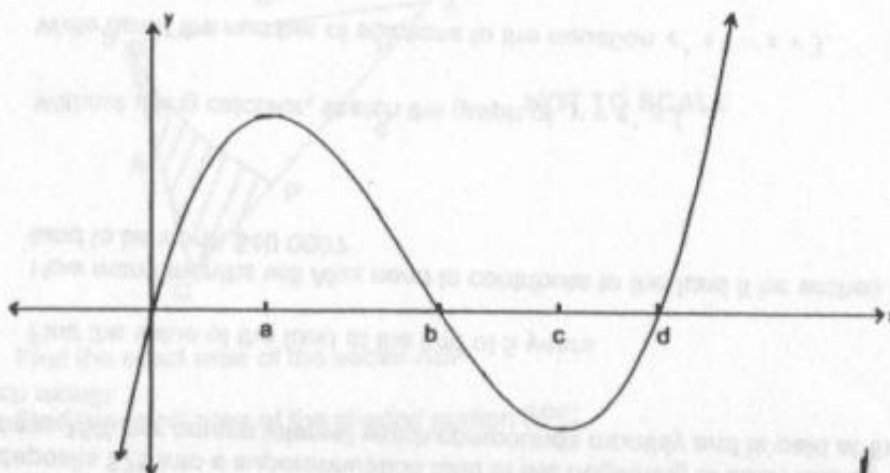
(ii) a total of 7

1

(b) Find all solutions to $4^x - 17 \times 2^x + 16 = 0$

3

(c) The following is the graph of a function $y = f(x)$.



(i) Over what domain is the function decreasing?

1

(ii) Draw a possible graph of the derivative of this function, $f'(x)$.

2

(d) A quadratic function is given by $f(x) = 2x^2 + mx + 5$ and $f(-2) = 3$.

(i) Find the value of m .

1

(ii) If the roots of the equation $f(x) = 0$ are α and β find the value of

(I) $\alpha + \beta$

1

(II) $\alpha^2 + \beta^2$

2

Solve $\tan^2 x = \frac{1}{3}$ for $0^\circ \leq x \leq 360^\circ$.

2

To calculate the area of the region bounded by the curve $y = x^2 - 2x$ and the x axis between $x = 0$ and $x = 4$, Essie used $\int_0^4 (x^2 - 2x) dx$.

- (i) Explain why Essie's method of calculating this area is incorrect.
- (ii) Find the area of the required region.

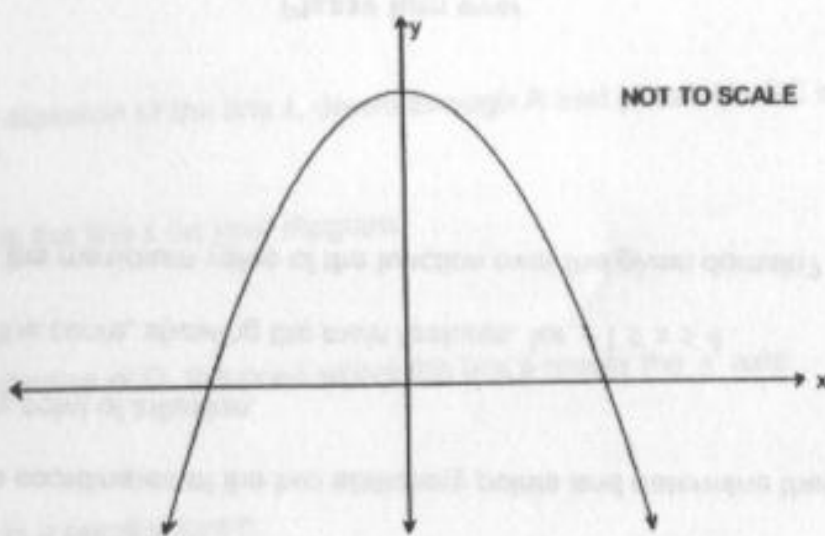
1

3

Find the gradient of the tangent to the curve $y = x - 2e^x$ at the point where $x = 1$.

2

The diagram shows the graph of $y = 4 - x^2$.



- (i) Show that the x intercepts are 2 and -2 .
- (ii) Find the exact volume of the solid formed when the region bounded by the curve $y = 4 - x^2$ and the x axis is rotated about the x axis.

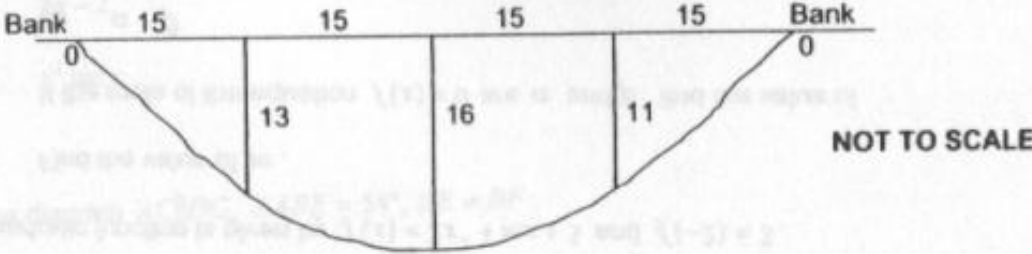
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3

Question 7 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) The diagram shows a cross-section of a river. The river is 60 m wide and its depth is recorded in metres at 15 m intervals across its width.



- (i) Find an estimate for the cross-sectional area of the river using Simpson's Rule with 5 function values. 3
- (ii) Water flows past this section at a rate of 5 metres per second. What is the volume of water passing this section per second? 1

- (b) Alex deposits \$75 into a superannuation fund at the beginning of each month. The fund pays 15% per annum interest which compounds monthly and is paid at the end of each month.

- (i) Find the value of the fund at the end of 5 years. 2
- (ii) How many months will Alex need to contribute to the fund if he wishes the fund to be worth \$40 000? 3

- (c) (i) Without using calculus, sketch the graph of $y = e^x + 1$. 1
 - (ii) Write down the number of solutions to the equation $e^x + 1 = x + 3$. 2
- Clearly indicate the solutions on your graph.

Please turn over

Question 8 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) Differentiate $y = 2^x$. 1
- (b) Given that $\log_a b = 2.57$ and $\log_a c = 0.35$, find the value of $\log_a (bc)^2$. 2
- (c) (i) Differentiate $y = \log(\cos x)$ 2
(ii) Hence find $\int \tan x \, dx$ 1
- (d) A particle is moving in a straight line with its displacement x metres from a fixed point at time t seconds given by $x = t^3 + t^2 - 5t + 2$, $t \geq 0$.
- (i) Find expressions for the velocity and acceleration of the particle. 2
- (ii) Comment on the acceleration of the particle throughout its motion. 1
- (iii) In what initial direction is the particle moving? 1
- (iv) At what time does the particle come to rest? 2

Please turn over

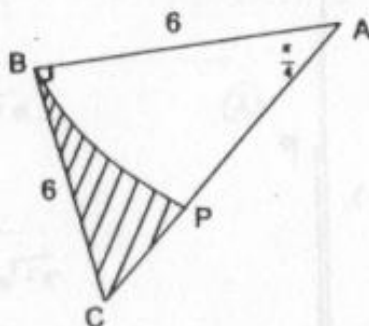
Question 9 (12 marks) Use a SEPARATE writing booklet.

Marks

- (a) In a jewel box there are 4 gold and 2 silver coins. Two coins are chosen at random from the box and the first coin is not replaced before the second coin is drawn. Find the probability that:

- (i) both coins are gold.
 (ii) at least one coin is gold.

- (b) In the diagram ABC is a triangle with a right angle at B, $AB = BC = 6$ cm and $\angle CAB = \frac{\pi}{4}$ radians. PB is an arc of a circle with centre A, radius AB, and meets AC at P.



NOT TO SCALE

- (i) Find the exact area of the sector ABP.
 (ii) Find the exact area of the shaded portion BPC.
- (c) (i) Show that $ke^0, ke^{-1}, ke^{-2}, \dots$ are the first terms of a geometric series.
 (ii) Explain why this series has a limiting sum.
- (d) The value, V , of a machine which loses value over time, t years, is given by $V = V_0 e^{-kt}$ where V_0 was the value of the machine when new and k is a constant, $k > 0$.
- (i) The new price of the machine was \$178 000 and it loses 16% of its value in the first year. Use this information to show that $k = 0.174$.
 (ii) The value of the machine continues to fall by 16% each year. Once the machine's value is less than 10% of its original price, the company can write the machine off.

After how many years can the machine be written off?

Question 10 (12 marks) Use a SEPARATE writing booklet.

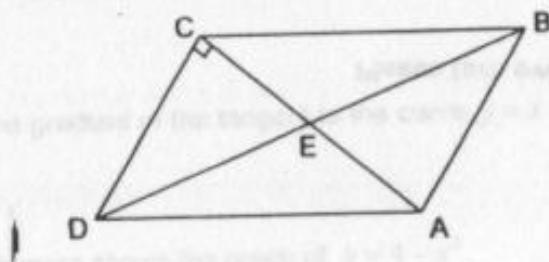
Marks

- (a) A cylindrical can is to be constructed in such a way that the sum of its height, h , and its diameter, $2r$, is 18 cm.

(i) Show that the volume of the can, V , is given by $V = 18\pi r^2 - 2\pi r^3$. 2

- (ii) Find the dimensions of the can that give the maximum volume. 3

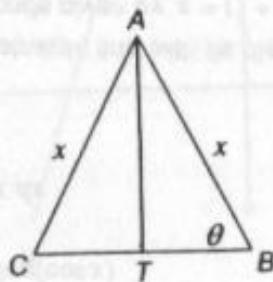
- (b) The quadrilateral $ABCD$ is a parallelogram with diagonal AC perpendicular to DC . The two diagonals intersect at E .



NOT TO SCALE

Show that $DE^2 + 3EA^2 = AD^2$. 3

- (c) The isosceles triangle ABC has equal sides, AB and AC , of length x metres. The perimeter of the triangle is 1 metre and $\angle ABC = \theta$ radians. T is the midpoint of BC .



(i) Show that $\sin \theta = \frac{\sqrt{x - \frac{1}{4}}}{x}$ 3

- (ii) Hence, or otherwise, show that the area of triangle ABC is given by $A = (\frac{1}{2} - x)\sqrt{x - \frac{1}{4}}$. 1

End of paper