ABBOTSLEIGH

AUGUST 2003
YEAR 12
ASSESSMENT 4
TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION

# Mathematics

## General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Write using blue or black pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.

**Total marks – 120**

- Attempt Questions 1-10.
- All questions are of equal value.



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**Attempt Questions 1-10**
**All questions are of equal value**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

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**Marks**

**Question 1 (12 marks) Use a SEPARATE writing booklet.**

(a) Evaluate $\dfrac{58.7}{31.4\times\sqrt{79.2}}$ correct to 3 significant figures. **2**

(b) Find the value of $\dfrac{7\pi}{6}$ in degrees. **1**

(c) Simplify $\dfrac{x^2-4}{x}+\dfrac{x^2-x-6}{x^2-3x}$ **3**

(d) Solve $9^t=\dfrac{1}{27}$ **2**

(e) Find a primitive function of $2-\dfrac{1}{x^2}$ **2**

(f) Solve $|3x+1|<5$ and graph your solution on a number line. **2**

End of Question 1

**Question 2** (12 marks) Use a SEPARATE writing booklet. **Marks**

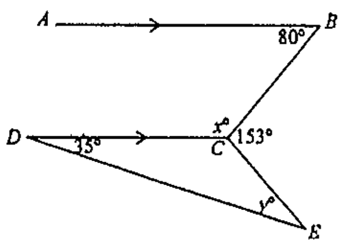
(a) Find the equation of the tangent to $y = \ln x$ at the point where $x = 1$. **3**

(b) Differentiate:

(i) $\dfrac{x}{\cos x}$ **2**

(ii) $e^{x^3}$ **1**

(c)




*Not to scale*

In the diagram $AB \parallel DC$, $\angle ABC = 80°$, $\angle CDE = 35°$, $\angle BCE = 153°$, $\angle BCD = x°$ and $\angle CED = y°$.

Find $x$ and $y$ giving full reasons for your answers. **3**

(d) Find:

(i) $\int \sec^2(3x+1)\,dx$ **1**

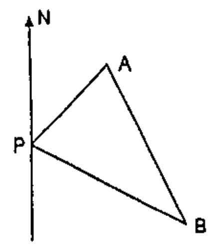
(ii) $\int \dfrac{dx}{2\sqrt{x}}$ **2**

**End of Question 2**



**Question 3** (12 marks) Use a SEPARATE writing booklet. **Marks**

(a)




*Not to scale*

Ship A is 20 kilometres from a port P and is on a bearing from P of 055°. Ship B is 27 kilometres from P on a bearing of 115°.

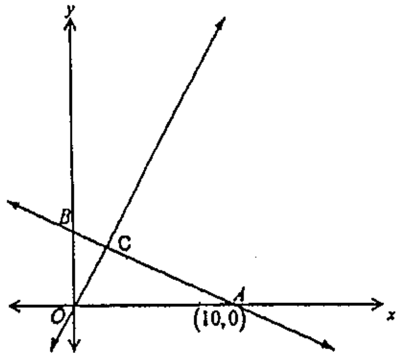
(i) Copy this diagram into your writing booklet, including on your diagram the given information and show that $\angle APB = 60°$. **2**

(ii) Determine the distance AB between the two ships, giving your answer to the nearest kilometre. **2**


**Question 3 continues on page 5**

Question 3 (continued) **Marks**

(b)




In the diagram, the equation of the line $OC$ is $y = 2x$. $OC$ is perpendicular to $AB$ and meets $AB$ at $C$. $A$ is the point $(10,0)$. Copy or trace the diagram into your writing booklet.

(i) Find the gradient of the line $AB$. **1**

(ii) Show that the equation of line $AB$ is $x + 2y - 10 = 0$. **1**

(iii) Show that the equation of the circle with diameter $OA$ is $(x-5)^2 + y^2 = 25$. **1**

(iv) Find the coordinates of $C$ and show that $C$ lies on the circumference of the circle in (iii). **3**

(v) Find the coordinates of $D$ if $OCDA$ is a parallelogram. **1**

(vi) Find the area of the parallelogram $OCDA$. **1**

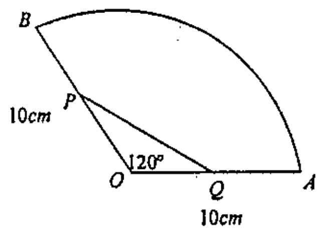
**End of Question 3**



**Marks**

Question 4 (12 marks) Use a SEPARATE writing booklet.

(a) The diagram below is not drawn to scale.




$O$ is the centre of the circle containing arc $AB$. $P$ is the midpoint of $OB$ and $Q$ is the midpoint of $OA$.
$\angle AOB = 120°$, $OA = OB = 10cm$

(i) Find the length of the arc $AB$. **2**

(ii) Find the shaded area $PQAB$. **2**

(b) Find:

(i) $\int_0^1 \left(e^{3x} + 2\right) dx$ **2**

(ii) $\int \frac{\cos 2x \, dx}{\sin 2x}$ **2**

(c) (i) Differentiate $x \sin x$. **2**

(ii) Hence, or otherwise, find the exact value of $\int_0^{\frac{\pi}{3}} x \cos x \, dx$ **2**

**End of Question 4**

**Question 5 (12 marks) Use a SEPARATE writing booklet.** **Marks**

(a) For the parabola $4y = x^2 - 6x + 1$, find

(i) the coordinates of the vertex and the focal length. **2**

(ii) the coordinates of the focus. **1**

(b) (i) Sketch the graph of $y = 1 + \cos 2x$ for $0 \le x \le 2\pi$. **2**

(ii) Use your graph to solve $1 + \cos 2x = 2$ for $0 \le x \le 2\pi$. **1**

(c) Evaluate $\int_0^{\sqrt{6}} \frac{x}{x^2+2}\,dx$, leaving your answer in exact simplified form. **3**

(d) The 'Feedum' dog pound is sheltering 50 dogs. Two cans of dog food are provided per dog per day. Each day after the first day, the pound is able to place a dog with a family. How many cans of dog food will remain from the pound's store of 3000 cans, after all the dogs have been placed? **3**

**End of Question 5**



**Question 6 (12 marks) Use a SEPARATE writing booklet.** **Marks**

(a) Two of the following real numbers are rational. Select them and write down their exact values, in simplest form.

$\pi,\quad e,\quad \log_e e,\quad \log_{10} 5,\quad 0.\dot{4},\quad \sqrt{2}$ **2**

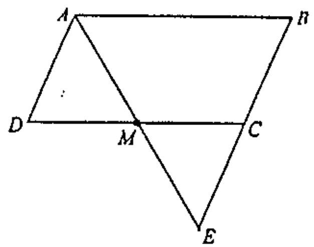
(b) (i) Sketch the graph of $y = \log_e x$ and state the domain. **2**

(ii) On your sketch, shade the region defined by $\int_1^5 \log_e x\,dx$ **1**

(iii) Use Simpson's rule with five function values to estimate $\int_1^5 \log_e x\,dx$ **2**

Give your answer correct to 2 decimal places.

(c)




***Not to scale***

$ABCD$ is a parallelogram and $M$ is the midpoint of $DC$. $AM$ produced meets $BC$ produced at $E$.

(i) Copy the diagram into your writing booklet and prove that $\Delta AMD \equiv \Delta EMC$, giving reasons. **3**

(ii) Prove that $ACED$ is a parallelogram, giving reasons. **2**

**End of Question 6**

**Question 7 (12 marks) Use a SEPARATE writing booklet.** **Marks**

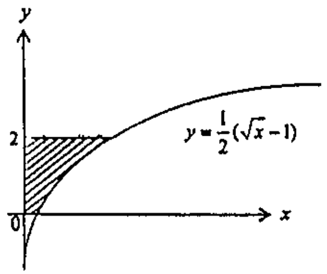
(a) What is the natural domain of $\sqrt{2-x}$? **1**

(b) In a raffle there are 100 tickets. The first prize is drawn and then the second prize is drawn without replacing the winning ticket. If George buys 15 tickets what is the probability that he:

(i) wins first prize? **1**

(ii) does not win a prize? **2**

(c)




The diagram shows the shaded region enclosed by the curve $y=\frac{1}{2}(\sqrt{x}-1)$, the $x$-axis, the $y$-axis and the line $y=2$. This region is rotated about the $y$-axis. Find the volume of this solid of revolution. **3**

(d) In the *C-The-World* travel agency, the ticket sales are declining by 8% every year, and 50 000 tickets were sold in 2002. Suppose the travel agency continues in business indefinitely.

(i) Find the number of tickets sold in 2003. **1**

(ii) What would the total sales, from the beginning of 2002 onwards, be eventually? **2**

(iii) What percentage of these sales would occur by the end of 2020? **2**

**End of Question 7**



**Question 8 (12 marks) Use a SEPARATE writing booklet.** **Marks**

(a) The gradient function of a curve is given by

$$f'(x)=x(x-2)(x+2)$$

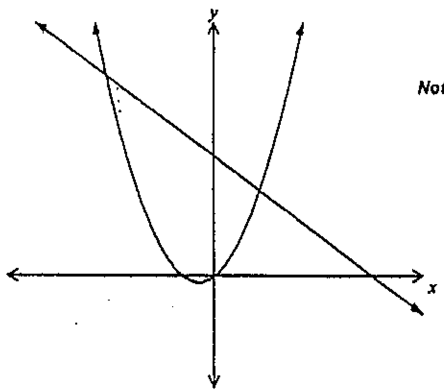
and the curve passes through the point $(0,4)$.

(i) Find the equation of the curve $y=f(x)$. **2**

(ii) Find any stationary points and determine their nature. **3**

(iii) Sketch the curve $y=f(x)$, labelling the turning points and the $y$-intercept. **2**

(b)




The graphs of $y=x^2+x$ and $y=3-x$ are drawn above.

(i) Find the coordinates of the points of intersection of the two graphs. **2**

(ii) Find the area of the shaded region in the diagram. **3**

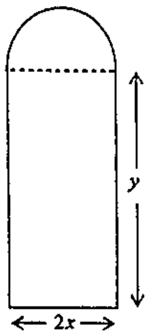
**End of Question 8**

**Question 9** (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Find the minimum value of $3\sin 2x$. **1**

(b) Solve $\ln(2x+15) = 2\ln x$. **3**

(c)




A window is in the shape of a rectangle and a semicircle as shown. The dimensions of the rectangle are $2x$ metres and $y$ metres. The perimeter of the window is $4$ metres.

(i) Show that $y = 2 - x - \dfrac{\pi x}{2}$. **2**

(ii) Show that the area, $A$, of the window is given by $A = 4x - 2x^2 - \dfrac{\pi x^2}{2}$ **2**

(iii) Find the exact value of $x$ which will make the area of the window a maximum. **3**

(iv) Show that this maximum area occurs when $x = y$. **1**

End of Question 9



**Question 10** (12 marks) Use a SEPARATE writing booklet. **Marks**

(a) Two coins are identical in appearance; however one is 'fair' but the other is biased, so that the probability of tossing a head with the biased coin is $\frac{4}{7}$. Using a tree diagram, or otherwise, answer the following:

(i) If a coin is chosen at random and tossed once, what is the probability that a head is obtained? **2**

(ii) If a coin is chosen at random and tossed twice, what is the probability of obtaining at least one head? **2**

(b) Julie invests \$3000 at the beginning of each year into a superannuation fund which pays 5% pa interest compounded annually. She does this for 10 years.

(i) Show that the amount accumulated at the end of the 10 years is \$39 620 to the nearest dollar. **3**

She then moves this amount to another fund which offers 5.4% pa compounded monthly. At the beginning of the second month she invests another $\$M$ into this fund.

(ii) Show that the amount now accumulated at the end of the second month is given by $39620(1.0045)^2 + M(1.0045)$ **2**

She continues to invest $\$M$ at the beginning of each month for a further 9 years and 10 months (a total of 10 years).

(iii) If she wishes to retire with \$500 000 at the end of this time, find the amount $M$. **3**

END OF PAPER

# ABBOTSLEIGH - MATHEMATICS TRIAL 2003 - SOLUTIONS

## QUESTION 1

(a) $0.21006\ldots$

$\doteqdot 0.210$

(b) $\frac{7\pi}{6} = \frac{7 \times 180^\circ}{6}$

$= 210^\circ$

(c) $\frac{x^2-4}{x} \div \frac{x^2-x-6}{x^2-3x}$

$= \frac{(x-2)(x+2)}{x} \times \frac{x(x-3)}{(x-3)(x+2)}$

$= x-2$

(d) $9^x = \frac{1}{27}$

$(3^2)^x = 3^{-3}$

$2x = -3$

$x = -1\frac{1}{2}$

(e) $\int \left(2 - \frac{1}{x^2}\right) dx$

$= \int (2 - x^{-2})\, dx$

$= 2x - \frac{x^{-1}}{-1} + C$

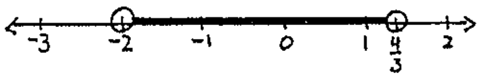
$= 2x + \frac{1}{x} + C$

(f) $|3x+1| < 5$

$-5 < 3x+1 < 5$

$-6 < 3x < 4$

$-2 < x < 4/3$




## QUESTION 2

(a) $y = \ln x$

$\frac{dy}{dx} = \frac{1}{x}$

at $x=1$, $y=0$, grad of tang $= m = \frac{dy}{dx} = \frac{1}{1} = 1$

$\therefore$ eq'n of tangent: $y - 0 = 1(x-1)$

$y = x-1$

(b) (i) $\frac{d}{dx}\left(\frac{x}{\cos x}\right) = \frac{\cos x \times 1 - x \times -\sin x}{\cos^2 x}$

$= \frac{\cos x + x \sin x}{\cos^2 x}$

(ii) $\frac{d}{dx}\left(e^{x^3}\right) = 3x^2 e^{x^3}$

(c) $\angle DCB + \angle ABC = 180^\circ$ (coint. angles, $AB \parallel DC$)

$\therefore x^\circ + 80^\circ = 180^\circ$

$x = 100$

$\angle DCE + 100^\circ + 153^\circ = 360^\circ$ (angles at a point)

$\therefore \angle DCE = 107^\circ$

$\angle DCE + \angle CDE + y^\circ = 180^\circ$ ($\angle$ sum of $\triangle$)

$\therefore 35^\circ + 107^\circ + y^\circ = 180^\circ$

$y = 38$

(d) (i) $\int \sec^2(3x+1)\, dx$

$= \frac{1}{3}\tan(3x+1) + C$

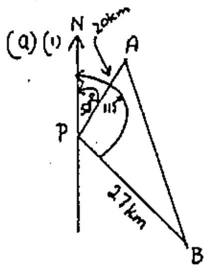
(ii) $\int \frac{dx}{2\sqrt{x}} = \frac{1}{2}\int x^{-1/2}\, dx$

$= \frac{1}{2} \times \frac{x^{1/2}}{1/2} + C$

$= \sqrt{x} + C$

## QUESTION 3

(a) (i)




$\angle APB = 115^\circ - 55^\circ$

$= 60^\circ$

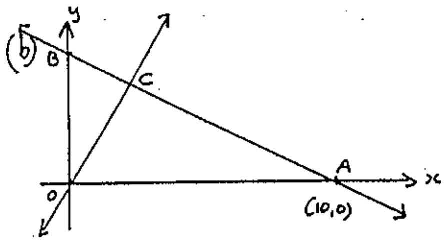
(ii) $AB^2 = 20^2 + 27^2 - 2 \times 20 \times 27 \times \cos 60^\circ$

$= 589$

$AB = 24.269\ldots$

$\doteqdot 24$ km

(b)




(i) gradient $AB \times$ gradient $OC = -1$ (perpendicular lines)

$\therefore$ gradient $AB = -\frac{1}{2}$

(ii) eq'n AB: $y - 0 = -\frac{1}{2}(x-10)$

$2y = -x+10$

$x + 2y - 10 = 0$

(iii) centre of circle = midpoint of OA $= (5,0)$

radius = 5 units

$\therefore$ equation is $(x-5)^2 + y^2 = 25$

(iv) Coordinates of C: $y = 2x$ ①

$x + 2y - 10 = 0$ ②

Subst ① into ② $x + 2(2x) - 10 = 0$

$5x = 10$

$x = 2$

Subst. $x = 2$ into ①

$y = 4$ $\therefore C(2,4)$

→ sub $(2,4)$ into $(x-5)^2 + y^2 = 25$ to show it lies on the circle

LHS $= (2-5)^2 + 4^2$

$= 9 + 16$

$= 25$

(v) $D(12,4)$

(vi) Area OCDA $= 2 \times$ area OCA

$= 2 \times \frac{1}{2} \times 10 \times 4$

$= 40$ square units

## QUESTION 4

(a) (i) $l = r\theta$

$\therefore l = 10 \times \frac{2\pi}{3}$

$= \frac{20\pi}{3}$ cm

$\theta = 120^\circ$

$= \frac{\pi}{180^\circ} \times 120^\circ$

$= \frac{2\pi}{3}$

(ii) Area $= \frac{1}{2}R^2\theta - \frac{1}{2}r^2\sin\theta$

$= \frac{1}{2} \times 10^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 5^2 \times \sin\frac{2\pi}{3}$

$= \frac{100\pi}{3} - \frac{25}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{100\pi}{3} - \frac{25\sqrt{3}}{4}$ cm²

(b) (i) $\int_0^1 (e^{3x} + 2)\, dx = \left[\frac{e^{3x}}{3} + 2x\right]_0^1$

$= \frac{e^3}{3} + 2 - \left(\frac{e^0}{3} + 0\right)$

$= \frac{e^3}{3} + 2 - \frac{1}{3}$

$= \frac{e^3 + 5}{3}$

(ii) $\int \frac{\cos 2x}{\sin 2x}\, dx$

$= \frac{1}{2}\int \frac{2\cos 2x}{\sin 2x}\, dx$

$= \frac{1}{2}\ln(\sin 2x) + C$

(c) (i) $\frac{d}{dx}(x\sin x) = x \times \cos x + 1 \times \sin x$

$= x\cos x + \sin x$

(ii) $\int_0^{\pi/3} x\cos x\, dx$

$= \int_0^{\pi/3} \frac{d}{dx}(x\sin x)\, dx - \int_0^{\pi/3} \sin x\, dx$

$= \left[x\sin x\right]_0^{\pi/3} + \left[\cos x\right]_0^{\pi/3}$

$= \frac{\pi}{3}\sin\frac{\pi}{3} - 0 \times \sin 0 + \cos\frac{\pi}{3} - \cos 0$

$= \frac{\pi}{3} \times \frac{\sqrt{3}}{2} + \frac{1}{2} - 1$

$= \frac{\sqrt{3}\pi}{6} - \frac{1}{2}$

$= \frac{\sqrt{3}\pi - 3}{6}$

## QUESTION 5

(a)(i) $4y = x^2 - 6x + 1$

$x^2 - 6x = 4y - 1$

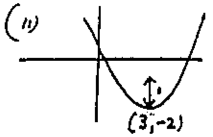
$x^2 - 6x + 9 = 4y + 8$

$(x-3)^2 = 4(y+2)$

Vertex : $(3, -2)$

focal length = 1

(ii)



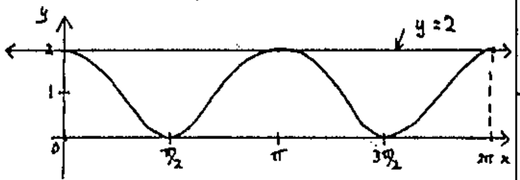

focus : $(3, -1)$

(b) (i) $y = 1 + \cos 2x$  $0 \le x \le 2\pi$

$0 \le 2x \le 4\pi$

period $= \frac{2\pi}{2}$

$= \pi$




(ii) Solution is $x = 0, \pi, 2\pi$

(c) $\int_0^{\sqrt{6}} \frac{x}{x^2+2}\, dx = \frac{1}{2}\int_0^{\sqrt{6}} \frac{2x}{x^2+2}\, dx$

$= \left[\frac{1}{2}\ln(x^2+2)\right]_0^{\sqrt{6}}$

$= \frac{1}{2}\ln((\sqrt{6})^2+2) - \frac{1}{2}\ln(0+2)$

$= \frac{1}{2}\ln 8 - \frac{1}{2}\ln 2$

$= \frac{1}{2}\ln 4$

$= \ln 4^{1/2}$

$= \ln 2$

## QUESTION 5 ctd.

(d) Total no. of cans used $= S_n$

$= 100 + 98 + 96 + \cdots + 2$

$a = 100, \ d = -2, \ T_n = 2 = l$

$T_n = a + (n-1)d$

$2 = 100 + (n-1) \times -2$

$2 = 100 - 2n + 2$

$2n = 100$

$n = 50$

∴ Need food for 50 days.

Total amount used $= S_n$

$= \frac{n}{2}(a + l)$

$= \frac{50}{2}(100 + 2)$

$= 2550$

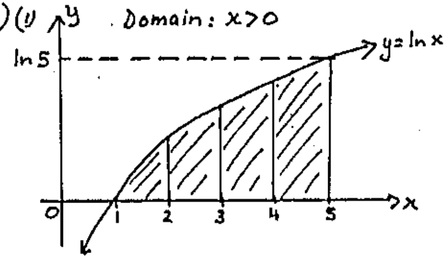
∴ There will be $3000 - 2550$

$= 450$ cans left.

## QUESTION 6

(a) Rational numbers are $\log_e e = 1$ and $0.\dot{4} = \frac{4}{9}$

(b)(i) Domain: $x > 0$




(ii) Shaded area above.

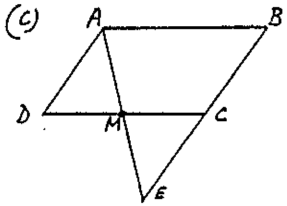
(iii) $\int_1^5 \log_e x\, dx \doteq \frac{1}{3}(0 + 4\ln 2 + \ln 3) + \frac{1}{3}(\ln 3 + 4\ln 4 + \ln 5)$

$= 4.041476\ldots$

$\doteq 4.04$

## QUESTION 6 ctd.

(c)




(i) ABCD is a parallelogram

∴ AB ∥ DC and AD ∥ BC

In △ ADM & △ EMC,

∠DAM = ∠MEC (alternate angles, AD ∥ BC)

∠ADM = ∠ABC (opp. angles of a parm)

∠ABC = ∠DCE (corresp. angles, AB ∥ DC)

∴ ∠ADM = ∠DCE

DM = MC (M is the midpoint of DC)

∴ △ADM ≡ △EMC (SAS test).

(ii) AM = ME (corresponding sides of congruent triangles in (i))

DM = MC (M is midpoint of DC)

∴ ACED is a parallelogram (diagonals bisect each other)

NB There are several different proofs

## QUESTION 7

(a) Domain: $2 - x \ge 0$

$-x \ge -2$

$x \le 2$

(b) (i) $P(1^{st}\text{ prize}) = \frac{15}{100}$

$= \frac{3}{20}$

(ii) P(Does not win either prize)

$= \frac{85}{100} \times \frac{84}{99}$

$= \frac{119}{165}$

## QUESTION 7 ctd.

(c) $y = \frac{1}{2}(\sqrt{x} - 1)$

$2y = \sqrt{x} - 1$

$2y + 1 = \sqrt{x}$

$x = (2y+1)^2$

$x^2 = (2y+1)^4$

Volume $= \int_0^2 \pi x^2\, dy$

$= \pi \int_0^2 (2y+1)^4\, dy$

$= \pi \left[\frac{(2y+1)^5}{2 \times 5}\right]_0^2$

$= \frac{\pi}{10}\left((2\times2+1)^5 - (2\times0+1)^5\right)$

$= \frac{\pi}{10} \times (5^5 - 1)$

$= \frac{1562\pi}{5}$ cubic units

(d) (i) In 2003, no. of tickets sold

$= 92\% \times 50000$

$= 46000$ tickets

(ii) $a = 50000, \ r = 0.92$  $S = \frac{a}{1-r}$

$S = \frac{50000}{1-0.92}$

$= 625000$

(iii) 19 years from 2002 to 2020 inclusive

$S_{19} = \frac{a(1-r^n)}{1-r}$

$= \frac{50000(1-0.92^{19})}{1-0.92}$

$= 496811.597$

$\doteq 496812$ tickets

% of sales $= \frac{496812}{625000} \times 100\%$

$= 79.48992\ldots\%$

$\doteq 79\%$

## QUESTION 8

(a) $f'(x) = x(x-2)(x+2)$ $\quad (0,4)$

$= x(x^2-4)$

$= x^3-4x$

(i) $f(x) = \frac{x^4}{4} - \frac{4x^2}{2} + C$ $\quad$ at $(0,4)$

$4 = 0 - 0 + C$

$\therefore C = 4$

$f(x) = \frac{x^4}{4} - 2x^2 + 4$

(ii) Stationary points at $\frac{dy}{dx} = 0$

$x(x-2)(x+2) = 0$

$x = 0, 2, -2$

$f''(x) = 3x^2 - 4$

at $x=0$, $y=4$, $f''(x) = 3\times0-4 = -4 < 0$

$\therefore$ max turning point at $(0,4)$

at $x=-2$, $y = \frac{(-2)^4}{4} - 2(-2)^2 + 4 = 0$

$f''(x) = 3(-2)^2 - 4 = 8 > 0$

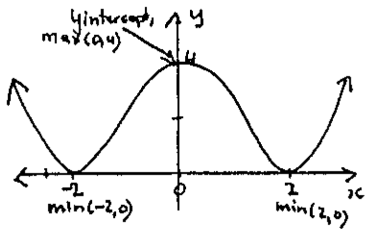
$\therefore$ min. turning point at $(-2,0)$

at $x=2$, $y = \frac{(2)^4}{4} - 2(2)^2 + 4 = 0$

$f''(x) = 3(2)^2 - 4 = 8 > 0$

$\therefore$ min. turning point at $(2,0)$

(iii)




## QUESTION 8 ctd

(b) (i) $y = x^2 + x$ ①

$y = 3 - x$ ②

Subst. ① into ②

$x^2 + x = 3 - x$

$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x = -3$ or $1$

at $x=-3$, $y=6$

at $x=1$, $y=2$

$\therefore$ points of intersection are $(-3,6)$ and $(1,2)$

(ii) Area $= \int_0^1 (x^2+x)\,dx + \int_1^3 (3-x)\,dx$

↑ area of $\Delta$.

$= \left[\frac{x^3}{3} + \frac{x^2}{2}\right]_0^1 + \left[3x - \frac{x^2}{2}\right]_1^3$

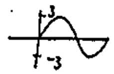
$= \frac{1}{3} + \frac{1}{2} - (0+0) + 9 - \frac{9}{2} - \left(3 - \frac{1}{2}\right)$

$= \frac{1}{3} + \frac{1}{2} + 9 - 4\frac{1}{2} - 3 + \frac{1}{2}$

$= \frac{17}{6}$ square units.

## QUESTION 9

(a) min value $= -3$




(b) $\ln(2x+15) = 2\ln x$ $\quad x > 0$

$\ln(2x+15) = \ln x^2$

$2x + 15 = x^2$

$x^2 - 2x - 15 = 0$

$(x+3)(x-5) = 0$

$\therefore x = -3$ or $5$

but $x > 0$ only for logs.

$\therefore x = 5$ is the only solution.

## QUESTION 9 ctd.

(c) Perimeter $= 4$

(i) $y + y + 2x + \frac{2\pi r}{2} = 4$ $\quad r = x$

$\therefore 2y + 2x + \pi x = 4$

$2y = 4 - 2x - \pi x$

$\therefore y = 2 - x - \frac{\pi x}{2}$

(ii) Area $= \frac{\pi r^2}{2} + BH$

$\therefore A = \frac{\pi x^2}{2} + 2xy$

$= \frac{\pi x^2}{2} + 2x\left(2 - x - \frac{\pi x}{2}\right)$

$= \frac{\pi x^2}{2} + 4x - 2x^2 - \pi x^2$

$= 4x - 2x^2 - \frac{\pi x^2}{2}$

(iii) $\frac{dA}{dx} = 4 - 4x - \pi x$

$\frac{d^2A}{dx^2} = -4 - \pi$

$< 0$ for all $x$

stat. points at $\frac{dA}{dx} = 0$

$4 - x(4+\pi) = 0$

$4 = x(4+\pi)$

$x = \frac{4}{4+\pi}$

$A$ is a quadratic in $x$ $\therefore$ the only stationary point is a maximum.

$\therefore x = \frac{4}{4+\pi}$ will make area a max.

(iv) $y = 2 - x - \frac{\pi x}{2}$

$= 2 - \frac{4}{4+\pi} - \frac{\pi}{2}\left(\frac{4}{4+\pi}\right)$

$= \frac{2(4+\pi) - 4 - 2\pi}{4+\pi}$

$= \frac{8 + 2\pi - 4 - 2\pi}{4+\pi}$

$= \frac{4}{4+\pi}$

$= x$

$\therefore$ Max area occurs when $x = y$.

## QUESTION 10

(a) (i) $P(\text{head}) = \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{4}{7}$

$= \frac{1}{4} + \frac{2}{7}$

$= \frac{15}{28}$

(ii) $P(\text{2 heads at least}) = 1 - P(\text{no heads})$

$= 1 - P(\text{2 tails})$

$= 1 - \left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{3}{7}\times\frac{3}{7}\right)$

$= 1 - \frac{1}{8} - \frac{9}{98}$

$= \frac{307}{392}$

(b) (i) $3000 \times 1.05 + 3000 \times 1.05^2 + \dots + 3000 \times 1.05^{10}$

$= 3000(1.05 + 1.05^2 + \dots + 1.05^{10})$

$= 3000\left\{\frac{1.05(1.05^{10} - 1)}{1.05 - 1}\right\}$

$= \$39620$

(ii) $39620 \times \left(1 + \frac{5.4\%}{12}\right) = 39620 \times 1.0045$

(at end of 1st month)

end of 2nd month: $\{39620(1.0045) + M\} \times \left(1 + \frac{5.4\%}{12}\right)$

$= \{39620 \times 1.0045 + M\} \times 1.0045$

$= 39620 \times 1.0045^2 + M(1.0045)$

(iii) for a further 118 months

$= 39620(1.0045)^{120} + M(1.0045)^{119} + M(1.0045)^{118} + \dots + M(1.0045)^1$

$500000 = 39620(1.0045)^{120} + M(1.0045 + 1.0045^2 + \dots + 1.0045^{119})$

$432094 = M(1.0045 + 1.0045^2 + \dots + 1.0045^{119})$

↑ $S_{119}$

$S_{119} = \frac{1.0045(1.0045^{119} - 1)}{1.0045 - 1}$

$= 157.6509792\dots$

$\therefore M = \frac{432094}{S_{119}}$

$= \$2740.81$