Student Number:__________________

ABBOTSLEIGH

AUGUST 2010

YEAR 12

ASSESSMENT 4

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Write using blue or black pen.
- Board-approved calculators may be used.
- A table of standard integrals is provided.
- All necessary working should be shown in every question.

Total marks **(120)**

- Attempt Questions 1-10.
- All questions are of equal value.

Outcomes assessed

Preliminary course

P1 demonstrates confidence in using mathematics to obtain realistic solutions to problems.

P2 provides reasoning to support conclusions that are appropriate to the context

P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities

P4 chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques

P5 understands the concept of a function and the relationship between a function and its graph

P6 relates the derivative of a function to the slope of its graph

P7 determines the derivative of a function through routine application of the rules of differentiation

P8 understands and uses the language and notation of calculus

HSC course

H1 seeks to apply mathematical techniques to problems in a wide range of practical contexts

H2 constructs arguments to prove and justify results

H3 manipulates algebraic expressions involving logarithmic and exponential functions

H4 expresses practical problems in mathematical terms based on simple given models

H5 applies appropriate techniques from the study of calculus, geometry, trigonometry and series to solve problems

H6 uses the derivative to determine the features of the graph of a function

H7 uses the features of a graph to deduce information about the derivative

H8 uses techniques of integration to calculate areas and volumes

H9 communicates using mathematical language, notation, diagrams and graphs

Question 1 (12 Marks) **Marks**

(a) Evaluate $\log_e 1.6$ correct to three decimal places. 2

(b) Solve $5 - 6x \leq 3$ 2

(c) Factorise fully: $24x^3 + 3y^3$ 2

(d) Simplify $\dfrac{\sin^2\theta + \cos^2\theta}{\tan^2\theta}$ 2

(e) Find the equation of the line passing through the point $(-1, 2)$ and parallel to $3x - y + 4 = 0$ 2

(f) Find the limiting sum of the geometric series $\dfrac{3}{2} + 1 + \dfrac{2}{3} + \ldots$ 2

Question 2 (12 Marks) **Start a new booklet** **Marks**

(a) Solve $\sqrt{3}\tan\theta + 1 = 0$ for $0 \leq \theta \leq 2\pi$ 3

(b) Differentiate with respect to x:

(i) $\dfrac{5x}{x^2 - 3}$ 2

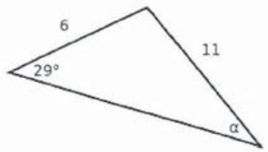
(ii) $(1 + \cos x)^5$ 2

(c) (i) Find $\displaystyle\int 1 + e^{5x}\,dx$ 2

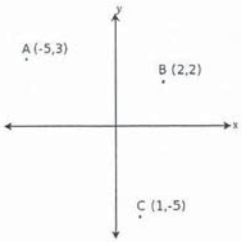
(ii) Evaluate $\displaystyle\int_0^{\frac{\pi}{6}} \sin 2x\,dx$ 3

Question 3 (12 Marks) **Start a new booklet** **Marks**

(a) Find the value of α in the diagram (Diagram is not to scale). Give your answer to the nearest degree. 2



(b) In the diagram A, B and C are the points $(-5,3)$, $(2,2)$ and $(1,-5)$ respectively (Diagram is not to scale).



(i) Calculate the gradient of AC. 1

(ii) Find the coordinates of X, the midpoint of AC. 1

(iii) Calculate the length of BX. 1

(iv) Hence, or otherwise, find the coordinates of D if X is the midpoint of BD. 1

(v) Show that $AC \perp BD$ 2

(vi) Explain why the quadrilateral $ABCD$ is a square. 2

(vii) Calculate the area of $ABCD$. 2

Question 4 (12 Marks) **Start a new booklet** **Marks**

(a) Find the equation of the normal to the curve $f(x) = 6\ln(x-1)$ at the point $(2,0)$. 3

(b) The number of seats in each row of a theatre increases by 4 as you go from the front row to the back row.

(i) If there are fifteen seats in the front row, show that there are $(4n+11)$ seats in the nth row. 1

(ii) If the theatre has 18 rows of seats, calculate the total number of seats in the theatre. 2

(c) A survey shows that if Australian voters were asked to choose who they preferred as Prime Minister, Julia Tillard or Tony Jabbot, 45% would choose Julia Tillard, 40% would choose Tony Jabbot and the remainder would be undecided.
If two voters were chosen at random and asked to make this choice:

(i) Find the probability that they would both be undecided. 1

(ii) Find the probability that at least one would be undecided. 2

(d) The pendulum arm on a grandfather clock is 1 metre in length.

(i) If it sweeps out a sector of area $\frac{\pi}{12}$ square metres as it swings, show that the angle through which the pendulum arm moves is $\frac{\pi}{6}$. 1

(ii) How far *apart* are the end points of the arm at the two extremes of the pendulum swing? 2

Question 5 (12 Marks) **Start a new booklet** **Marks**

(a) The roots of the quadratic equation $px^2 - x + q = 0$ are -1 and 3. Find p and q. 2

(b) A function is defined by $g(x) = (x^2 - 4)(2 - x)$

(i) Find all solutions of $g(x) = 0$ 2

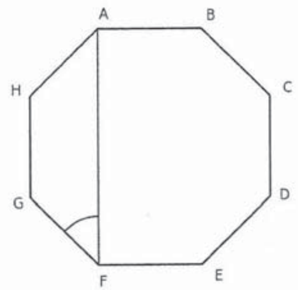
(ii) Find the coordinates of the turning points of the graph of $y = g(x)$, and determine their nature. 3

(iii) Sketch the graph of $y = g(x)$ showing the turning points and the x and y intercepts. 2

(iv) For what values of x is the graph concave up? 1

(c) Use Simpson's rule with three function values to find an approximation to the value of $\int_0^2 \sqrt{4 - x^2}\, dx$. Give your answer correct to three significant figures. 2

Question 6 (12 Marks) **Start a new booklet** **Marks**



(a) $ABCDEFGH$ is a regular octagon (Diagram is not to scale). Find the size of $\angle AFG$ giving all reasons. 3

(b) Find the coordinates of the focus of the parabola $x^2 = -12(y - 3)$. 2

(c) Sunlight transmitted into water loses intensity as it penetrates to greater depths. Intensity I at depth d metres below the surface is given by $I = Ae^{-kd}$. If the intensity of sunlight 300 metres below the surface of the water is three-tenths of the original intensity find:

(i) the value of k 2

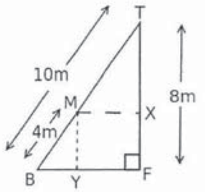
(ii) the depth below the surface (to the nearest metre) at which the intensity would be halved. 2

(d) Solve for x: 3

$$3e^{-2x} - e^{-x} - 2 = 0$$

Question 7 (12 Marks) **Start a new booklet** **Marks**

(a) A ladder TB, 10 metres long, rests against a vertical wall with the top, T, 8 metres up the wall (Diagram is not to scale).



(i) How far is the base of the ladder, B, from the foot of the wall, F? 1

(ii) Milli stands at the point M, 4 metres from the base of the ladder, with Y vertically below her on the ground and X horizontally across from her on the wall. Prove that ΔMYB is similar to ΔTFB. 2

(iii) How far above ground level is Milli? 2

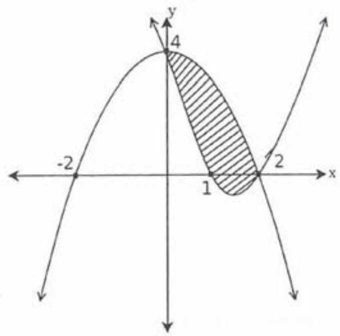
(b) Merrilyn, a graduate engineer, earns \$60 000 in her first year of employment and in each of the following years her annual salary is increased by 5% of the previous year's salary.

(i) What is Merrilyn's annual salary in her 5th year of employment? 2

(ii) Calculate her total earnings for the first ten years of her employment. 2

(iii) At the *end* of each year of employment Merrilyn invests \$5 000. Her investment earns interest at a rate of 4% per annum, compounded annually. Calcuate the total amount she has accrued by the end of her first 10 years of employment. 3

Question 8 (12 Marks) **Start a new booklet** **Marks**



(a) The shaded region is bounded by two parabolae. (Diagram is not to scale) The parabola $y = 4 - x^2$ cuts the x axis at $(-2, 0)$ and $(2, 0)$ and the y axis at $(0, 4)$. The other parabola cuts the x axis at $(1, 0)$ and $(2, 0)$ and the y axis at $(0, 4)$.

(i) State the roots of the other parabola. 1

(ii) Hence, or otherwise, show that the equation of the other parabola is

$$y = 2x^2 - 6x + 4$$

1

(iii) Use calculus to find the area of the shaded region. 3

(b) $\displaystyle\int_0^1 \frac{e^x}{1 + e^x}\, dx = \log_e c$. Find c. 3

(c) Consider the equation in x:

$$(2k - 3)\, x^2 + (k + 1)\, x - 1 = 0$$

(i) For what value of k is this a linear equation? 1

(ii) Find all values of k for which this is a quadratic equation with two distinct roots. 3

Question 9 (12 Marks) **Start a new booklet** **Marks**

(a) (i) Draw a neat sketch of the curve $y = \sec \frac{x}{2}$ for $-2\pi \leq x \leq 2\pi$ 2

(ii) A section of the curve $y = \sec \frac{x}{2}$ from $x = -\frac{\pi}{2}$ to $x = \frac{\pi}{2}$ is rotated about the x axis to form a solid of revolution. Calculate the exact volume of this solid. 3

(b) The gradient of a curve at any point is given by $f'(x) = e^{-2x}$.

(i) Explain why the curve $y = f(x)$ is an increasing function for all values of x. 1

(ii) Given that the curve has a y intercept of 4.5, use $f'(x)$ to show that

$$f(x) = 5 - \frac{e^{-2x}}{2}$$

2

(iii) What is $\lim_{x \to \infty} f(x)$? 1

(iv) Calculate the exact value of the x intercept of the curve. 2

(v) Draw a neat sketch of $f(x)$, showing the asymptote. 1

Question 10 (12 Marks) **Start a new booklet** **Marks**

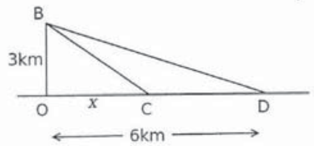
(a) $A\ (4, 2)$ and $B\ (-2, -8)$ are two points on the number plane. The point $P\ (x, y)$ moves so that PA is always perpendicular to PB.

(i) Find the gradients of PA and PB in terms of x and y. 1

(ii) Hence, show that the equation of the locus of P is $(x-1)^2 + (y+3)^2 = 34$ 3

(iii) Describe the locus of P geometrically. 1

(b) A man in a boat at B is 3km from the nearest point O on a straight beach (Diagram is not to scale). He wants to get to his beachhouse at D, 6 kilometres along the beach from O.



(i) He decides to row to a point C, between O and D, and then walk from C to D. If C is x kilometres from O, find an expression for the distance BC he will row. 1

(ii) If he can row at 4 km/h and walk at 5 km/h, show that the total time in hours, $T(x)$, he will take to row to C then walk to D is given by

$$T(x) = \frac{\sqrt{x^2+9}}{4} + \frac{6-x}{5} \text{ for } 0 \leq x \leq 6$$

2

(iii) Find the value of x for which the total time he will take to get to his beach house is a minimum. 3

(iv) What is the least amount of time he will take to reach his beach house? 1

END OF PAPER

TRIAL MATHEMATICS 2010

1(a) $\log_e 1.6 = 0.47000\ldots$

$\doteq 0.470$

(b) $5 - 6x \le 3$

$-6x \le -2$

$x \ge \frac{1}{3}$

(c) $24x^3 + 3y^3 = 3(8x^3 + y^3)$

$= 3(2x+y)(4x^2 - 2xy + y^2)$

(d) $\dfrac{\sin^2\theta + \cos^2\theta}{\tan^2\theta} = \dfrac{1}{\tan^2\theta}$

$= \cot^2\theta$

(e) par. to $3x - y + 4 = 0$

$y = 3x + 4$

grad $m = 3$, $(-1, 2)$ on line

$y - y_1 = m(x - x_1)$

$y - 2 = 3(x + 1)$

$y - 2 = 3x + 3$

$3x - y + 5 = 0$

(f) $\frac{3}{2} + 1 + \frac{2}{3} + \ldots$

geom $a = \frac{3}{2}$, $r = \frac{2}{3}$

$S_\infty = \dfrac{a}{1-r}$

$= \dfrac{\frac{3}{2}}{1 - \frac{2}{3}}$

$= \dfrac{\frac{3}{2}}{\frac{1}{3}}$

$= \dfrac{9}{2}$

2(a) $\sqrt{3}\tan\theta + 1 = 0$, $0 \le \theta \le 2\pi$

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\theta = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$ or $\frac{11\pi}{6}$

(b)(i) $\dfrac{d}{dx}\left(\dfrac{5x}{x^2-3}\right) = \dfrac{(x^2-3)\times 5 - 5x \times 2x}{(x^2-3)^2}$

$= \dfrac{5x^2 - 15 - 10x^2}{(x^2-3)^2}$

$= \dfrac{-5x^2 - 15}{(x^2-3)^2}$

$= \dfrac{-5(x^2+3)}{(x^2-3)^2}$

(ii) $\dfrac{d}{dx}(1+\cos x)^5 = 5(1+\cos x)^4 \times -\sin x$

$= -5\sin x(1+\cos x)^4$

(c)(i) $\int (1 + e^{5x})\,dx = x + \frac{1}{5}e^{5x} + c$

(ii) $\int_0^{\frac{\pi}{6}} \sin 2x\,dx = \left[-\frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{6}}$

$= \left[-\frac{1}{2}\cos\frac{\pi}{3} + \frac{1}{2}\cos 0\right]$

$= -\frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times 1$

$= \frac{1}{4}$

3(a)

$\dfrac{\sin\alpha}{6} = \dfrac{\sin 29^\circ}{11}$

$\sin\alpha = \dfrac{6\sin 29^\circ}{11}$

$= 0.2644\ldots$

$\alpha = 15.33\ldots$

$= 15^\circ$ (n. deg).

(b)(i) $A(-5, 3)$ $B(2, 2)$ $C(1, -5)$

$m_{AC} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{-5 - 3}{1 + 5}$

$= -\dfrac{8}{6}$

$= -\dfrac{4}{3}$

(ii) $X = \left(\dfrac{-5+1}{2}, \dfrac{3-5}{2}\right)$

$= \left(-\frac{4}{2}, -\frac{2}{2}\right)$

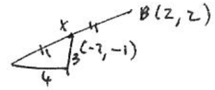
$= (-2, -1)$

(iii) $BX = \sqrt{(2+2)^2 + (2+1)^2}$

$= \sqrt{16 + 9}$

$= 5$

(iv) $D(-6, -4)$



(v) $m_{BD} = \dfrac{2+4}{2+6}$

$= \dfrac{6}{8}$

$= \dfrac{3}{4}$

so $m_{BD} \times m_{AC} = \frac{3}{4} \times -\frac{4}{3}$

$= -1$

$\therefore BD \perp AC$

(vi) It is a rhombus as diagonals bisect at 90°. $m_{AB} = \dfrac{2-3}{2+5}$ $m_{BC} = \dfrac{2+5}{2-1}$

$= -\frac{1}{7}$ $= 7$

so $m_{AB} \times m_{BC} = -\frac{1}{7} \times 7$

$= -1$

so $AB \perp BC$

so square as adjacent sides right∠ between them.

3(b)(vii) equal diagonals since square.

$A = \frac{1}{2}xy$ (rhombus)

$= \frac{1}{2} \times 10 \times 10$

$= 50$

so 50 units^2

4(a) $f(x) = 6\ln(x-1)$
$f'(x) = \frac{6}{x-1}$
grad of tgt at (2, 0)
$f'(2) = \frac{6}{2-1}$
$= 6$
∴ norml grad $-\frac{1}{6}$
eq'n of norml $y - 0 = -\frac{1}{6}(x-2)$
$y = -\frac{1}{6}(x-2)$
$6y = -x+2$
$x+6y-2=0$.

(b)(i) 15, 19, 23, ...
arith., $a = 15$, $d = 4$
$T_n = 15 + (n-1)\times 4$
$= 15+4n-4$
$= 11 + 4n$
so $(4n+11)$ seats in nth row

(ii) 18 rows $S_{18} = \frac{18}{2}(2\times 15 + (18-1)\times 4)$
$= 9(30+68)$
$= 882$

There are 882 seats.

(c)(i) $(0.15)^2 = 0.0225$
(or 2.25%)

(ii) P(at least 1 under) = 1 − P(neither under).
$= 1-(0.85)^2$
$= 0.2775$
(or 27.75%)

(d)(i) $A = \frac{1}{2}r^2\theta$, $r=1$, $A = \frac{\pi}{12}$
$\frac{\pi}{12} = \frac{1}{2}\theta$
$\theta = \frac{\pi}{6}$ so $\frac{\pi}{6}$ rad.

(ii) $a^2 = 1^2+1^2-2\times 1\times 1\cos\frac{\pi}{6}$.

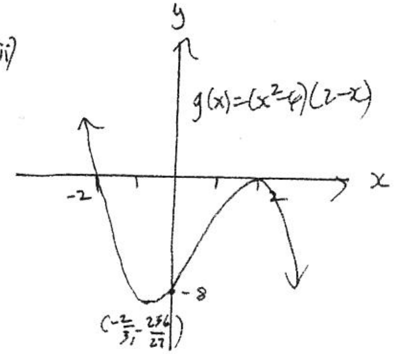
$= 2-2\frac{\sqrt{3}}{2}$
$= 2-\sqrt{3}$ so $\sqrt{2-\sqrt{3}}$ m apart.

5(a) $px^2 - x + q = 0$ roots −1, 3
$p+1+q=0$ ①
$9p-3+q=0$ ②.
② − ① $8p-4=0$
$p = \frac{1}{2}$
sub in ① $\frac{1}{2}+1+q=0$
$q = -\frac{3}{2}$
∴ $p = \frac{1}{2}$, $q = -\frac{3}{2}$.

or $-\frac{b}{a} = \alpha+\beta$, $\frac{c}{a} = \alpha\beta$.
$\frac{1}{p} = -1+3$ $\frac{q}{p} = -3$
$\frac{1}{p} = 2$ $2q = -3$
$p = \frac{1}{2}$ $q = -\frac{3}{2}$

(b) $g(x) = (x^2-4)(2-x)$
(i) $g(x) = 0$ $(x^2-4)(2-x)=0$
$(x-2)(x+2)(2-x)=0$
∴ $x = 2$ or -2 or 2
so $x = 2$ or -2.

(ii) $g(x) = 2x^2-x^3-8+4x$
$g'(x) = 4x-3x^2+4$.
$g''(x) = 4-6x$
tps $g'(x) = 4x-3x^2+4=0$.
$3x^2-4x-4=0$.
$(3x+2)(x-2)=0$.
$x = -\frac{2}{3}$ or 2
$g(-\frac{2}{3}) = -\frac{256}{27}$ $g(2) = 0$.
$(-\frac{2}{3}, -\frac{256}{27})$, $(2, 0)$
nature? $g''(-\frac{2}{3}) = 4-6\cdot -\frac{2}{3}$ $g''(2) = 4-6\times 2$
> 0. $= -8$
   < 0
so conc up ∪ so   so conc dn ∩
min tpt at $(-\frac{2}{3}, -\frac{256}{27})$ max tpt at (2, 0)

5b(iii)


(iv) $g''(x) > 0$.
$4-6x > 0$.
$-6x > -4$
$x < \frac{2}{3}$.
So conc up for $x < \frac{2}{3}$.

(c) $\int_0^2 \sqrt{4-x^2}\,dx$

x	0	1	2
y	2	$\sqrt{3}$	0

$\doteqdot \frac{h}{6}(y_0+4y_1+y_2)$
$= \frac{2}{6}(2+4\sqrt{3}+0)$
$= 2.9760...$
$= 2.976$ (3dp)

6(a) octagon. int < sum $(n-2)\times 180°$
$= 6\times 180°$
$= 1080°$
each < of reg oct $= \frac{1080°}{8}$
$= 135°$
AFGH is a quad so < sum 360°
isos trapezium as AH = GF
(reg. octagon)
∴ $\angle AFG = \frac{360°-2\times 135°}{2}$
$= 45°$.
($\angle HAF = \angle AFG$ in isos. trap.)

(b) $x^2 = -12(y-3)$ $x^2 = -4a(y-k)$
vertex (0, 3)

focal length $a = 3$
∴ focus (0, 0)

(c)(i) $I = Ae^{-kd}$ orig. intensity is $I = A$.
when $d = 300$, $I = 0.3A$
so $0.3A = Ae^{-300k}$.
$0.3 = e^{-300k}$.
$-300k = \ln 0.3$
$k = -\frac{\ln 0.3}{300}$.

(ii) Solve $I = 0.5A$
$0.5A = Ae^{\frac{d\ln 0.3}{300}}$
$\frac{d\ln 0.3}{300} = \ln(0.5)$
$d = \frac{300\ln 0.5}{\ln 0.3}$
$= 172.714...$
So approx 173 m below.

6(d) $3e^{-2x} - e^{-x} - 2 = 0$

let $u = e^{-x}$

$3u^2 - u - 2 = 0$

$(3u+2)(u-1) = 0$

$u = -\frac{2}{3}$ or 1

$e^{-x} = -\frac{2}{3}$ or $e^{-x} = 1$

no sol'n $\quad x = 0$

$(\because e^{-x} > 0)$

$\therefore x = 0$.

7(a)(i) 6m (Pythagorean)

(ii) in ΔMYB and ΔTXM

$\angle MYB = \angle TXM = 90^\circ$

$(MY \perp BF,\ TX \perp BF)$

$\angle MBY = \angle TMX$ (corresp. ∠s, $MX \parallel BF$)

$\therefore \Delta MXB \parallel\!\parallel \Delta TXM$ (equiangular)

(iii) $\frac{MB}{TB} = \frac{MY}{TF}$ (corresp. sides of sim Δs in prop)

$\frac{4}{10} = \frac{MY}{8}$

$MY = \frac{8 \times 4}{10}$

$= 3.2$

So 3.2m above ground

(b)(i) $T_5 = 60000 \times 1.05^4$

$= 72930.375\ldots$

So approx \$72930.38.

(ii) $S_{10} = \frac{60000(1.05^{10} - 1)}{1.05 - 1}$

$= 754\,673.552\ldots$

Approx \$754 673.55

(iii) At end 1st yr has A_1

$A_1 = 5000$

end 2nd $A_2 = 1.04 \times 5000 + 5000$

$A_3 = 1.04^2 \times 5000 + 1.04 \times 5000 + 5000$

$\vdots$

A_n geom. sum $a = 5000$, $r = 1.04$.

end of 10th y'r.

$A_{10} = \frac{5000(1.04^{10} - 1)}{1.04 - 1}$

$= 60\,030.535\ldots$

So approx \$60 030.54

8(a)(i) 1, 2

(ii) $\alpha + \beta = 3 = -\frac{b}{a}$ $\quad \alpha\beta = 2 = \frac{c}{a}$ etc

or $y = a(x-1)(x-2)$

sub in (0, 4) $\quad 4 = a \times -1 \times -2$

$2a = 4$

$a = 2$

$\therefore y = 2(x-1)(x-2)$

$= 2(x^2 - 3x + 2)$

$= 2x^2 - 6x + 4$ QED.

(iii) area $= \int_0^2 4 - x^2 - (2x^2 - 6x + 4)\,dx$

$= \int_0^2 (-3x^2 + 6x)\,dx$

$= [-x^3 + 3x^2]_0^2$

$= -8 + 12 - (-0 + 0)$

$= 4$

so 4 units²

(b) $\int_0^1 \frac{e^x}{1+e^x}\,dx = \log_e c$

$\int_0^1 \frac{e^x}{1+e^x}\,dx = [\ln(1+e^x)]_0^1$

$= \ln(1+e^1) - \ln(1+e^0)$

$= \ln(1+e) - \ln 2$

$= \ln \frac{1+e}{2}$

$\therefore c = \frac{1+e}{2}$.

(c) $(2k-3)x^2 + (k+1)x - 1 = 0$

(i) linear if $2k - 3 = 0$

$\therefore k = \frac{3}{2}$

(ii) quad. w. dist roots if $k \neq \frac{3}{2}$ and $\Delta > 0$.

so $(k+1)^2 - 4(2k-3) \times -1 > 0$.

$k^2 + 2k + 1 + 8k - 12 > 0$

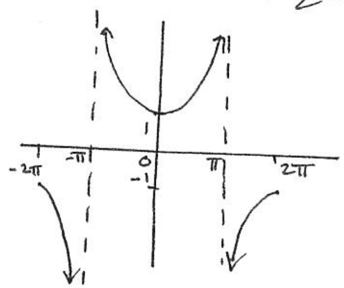
$k^2 + 10k - 11 > 0$

$(k+11)(k-1) > 0$.

$\therefore k > 1$ $(k \neq \frac{3}{2})$ or $k < -11$

9(a)(i) $y = \sec \frac{x}{2}$ $\quad -2\pi \le x \le 2\pi$

$-\pi \le \frac{x}{2} \le \pi$



(ii) $V = 2\pi \int_0^{\frac{\pi}{2}} (\sec \frac{x}{2})^2\,dx$

$= 2\pi \int_0^{\frac{\pi}{2}} \sec^2 \frac{x}{2}\,dx$

$= 2\pi [2\tan \frac{x}{2}]_0^{\frac{\pi}{2}}$

$= 2\pi [2\tan \frac{\pi}{4} - 2\tan 0]$

$= 2\pi [2 - 0]$

$= 4\pi$ so 4π units³

(b) $f'(x) = e^{-2x}$

(i) increasing $\because e^{-2x} > 0$ for all real x, so grad pos for all real x

(ii) (0, 4.5) on curve show $f(x) = 5 - \frac{e^{-2x}}{2}$

$f'(x) = e^{-2x}$

$f(x) = -\frac{1}{2} e^{-2x} + c$

$4.5 = -\frac{1}{2} e^0 + c$

$4.5 = -\frac{1}{2} + c$

$c = 5$

$\therefore f(x) = -\frac{1}{2} e^{-2x} + 5$ QED

(iii) $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} 5 - \frac{e^{-2x}}{2}$

$= 5 - 0$

$= 5$ $\quad (\because \lim_{x \to \infty} e^{-2x} = 0)$

9(b)(iv) x-int. $f(x) = 0$

$$\frac{e^{-2x}}{2} = 5$$

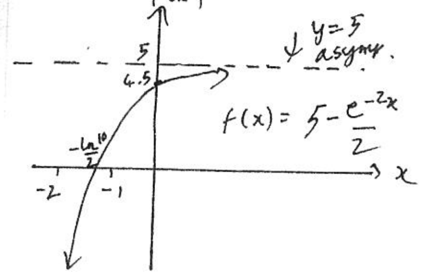
$$e^{-2x} = 10$$

$$-2x = \ln 10$$

$$x = -\frac{\ln 10}{2}$$

so $\left(-\frac{\ln 10}{2}, 0\right)$

(v)



10(a)(i) $m_{PA} = \frac{y-2}{x-4}$ $m_{PB} = \frac{y+8}{x+2}$

$PA \perp PB$ ∴ $\frac{y-2}{x-4} \times \frac{y+8}{x+2} = -1$

∴ $(y-2)(y+8) = -(x-4)(x+2)$

$y^2 + 6y - 16 = -(x^2 - 2x - 8)$

$x^2 - 2x - 8 + y^2 + 6y - 16 = 0$

$x^2 - 2x + 1 + y^2 + 6y + 9 = 8 + 16 + 1 + 9$

$(x-1)^2 + (y+3)^2 = 34$

(ii) circle centre $(1, -3)$ rad $\sqrt{34}$ u.

(b) (i) $BC = \sqrt{x^2+9}$ (Pythag)

(ii) $T = \frac{D}{S}$ ∴ rowing time $= \frac{\sqrt{x^2+9}}{4}$

walking time $= \frac{6-x}{5}$

∴ total time $T(x) = \frac{\sqrt{x^2+9}}{4} + \frac{6-x}{5}$ QED

noting $0 \le x \le 6$

(iii) min $T(x)$

$$T(x) = \frac{(x^2+9)^{\frac{1}{2}}}{4} + \frac{6}{5} - \frac{x}{5}$$

$$T'(x) = \frac{\frac{1}{2}(x^2+9)^{-\frac{1}{2}} \times 2x}{4} - \frac{1}{5}$$

$$= \frac{x}{4\sqrt{x^2+9}} - \frac{1}{5}$$

for max/min $T'(x) = 0$.

so $\frac{x}{4\sqrt{x^2+9}} = \frac{1}{5}$.

∴ $\frac{x}{\sqrt{x^2+9}} = \frac{4}{5}$

$\frac{x^2}{x^2+9} = \frac{16}{25}$.

$25x^2 = 16x^2 + 9 \times 16$

$9x^2 = 9 \times 16$

$x^2 = 16$

$x = 4$ $(x \geq 0)$

so $x = 4$ is poss max/min

10(b)(iii)

(iii) $T'(4^+) > 0$, $T'(4^-) < 0$.

$-$ _ 0 _ $+$ (at 4)

so min at $x = 4$.

$$T(4) = \frac{\sqrt{4^2+9}}{4} + \frac{6-4}{5}$$

$$= \frac{5}{4} + \frac{2}{5}$$

$$= \frac{33}{20}$$

so $\frac{33}{20}$ hours.

= 1h 39min.