Student's Name: ____________________

Student Number: | | | | | | | | | |

Teacher's Name: ____________________

ABBOTSLEIGH

2016
HIGHER SCHOOL CERTIFICATE
Assessment 4

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black pen.
- **Board-approved** calculators may be used.
- A reference sheet is provided.
- In Questions 11–16, show relevant mathematical reasoning and/ or calculations.
- Make sure your HSC candidate Number is on the front cover of each booklet.
- Start a new booklet for Each Question.
- Answer the Multiple Choice questions on the answer sheet provided.
- If you do not attempt a whole question, you must still hand in the Writing Booklet, with the words '**NOT ATTEMPTED**' written clearly on the front cover.

Total marks - 100

- Attempt Sections 1 and 2.

Section I — Pages 3 - 6

10 marks
- Attempt Questions 1–10.
- Allow about 15 minutes for this section.

Section II — Pages 7 - 15

90 marks
- Attempt Questions 11- 16.
- Allow about 2 hrs and 45 minutes for this section.

Outcomes to be assessed:

Mathematics

Preliminary Outcomes:

P2 Provides reasoning to support conclusions which are appropriate to the context
P3 Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
P4 Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques
P5 Understands the concept of a function and the relationship between a function and its graph
P6 Relates the derivative of a function to the slope of its graph
P7 Determines the derivative of a function through routine application of the rules of differentiation
P8 Understands and uses the language and notation of calculus

HSC Outcomes:

H1 Seeks to apply mathematical techniques to problems in a wide range of practical contexts
H2 Constructs arguments to prove and justify results
H3 Manipulates algebraic expressions involving logarithmic and exponential functions
H4 Expresses practical problems in mathematical terms based on simple given models
H5 Applies appropriate techniques from the study of calculus, geometry, trigonometry and series to solve problems
H6 Uses the derivative to determine the features of the graph of a function
H7 Uses the features of a graph to deduce information about the derivative
H8 Uses techniques of integration to calculate areas and volumes
H9 Communicates using mathematical language, notation, diagrams and graphs

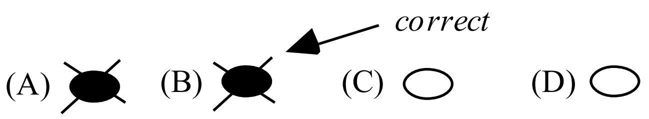
SECTION I

10 marks
Attempt Questions 1 – 10
Use the multiple-choice answer sheet
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.



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1. Evaluate $4e^3 + \ln(2016)$ correct to 3 significant figures.

   (A) 87.9

   (B) 87.951

   (C) 88.0

   (D) 88.951

2. The equation of the line passing through the point (0, 2) and perpendicular to the line $2x - 3y = 10$ is:

   (A) $3x + 2y - 4 = 0$

   (B) $3x + 2y - 6 = 0$

   (C) $2x - 3y + 6 = 0$

   (D) $2x - 3y - 6 = 0$

3. Flora notices that her household expenses are increasing by $10.50 each month. If in July 2016 her expenses were $455, then her anticipated expenses for the month of August 2017 will be?

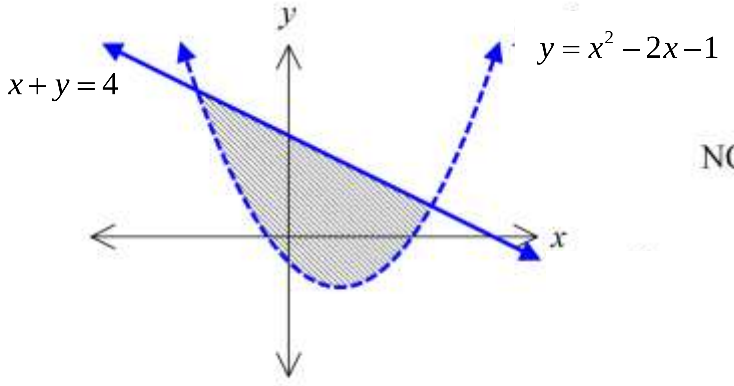
(A) $465.50

(B) $570.50

(C) $581

(D) $591.50

4. Which set of inequations represent the shaded region shown below?



NOT TO SCALE

(A) $x+y\geq 4$ and
$y<x^2-2x-1$

(B) $x+y\geq 4$ and
$y>x^2-2x-1$

(C) $x+y\leq 4$ and
$y<x^2-2x-1$

(D) $x+y\leq 4$ and
$y>x^2-2x-1$

5. Consider the function $f(x)=\dfrac{x+2}{\sqrt{x-3}}$.

Which expression represents the largest possible domain for $f(x)$?

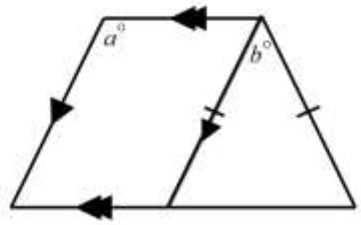
(A) $x>3$

(B) $x\geq 3$

(C) $x>2$

(D) $x\geq 2$

6. A composite shape is made up of a parallelogram and an isosceles triangle, as shown in the diagram.



Which of the following is correct?

(A) $a+2b=180$

(B) $2a-b=180$

(C) $a+b=180$

(D) $b=2a$

7. If $f(x-1)=x^2-2x+3$, then $f(x)$ is equal to?

(A) x^2-2

(B) x^2+2

(C) x^2-2x+2

(D) x^2-2x+4

8. The graph of $y = kx - 4$ intersects the graph of $y = x^2 + 2x$ at two distinct points. Which of the following statements is true?

(A) $k^2 - 4k + 16 > 0$

(B) $k^2 - 4k + 16 < 0$

(C) $k^2 - 4k - 12 > 0$

(D) $k^2 - 4k - 12 < 0$

9. Consider the tangent to the graph $y = x^2$ at the point $(2,\ 4)$.

Which of the following lines is parallel to the tangent?

(A) $x - 4y + 1 = 0$

(B) $4x - y + 1 = 0$

(C) $y = -4x + 6$

(D) $y = 2x + 4$

10. The limiting sum of $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \ldots$ is?

(A) $\frac{3}{5}$

(B) $-\frac{2}{3}$

(C) $\frac{3}{2}$

(D) 3

End of Section 1

SECTION II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Factorise fully $4x^3 - 108$. **2**

(b) Show that the derivative of $(x-3)e^{2x}$ is $e^{2x}(2x-5)$. **2**

(c) Find the derivative of $y = 2 + \tan 3x$. **1**

(d) If $y = \dfrac{\sin x}{1+\cos x}$, show that $\dfrac{dy}{dx} = \dfrac{1}{1+\cos x}$. **3**

(e) (i) Express $\sqrt{48}$ in the form $k\sqrt{3}$ where k is an integer. **1**

(ii) Hence or otherwise, simplify $\dfrac{\sqrt{48}+2\sqrt{27}}{\sqrt{6}}$, giving your answer in simplified surd form. **2**

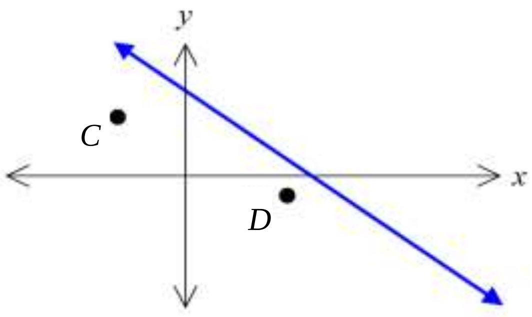
(f) Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \cos 2x \; dx$. **2**

(g) Find $\displaystyle\int \frac{x^2}{x^3-6}\, dx$. **2**

End of Question 11

Question 12 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The line l has the equation $7x+3y-13=0$ and the points C and D are $(-1, 3)$ and $\left(\frac{3}{2}, -1\right)$ respectively.



(i) Find the gradient of line l. **1**

(ii) Find the equation of the line which passes through C and is parallel to l. **2**

(iii) The point A lies on l and D is the midpoint of AC. Find the coordinates of A. **2**

(iv) Without finding the point of intersection, find the equation of the line which passes through the point of intersection of $7x+3y-13=0$ and $3x+2y-12=0$ and also passes through D. **3**

(b) The equation $4x^2+6x+3=0$ has roots α and β.

(i) Write down the values of $\alpha+\beta$ and $\alpha\beta$. **2**

(ii) Show that $\alpha^2+\beta^2=\frac{3}{4}$. **2**

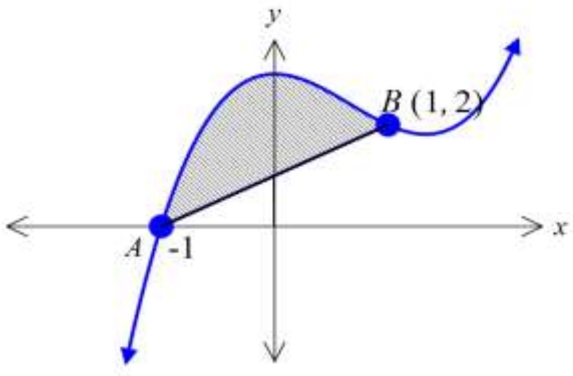
(iii) Show that $(3\alpha-\beta)(3\beta-\alpha)=\frac{21}{4}$. **2**

(iv) Explain why $4x^2+12x+21=0$ has the roots $(3\alpha-\beta)$ and $(3\beta-\alpha)$. **1**

End of Question 12

Question 13 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The curve with the equation $y = x^3 - 2x^2 + 3$ is shown in the diagram.



NOT TO SCALE

The curve cuts the x-axis at the point $A\ (-1, 0)$ and passes through the point $B\ (1,\ 2)$.

(i) Evaluate $\int_{-1}^{1} x^3 - 2x^2 + 3 \ \ dx$. **2**

(ii) Hence, find the area of the shaded region bounded by the curve $y = x^3 - 2x^2 + 3$ and the line AB. **1**

(b) The displacement x metres from the origin at time t seconds, of a particle travelling in a straight line is given by the formula $x = t^3 - 21t^2$.

(i) Find the acceleration of the particle at time t seconds. **2**

(ii) Find the time(s) at which the particle is stationary. **2**

(c) The volume, V m^3, of water in a tank after time t seconds is given by the equation

$$V = 3\ln(2t+1) - 5t + 2$$

(i) Find $\dfrac{dV}{dt}$. **2**

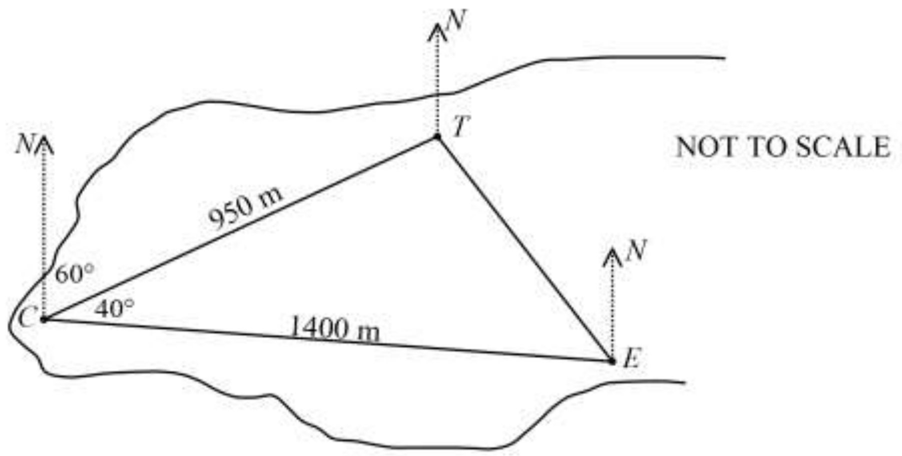
(ii) Explain why the volume of water is decreasing at $t = 1$. **2**

Question 13 continues on the next page.

Question 13 continued.

(d) There are plans to construct a series of straight paths on the flat top of a mountain. A straight path will connect the cable car station at C to a communication tower at T, as shown in the diagram below.

The bearing of the communication tower to the cable car station is $060°$.
The length of the straight path between the communication tower and the cable car station is 950 m.



Paths will also connect the cable car station and the communication tower to the camp site at E. The length of the straight path between the cable car station and the camp site is 1400 m. The angle TCE is $40°$.

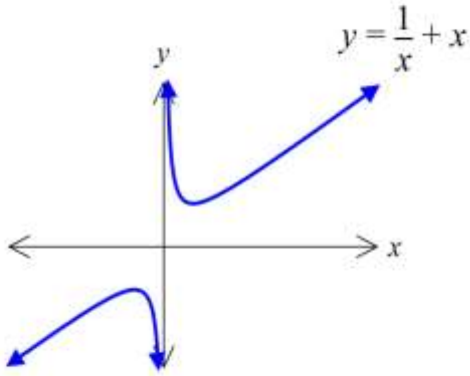
(i) Calculate the length of the path between the communication tower and the camp site, correct to the nearest metre. **2**

(ii) Find the bearing of the camp site from the communication tower, correct to the nearest degree. **2**

End of Question 13

Question 14 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The area bounded by the curve $y = \frac{1}{x} + x$ and the lines $x = 1$ and $x = 3$ is rotated about the x-axis. **3**



Find the volume of the solid of revolution formed.
Leave your answer in exact form.

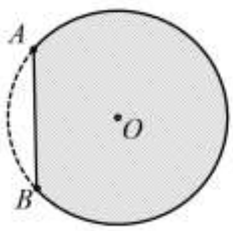
(b) A particle moves in a straight line. At time t seconds its displacement x metres from a fixed point O on the line is given by $x = 3 - \cos 2t,\ \ 0 \le t \le 2\pi$.

(i) Sketch the graph of x as a function of t, showing all the important features. **2**

(ii) Explain how you can use your graph to determine the times that the particle is at rest. **1**

(iii) Find the time when the particle first reaches its maximum speed. **2**

(c) A company is designing a new logo in the shape of a circle with a small segment taken out as shown to the right. The radius of the circle is 4 cm and the length of AB is also 4 cm.



NOT TO SCALE

(i) Explain why $\angle AOB = \frac{\pi}{3}$. **1**

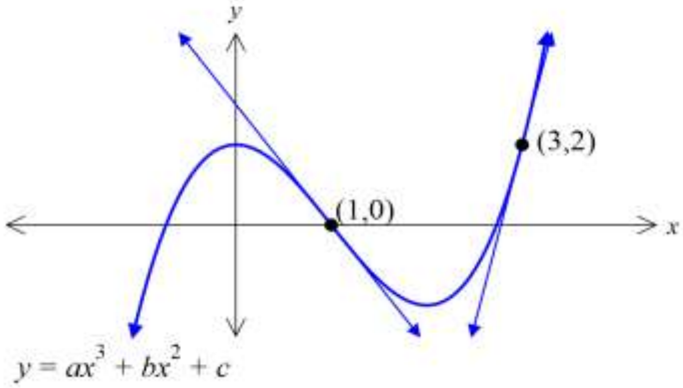
(ii) Find the area of the logo correct to 3 significant figures. **2**

Question 14 continues on the next page.

Question 14 continued.

(d) The cubic function $y = ax^3 + bx^2 + c$ where a, b and c are real constants with $a \neq 0$ is shown in the diagram.

The derivative of this function is $f'(x) = 3ax^2 + 2bx$.



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Two tangents are drawn to this function such that their equations are:

- $y = -3x + 3$ at the point $(1,\ 0)$ and
- $y = 9x - 25$ at the point $(3,\ 2)$.

(i) Show that $9a + 2b = 3$ and $3a + 2b = -3$. **2**

(ii) Hence find the values of a, b and c. **2**

End of Question 14

Question 15 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) The rate of increase of a population $P(t)$ of people in a certain country is determined by the equations $\frac{dP}{dt} = kP$ and $P = P_0 e^{kt}$, where k is a constant, P_0 is the original population and t is the time in years. The population of the country doubles every 20 years.

(i) Show that $k = \frac{1}{20}\ln 2$. **2**

(ii) Data is first collected about the population of this country in the year 2000. In which year will the country reach a population three times that it had at the beginning of 2000? **2**

(iii) Given that at the beginning of the year 2000 the population was 15.1 million, what will be the population of the country at the beginning of the year 2050? Give your answer correct to 3 significant figures. **2**

(b) On the 1st of January 2016 the population of a particular country town was 30 000. At the end of each year 2500 people leave the town to live in the city. During the period between January and the people leaving in December each year, the population increases by 5%.

(i) Show that the number of people in the country town just after the first group of 2500 left in December 2016 is 29 000. **1**

(ii) Show that the expression for the number of people in the country town just after the second group of 2500 left in December 2017 is given by **1**

$$P_2 = 30000 \times 1.05^2 - 2500(1.05 + 1).$$

(iii) Show that P_n, the population after the nth group left is given by **2**

$$P_n = 30000 \times 1.05^n - 50000 \times (1.05^n - 1).$$

(iv) Hence, determine in which year the population of the town will be zero. **3**

(c) A parabola has its focus at $(5, 1)$ and vertex at $(3, 1)$. **2**

Show that the equation of the parabola is $y^2 - 2y - 8x + 25 = 0$.

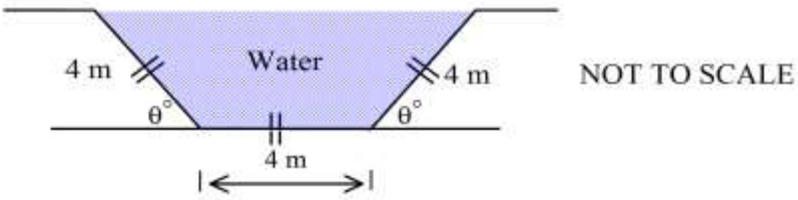
End of Question 15

Question 16 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) An irrigation channel has a cross-section in the shape of a trapezium as shown in the diagram.

The bottom and sides of the trapezium are 4 metres long.

Suppose that the sides of the channel make an angle of θ with the horizontal where $\theta \leq \frac{\pi}{2}$.



(i) Show that the cross-sectional area is given by $A = 16(\sin\theta + \cos\theta\sin\theta)$. **2**

(ii) Show that $\frac{dA}{d\theta} = 16(2\cos^2\theta + \cos\theta - 1)$. **2**

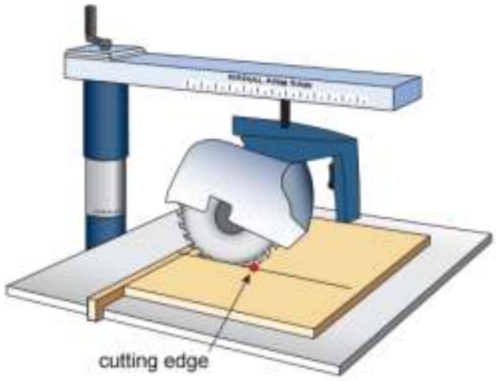
(iii) Hence, show that the maximum cross-sectional area occurs when $\theta = \frac{\pi}{3}$. **3**

(iv) Hence, find the maximum area of the irrigation channel, correct to the nearest square metre. **1**

Question 16 continues on the next page.

Question 16 continued.

(b) When a radial arm saw (as shown on the right) is used, its cutting edge (as indicated by the dot) moves forwards and then backwards along a straight line.



During a particular cutting procedure, the velocity of the cutting edge of the saw, in metres per second, can be modelled by the function

$$v(t) = 0.05t^3 - 0.38t^2 + 0.624t \ ,$$

where t represents the time in seconds from the start of the cutting procedure and $0 \le t \le 5.2$.

(i) For what values of t is the cutting edge of the saw at rest? **2**

(ii) Calculate $\int_0^{5.2} v(t)\ dt$, correct to 3 decimal places. **2**

(iii) Interpret your answer to part (ii) in the context of the motion of the cutting edge of the saw. **1**

(iv) Write an expression to find the total distance travelled by the cutting edge of the saw during the cutting procedure. (There is no need to evaluate this). **2**

End of Paper

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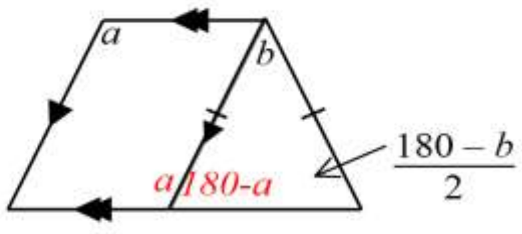
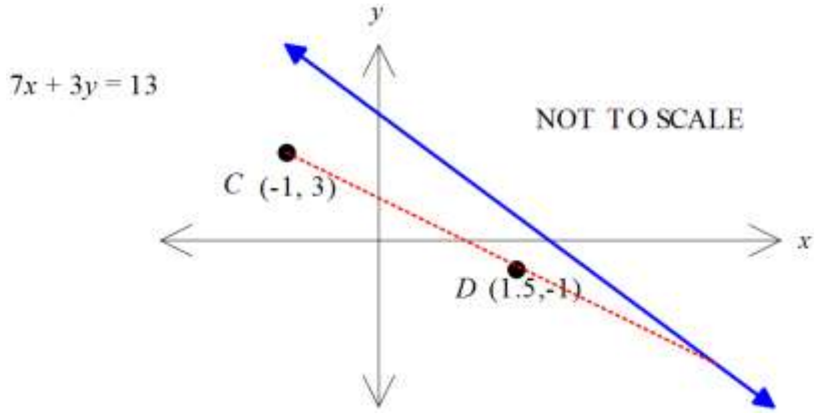
Abbotsleigh 2 unit Mathematics Task 4 2016 Solutions:

Question	Working	Solution
1	88.0	C
2	$y=\frac{2x-10}{3}$ $m=\frac{2}{3}$ $\therefore m\perp \text{line}=-\frac{3}{2}$ Equ of line is $y=-\frac{3}{2}x+2$ $\Rightarrow 2y+3x-4=0$	A
3	$a=455,\ \ d=10.50,\ \ n=14$ $T_{14}=455+13\times 10.50$ $=\$591.50$	D
4	Test pointt $(0,0)$ $x+y\le 4$ $y>x^2-2x-1$	D
5	$x-3>0$ $x>3$	A
6	 $180-a=\frac{180-b}{2}$ $360-2a=180-b$ $2a-b=180$	B

7	Trial and error for B $f(x-1)=(x-1)^2+2$ $=x^2-2x+1+2$ $=x^2-2x+3$ OR sub in $(x+1)$ $(x+1)^2-2(x+1)+3$ $=x^2+2x+1-2x-2+3$ $=x^2+2$	B
8	Solve $\left.\begin{array}{l} y=kx-4 \\ y=x^2+2x \end{array}\right\}$ $kx-4=x^2+2x$ $x^2+x(2-k)+4=0$ For two distinct roots, $\Delta>0$ $(2-k)^2-4(1)(4)>0$ $4-4k+k^2-16>0$ $k^2-4k-12>0$	C
9	$y=x^2$ $y'=2x$ At $x=2, y'=4$ m of tangent = 4 $\therefore$ parallel line has same gradient ie, $4x-y+1=0$	B
10	$a=1,\ r=-\frac{2}{3}$ $S_\infty=\frac{a}{1-r}$ $=\frac{1}{1-\left(-\frac{2}{3}\right)}$ $=\frac{3}{5}$	A

Question	Working	Marks
11(a)	$4x^3 - 108$ $= 4\left(x^3 - 27\right)$ ☑ $= 4\left(x-3\right)\left(x^2 + 3x + 9\right)$ ☑	**2**
11(b)	RTShow: derivative of $(x-3)e^{2x}$ is $e^{2x}(2x-5)$ Proof: $\frac{d}{dx}\left((x-3)e^{2x}\right) = (x-3)2e^{2x} + e^{2x}(1)$ ☑ $= e^{2x}\left(2(x-3)+1\right)$ ☑ $= e^{2x}\left(2x-6+1\right)$ $= e^{2x}(2x-5)$ as required	**2**
11(c)	$y = \tan 3x + 2$ $\frac{dy}{dx} = 3\sec^2 3x$	**1**
11(d)	RTShow: If $y = \frac{\sin x}{1+\cos x}\frac{dy}{dx}$ then $\frac{dy}{dx} = \frac{1}{1+\cos x}$ Proof: $\frac{dy}{dx} = \frac{(1+\cos x)(\cos x) - \sin x\ (-\sin x)}{(1+\cos x)^2}$ ☑ $= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ ☑ $= \frac{1+\cos x}{(1+\cos x)^2}$ (since $\cos^2 x + \sin^2 x = 1$) ☑ $= \frac{1}{1+\cos x}$ as required	**3**
11(e)(i)	$\sqrt{48} = \sqrt{16} \times \sqrt{3}$ $= 4\sqrt{3}$ ☑	**1**

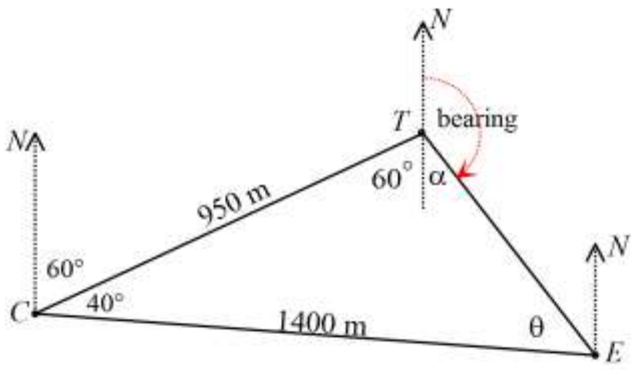
11(e)(ii)	$\dfrac{\sqrt{48}+2\sqrt{27}}{\sqrt{6}}$ $=\dfrac{4\sqrt{3}+2\times 3\sqrt{3}}{\sqrt{6}}$ ☑ $=\dfrac{10\sqrt{3}}{\sqrt{6}}$ $=\dfrac{10}{\sqrt{2}} \quad \left(\times\dfrac{\sqrt{2}}{\sqrt{2}}\right)$ $=\dfrac{10\sqrt{2}}{2}$ $=5\sqrt{2}$ ☑	**2**
11(f)	$\int_0^{\frac{\pi}{4}} \cos 2x \ dx$ $=\left[\dfrac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$ ☑ $=\dfrac{1}{2}\left(\sin(2\times\dfrac{\pi}{4})-\sin 0\right)$ $=\dfrac{1}{2}\sin\dfrac{\pi}{2}$ $=\dfrac{1}{2}$ ☑	**2**
11(g)	$\int \dfrac{x^2}{x^3-6}\ dx$ $=\dfrac{1}{3}\int \dfrac{3x^2}{x^3-6}\ dx$ $=\dfrac{1}{3}\ln\left(x^3-6\right)+C$ ☑ recognising log ☑ $\dfrac{1}{3}$	**2**

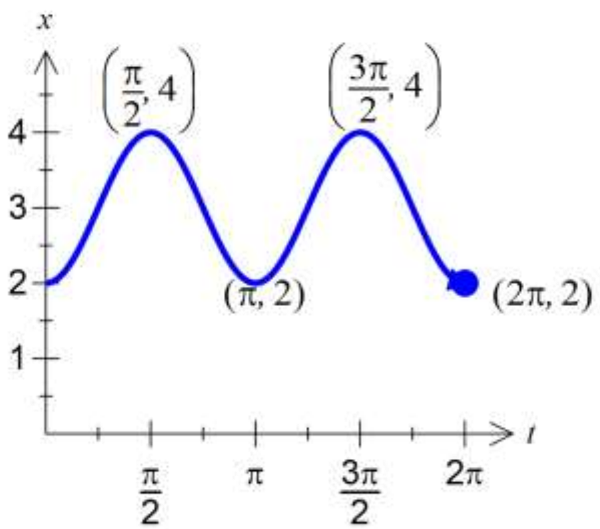
Question	Working	Marks
12(a)(i)	 $7x+3y=13$ $3y=13-7x$ $y=-\frac{7}{3}x+\frac{13}{3}$ $m=-\frac{7}{3}$ ☑	1
12(a)(ii)	Since parallel to l, gradient $=-\frac{7}{3}$ $C(-1,3)$ Equ of line is $y-3=-\frac{7}{3}(x+1)$ ☑ $3y-9=-7x-7$ $7x+3y-2=0$ ☑	2
12(a)(iii)	D is the midpoint of AC $\therefore (1.5,-1)=\left(\frac{x-1}{2},\frac{y+3}{2}\right)$ $\therefore 1.5=\frac{x-1}{2}$ and $-1=\frac{y+3}{2}$ $3=x-1$ $-2=y+3$ $x=4$ ☑ $y=-5$ ☑ $\therefore A$ is $(4,-5)$	2

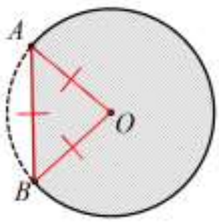
12(a)(iv)	Equ of line is given by $7x+3y-13+k(3x+2y-12)=0$ ✓ Sub in $(1.5,-1)$ $7(1.5)+3(-1)-13+k(3(1.5)+2(-1)-12)=0$ $-5.5-9.5k=0$ $k=\frac{5.5}{-9.5}$ $=-\frac{11}{19}$ ✓ $\therefore$ Equ is $7x+3y-13-\frac{11}{19}(3x+2y-12)=0$ $133x+57y-247-33x-22y+132=0$ $100x+35y-115=0$ ✓ $20x+7y-23=0$	**3**
12(b)(i)	$4x^2+6x+3=0$ $a=4,\quad b=6,\quad c=3$ $\alpha+\beta=-\frac{b}{a}$ $=-\frac{6}{4}$ ✓ $=-\frac{3}{2}$ $\alpha\beta=\frac{c}{a}$ $=\frac{3}{4}$ ✓	**2**
12(b)(ii)	RTShow: $\alpha^2+\beta^2=\frac{3}{4}$ Proof: $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$ ✓ $=\left(-\frac{3}{2}\right)^2-2\left(\frac{3}{4}\right)$ ✓ $=\frac{9}{4}-\frac{3}{2}$ $=\frac{3}{4}$ as required	**2**

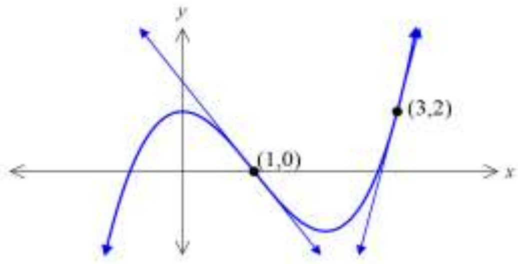
12(b)(iii)	RTShow: $\left(3\alpha-\beta\right)\left(3\beta-\alpha\right)=\dfrac{21}{4}$ Proof: $\left(3\alpha-\beta\right)\left(3\beta-\alpha\right) = 9\alpha\beta-3\alpha^2-3\beta^2+\alpha\beta$ $=10\alpha\beta-3\left(\alpha^2+\beta^2\right)$ ☑ $=10\times\dfrac{3}{4}-3\times\dfrac{3}{4}$ ☑ $=\dfrac{21}{4}$ as required	**2**
12(b)(iv)	For $4x^2+12x+21=0$, product of roots $=\dfrac{c}{a}$ $=\dfrac{21}{4}$ From part (iii), $(3\alpha-\beta)(3\beta-\alpha)=\dfrac{21}{4}$ $\therefore$ Roots of $4x^2+12x+21=0$ are $(3\alpha-\beta)$ and $(3\beta-\alpha)$.	**1**

Question	Working	Marks
13(a)(i)	$\int_{-1}^{1} x^3 - 2x^2 + 3 \; dx$ $= \left[\frac{x^4}{4} - \frac{2x^3}{3} + 3x \right]_{-1}^{1}$ ✓ $= \frac{(1)^4}{4} - \frac{2(1)^3}{3} + 3(1) - \left(\frac{(-1)^4}{4} - \frac{2(-1)^3}{3} + 3(-1) \right)$ $= \frac{14}{3}$ ✓	**2**
13(a)(ii)	Shaded area = area under curve (i) - area of triangle $= \frac{14}{3} - \frac{1}{2} \times 2 \times 2$ ✓ $= \frac{8}{3}$ units^2	**1**
13(b)(i)	$x = t^3 - 21t^2$ $\dot{x} = \frac{dx}{dt}$ $= 3t^2 - 42t$ ✓ $\ddot{x} = \frac{d}{dx}\left(\frac{dx}{dt}\right)$ $= 6t - 42$ ✓	**2**
13(b)(ii)	Particle is stationary when $\dot{x} = 0$ i.e. $3t^2 - 42t = 0$ ✓ $3t(t-14) = 0$ $t = 0,\; t = 14$ ✓ $\therefore$ Particle is stationary at $t = 0$ and $t = 14$ seconds.	**2**
13 (c)(i)	$V = 3\ln(2t+1) - 5t + 2$ $\frac{dV}{dt} = \frac{3 \times 2}{2t+1} - 5$ ✓ $= \frac{6}{2t+1} - 5$	**1**
13(c)(ii)	At $t = 1$, $\frac{dV}{dt} = \frac{6}{2(1)+1} - 5$ ✓ $= 2 - 5$ $= -3$ Since $\frac{dV}{dt} < 0$, the volume is decreasing ✓	**2**

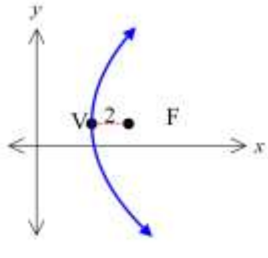
13(d)(i)	 $TE^2 = 950^2 + 1400^2 - 2(950)(1400)\cos 40$ ✓ $= 824821.7813$ 4636555.2... if used radians $TE = \sqrt{824821.7813}$ $= 908.1969948$ 2153 if used radians $= 908$ m (to nearest metre) ✓ $\therefore$ Distance from communication tower to the camp site is 908 m	**2**
13(d)(ii)	Find θ: $\frac{\sin\theta}{950} = \frac{\sin 40}{TE}$ $\sin\theta = \frac{\sin 40}{TE} \times 950$ $\theta = 42.25057234$ $= 42°15'$ ✓ or equivalent step $\angle CTE = 180 - \theta - 40$ $= 97°45'$ $\therefore \alpha = 97°45' - 60$ $= 37°45'$ Bearing = $180 - \alpha$ $= 142°$ (to mearest degree) ✓ $\therefore$ Bearing of the camp site form the communication tower is $142°$ NOTE if students use the sine rule to find $\angle CTE$, they have to be careful to use the obtuse angle and justify why. This is a lot trickier. They could use the cosine rule to find $\angle CTE$.	**2**

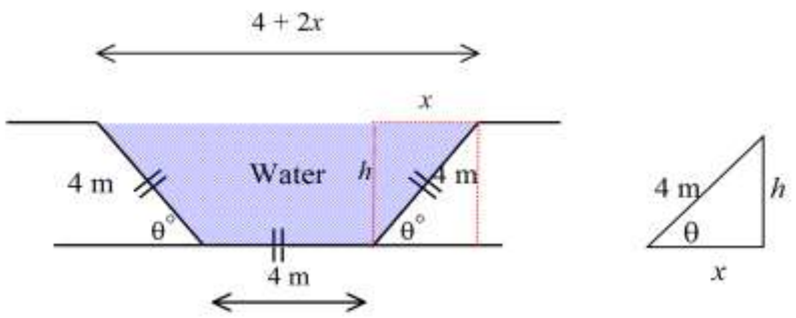
Question	Working	Marks
14(a)(i)	$y=\frac{1}{x}+x$ $y^2=\left(\frac{1}{x}+x\right)^2$ $=\left(x^{-1}+x\right)^2$ $=x^{-2}+2+x^2$ ✓ $V=\pi\int_1^3 x^{-2}+2+x^2\ dx$ ✓ $=\pi\left[\frac{x^3}{3}+2x-x^{-1}\right]_1^3$ $=\pi\left(\frac{3^3}{3}+2(3)-(3)^{-1}-(\frac{1^3}{3}+2(1)-(1)^{-1})\right)$ $=\frac{40\pi}{3}$ units ^{3} ✓	**3**
14(b)(i)	$x=3-\cos 2t,\ \ 0\le t\le 2\pi$  ✓ for period halved and vertical translation ✓ labelling correct max and min points and intercept	**2**
14(b)(ii)	Particle is at rest when the velocity is 0, that is the gradient of the tangents is 0. Therefore, the maximum and minimum turning points will indicate when the particle is at rest.	**1**

14(b)(iii)	Max speed when acceleration =0 ☑ $x = 3 - \cos 2t$ $v = 2\sin 2t$ $a = 4\cos 2t$ When $a = 0$, $4\cos 2t = 0$ $\cos 2t = 0$ $2t = \frac{\pi}{2}$ $t = \frac{\pi}{4}$ $\therefore$ Max speed occurs at $\frac{\pi}{4}$ seconds ☑ OR students could say this occurs at the point of inflection on the graph and find their answer this way.	**2**
14 (c)(i)	 $AB = 4$ (given) $OA = OB = 4$ cm radii of circle $\therefore \Delta OAB$ is an equilateral triangle ☑ $\therefore \angle AOB = \frac{\pi}{3}$	**1**
14(c)(ii)	Area of logo = area of circle - area of segment Area of segment $= \frac{1}{2}(4)^2\left(\frac{\pi}{3}\right) - \frac{1}{2}(4)^2 \sin\frac{\pi}{3}$ ☑ $= \frac{8\pi}{3} - \frac{8\sqrt{3}}{2}$ $\therefore$ Area of logo $= \pi(4)^2 - \left(\frac{8\pi}{3} - \frac{8\sqrt{3}}{2}\right)$ ☑ $= 48.81610528$ $= 48.8$ cm^2 (to 3 sig figs)	**2**

14(d)(i)	 $y = ax^3 + bx^2 + c$ $y' = 3ax^2 + 2bx$ At (1, 0) gradient of the tangent is -3 Sub (1, 0) into $y' = 3ax^2 + 2bx$ $y' = 3a(1)^2 + 2b(1)$ $\therefore -3 = 3a + 2b$ $\Rightarrow 3a + 2b = -3$ ☑ At (3, 2) gradient of the tangent is 9 Sub (3,2) into $y' = 3ax^2 + 2bx$ $y' = 3a(3)^2 + 2b(3)$ $\therefore 9 = 27a + 6b$ $\Rightarrow 9a + 2b = 3$ ☑	**2**
14(d)(ii)	Solve $\left.\begin{array}{ll} 3a + 2b = -3 & (1) \\ 9a + 2b = 3 & (2) \end{array}\right\}$ (2) − (1) $6a = 6$ $a = 1$ Sub $a = 1$ into (1) $3(1) + 2b =$ -3 $2b = -6$ $b = -3$ Sub $a = 1, b = -3$ into $y = ax^3 + bx^2 + c$ $y = x^3 - 3x^2 + c$ and sub (1,0) $0 = 1^3 - 3(1)^2 + c$ $c = 2$ $\therefore a = 1,\ b = -3,\ c = 2$ ☑ for one of the correct values ☑ for all vallues correct	**2**

Question	Working	Marks
15(a)(i)	$P = P_0 e^{kt}$ After 20 years $P = 2P_0$ $\therefore 2P_0 = P_0 e^{20k}$ ☑ $2 = e^{20k}$ $\ln 2 = 20k$ ☑ $k = \frac{1}{20}\ln 2 \ (0.03465...)$	**2**
15(a)(ii)	$3P_0 = P_0 e^{\frac{1}{20}\ln 2t}$ ☑ $3 = e^{\frac{1}{20}\ln 2\ t}$ $\ln 3 = \frac{1}{20}\ln 2\ \ t$ $t = \frac{20\ln 3}{\ln 2}$ $t = 31.69925001$ ☑ $\therefore$ Population trebles 32 years later, that is 2032	**2**
15(a)(iii)	$P_0 = 15.1$ million $P = 15.1e^{\frac{1}{20}\ln 2\times 50}$ ☑ $= 85.41849917$ ☑ $\therefore$ Population is 85.4 million (to 3 sig figs)	**2**
15(b)(i)	1^{st} Jan 2016, poulation = 30 000 Each year pop grows by 5% and 2500 leave RTShow: Pop in Dec 2016 = 29 000 Proof: $P_1 = 30000\times 1.05 - 2500$ ☑ $= 29000$ as required	**1**
15(b)(ii)	RTShow: $P_2 = 30000\times 1.05^2 - 2500(1.05+1)$ Proof: $P_1 = 30000\times 1.05 - 2500$ $P_2 = P_1\times 1.05 - 2500$ $= \left(30000\times 1.05 - 2500\right)\times 1.05 - 2500$ ☑ $= 30000\times 1.05^2 - 2500\times 1.05 - 2500$ $= 30000\times 1.05^2 - 2500(1.05+1)$	**1**

15(b)(iii)	RTShow: $P_n = 30000\times1.05^n - 2500(1.05^n - 1)$ Proof: $P_1 = 30000\times1.05 - 2500$ $P_2 = 30000\times1.05^2 - 2500(1.05+1)$ $P_3 = P_2\times1.05 - 2500$ $= \left(30000\times1.05^2 - 2500(1.05+1)\right)\times1.05 - 2500$ $= 30000\times1.05^3 - 2500(1.05^2+1.05) - 2500$ $= 30000\times1.05^3 - 2500(1.05^2+1.05+1)$ . . (Pattern continues) . $P_n = 30000\times1.05^n - 2500(1.05^{n-1}+1.05^{n-2}+....+1.05^2+1.05+1)$ ✓ setting up to here $= 30000\times1.05^n - 2500\,\underbrace{(1+1.05+1.05^2+...+1.05^{n-2}+1.05^{n-1})}_{\text{GP } a=1,\ r=1.05,\ n=n}$ $S_n = \frac{1(1.05^n-1)}{1.05-1}$ $= 30000\times1.05^n - \frac{2500(1.05^n-1)}{0.05}$ ✓ $= 30000\times1.05^n - 50000(1.05^n-1)$ as required	**2**
15(b)(iv)	Population is zero $0 = 30000\times1.05^n - 50000(1.05^n-1)$ $0 = 30000\times1.05^n - 50000\times1.05^n + 50000$ ✓ $0 = -20000\times1.05^n + 50000$ $20000\times1.05^n = 50000$ $1.05^n = \frac{50000}{20000}$ $1.05^n = \frac{5}{2}$ ✓ $n\ln(1.05) = \ln\left(\frac{5}{2}\right)$ $n = \frac{\ln\left(\frac{5}{2}\right)}{\ln 1.05}$ $n = 18.78023465$ $\therefore$ Population will be zero in the year 2035 ✓	**3**
15(c)	Focus (5 ,1) Vertex (3, 1) Parabola is sideways opening to right Focal length = 2 Equ of parabola is  $(y-g)^2 = 4a(x-h)$ Vertex (h,g) $(y-1)^2 = 4(2)(x-3)$ ✓ $(y^2-2y+1) = 8(x-3)$ $y^2-2y+1 = 8x-24$ ✓ $y^2-2y-8x+25 = 0$	**2**

Question	Working	Marks
16(a)(i)	 RTShow: $A=16(\sin\theta+\cos\theta\sin\theta)$ Proof: $\cos\theta=\frac{x}{4} \quad \Rightarrow x=4\cos\theta$ $\sin\theta=\frac{h}{4} \quad \Rightarrow h=4\sin\theta$ ☑ one of these statements Cross-section is a trapezium $A=\frac{h}{2}(a+b)$ $=\frac{4\sin\theta}{2}(4+(4+2x))$ $=2\sin\theta(4+4+2(4\cos\theta))$ ☑ $=2\sin\theta(8+8\cos\theta)$ $=16\sin\theta+16\sin\theta\cos\theta$ $=16(\sin\theta+\sin\theta\cos\theta)$	**2**
16(a)(ii)	RTShow: $\frac{dA}{d\theta}=16(2\cos^2\theta+\cos\theta-1)$ Proof: $A=16(\sin\theta+\sin\theta\cos\theta)$ $\frac{dA}{d\theta}=16(\cos\theta+\sin\theta(-\sin\theta)+\cos\theta\cos\theta)$ ☑ $=16(\cos\theta-\sin^2\theta+\cos^2\theta)$ $=16(\cos\theta-(1-\cos^2\theta)+\cos^2\theta)$ $=16(\cos\theta-1+\cos^2\theta+\cos^2\theta)$ ☑ $=16(\cos\theta-1+2\cos^2\theta)$ $=16(2\cos^2\theta+\cos\theta-1)$	**2**

16(a)(iii)	RTShow: Max area when $\theta = \frac{\pi}{3}$ Proof: $\frac{dA}{d\theta} = 16\left(2\cos^2\theta + \cos\theta - 1\right)$ For max $\frac{dA}{d\theta} = 0$ $16\left(2\cos^2\theta + \cos\theta - 1\right) = 0$ $(2\cos\theta - 1)(\cos\theta + 1) = 0$ ✓ $\cos\theta = \frac{1}{2}$ or $\cos\theta = 1$ and $\theta < \frac{\pi}{2}$ $\theta = \frac{\pi}{3}$ or $\theta = 0$ ✓ not possible Test for Max: ✓ x: $\frac{\pi}{6}$ | $\frac{\pi}{3}$ | $\frac{3\pi}{8}$ $\frac{dA}{d\theta}$: $16\left(2\cos^2\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right) - 1\right)$ | $16\left(2\cos^2\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right) - 1\right)1$ | $6\left(2\cos^2\left(\frac{3\pi}{8}\right) + \cos\left(\frac{3\pi}{8}\right) - 1\right)$ $1.36 > 0$ | 0 | $-0.324 < 0$ Change in $\frac{dA}{d\theta}$ $\therefore$ Max at $\frac{\pi}{3}$	**3**
16(a)(iv	$A = 16\left(\sin\theta + \sin\theta\cos\theta\right)$ when $\theta = \frac{\pi}{3}$ $A = 16\left(\sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)\right)$ $= 16\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}\right)$ $= 16\left(\frac{2\sqrt{3}}{4} + \frac{\sqrt{3}}{4}\right)$ $= 16\left(\frac{3\sqrt{3}}{4}\right)$ $= 12\sqrt{3}$ $= 20.78460969$ $= 21\text{m}^2$ (to nearest m^2) ✓	**1**

16(b)(i)	$v(t) = 0.05t^3 - 0.38t^2 + 0.624t \qquad 0 \le t \le 5.2$ At rest $v(t) = 0$ $0.05t^3 - 0.38t^2 + 0.624t = 0$ $50t^3 - 380t^2 + 624t = 0$ $25t^3 - 190t^2 + 312t = 0$ $t(25t^2 - 190t + 312) = 0$ ☑ $t = 0$ or $t = \frac{190 \pm \sqrt{(-190)^2 - 4(25)(312)}}{25 \times 2}$ $= \frac{190 \pm \sqrt{4900}}{50}$ $= \frac{190 \pm 70}{50}$ $= \frac{190+70}{50}, \frac{190-70}{50}$ $= 5.2, \; 2.4$ Cutting saw is a rest at 0, 2.4 and 5.2 seconds ☑	**2**
16(b)(ii)	$\int_0^{5.2} 0.05t^3 - 0.38t^2 + 0.624t \, dt$ $= \left[\frac{0.05t^4}{4} - \frac{0.38t^3}{3} + \frac{0.624t^2}{2}\right]_0^{5.2}$ ☑ $= \frac{0.05(5.2)^4}{4} - \frac{0.38(5.2)^3}{3} + \frac{0.624(5.2)^2}{2} - \left(\frac{0.05(0)^4}{4} - \frac{0.38(0)^3}{3} + \frac{0.624(0)^2}{2}\right)$ $= -0.2343466667$ $= -0.234$ (to 3 decimal places) ☑	**2**
16(b)(iii)	The saw starts moving and at 2.4 seconds it as at rest and changes direction. At 5.2 seconds it is at rest again. The integral of *v(t)* will give the distance travelled. As the answer to part (ii) is negative, the saw must have travelled further in the second stage of its movement than the first.	**1**
16(b)(iv)	To find the total distance travelled, find the distance travelled (forward) in the first 2.4 seconds and add this to the distance travelled (backwards) in the time from 2.4 to 5.2 seconds. $\therefore$ Total distance $= \int_0^{2.4} 0.05t^3 - 0.38t^2 + 0.624t \, dt + \left\| \int_{2.4}^{5.2} 0.05t^3 - 0.38t^2 + 0.624t \, dt \right\|$ ☑ For recognising two integrals separated by $t = 0, \; 2.4, \; 5.2$ ☑ Correct absolute value or equivalent	**2**