ASCHAM SCHOOL

FORM 6 2 UNIT MATHEMATICS TRIAL EXAMINATION

2001

July 2001

Time allowed: 3 hours
Plus 5 minutes reading time

Instructions

1. Attempt all questions
2. All questions are of equal value
3. All necessary working should be shown in each question.
   Marks may be deducted for careless or badly presented work.
4. Standard integrals are provided at the back of the paper.
5. Board approved calculators may be used.
6. Answer each question in a separate writing booklet

QUESTION 1

a) Solve $5 - 3x \geq 7$ and graph your solution on a number line. (2)

b) Solve the equations $x - 2y = 3$

$2x + y = 1$ (2)

c) Evaluate $\log_e 5$ to 2 decimal places. (1)

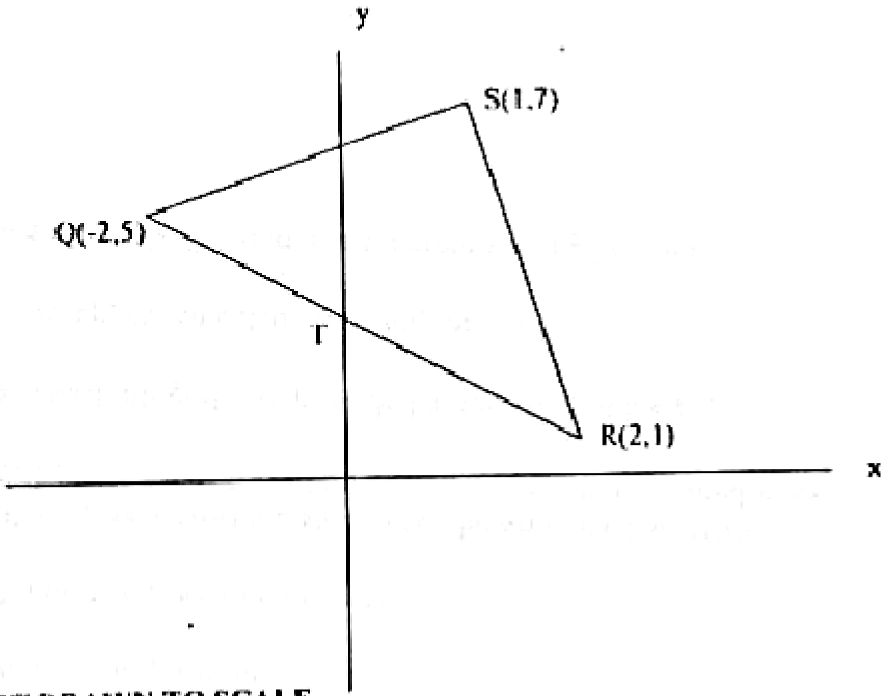
d) Solve for x: $|3x + 1| = 4$ (2)

e) Fnd the exact value of $\cos\frac{5\pi}{6}$ (1)

f) Find p and q if $\frac{6}{2 - \sqrt{3}} = p + q\sqrt{3}$ and p and q are integers (2)

g) Sketch $y = x(x - 1)$ on the Cartesian plane indicating intercepts on axes and vertex. (2)

QUESTION 2



NOT DRAWN TO SCALE

Draw the diagram in your answer booklet

a) Show that the equation of the line QR is $x + y - 3 = 0$ **(2)**

b) Find the perpendicular distance from S to QR. **(2)**

c) Hence find the area of triangle SQR. **(2)**

d) If T lies on the y axis, show that T is the midpoint of QR. **(2)**

e) Find the co-ordinates of the point P such that

QPRS is a parallelogram. **(1)**

f) Find the equation of the circle on QR as diameter **(2)**

g) Shade the region defined by $x + y - 3 \geq 0$ and $x \geq 0$ **(1)**

QUESTION 3

a) Differentiate and simplify:

i) $\frac{1}{x}$ **(1)**

ii) $\tan(2 - 3x)$ **(2)**

iii) $\frac{e^{2x}}{3x+2}$ **(3)**

iv) $\sin^2 x$ **(1)**

b) ABCD is a rhombus. $\angle DBC = (x + 20)^\circ$ and $\angle ABD = 3x^\circ$.
Find x. **(2)**

c) The third term of a geometric sequence is -3 and the eighth term is 96. Find the first term and the common ratio of the sequence. **(3)**

QUESTION 4

a) Find i) $\int \frac{x^2+3}{x}dx$ **(2)**

ii) $\int \frac{x}{x^2+3}dx$ **(2)**

b) Find the largest angle of ΔABC to the nearest minute

if AB = 6cm, BC = 8 cm and AC = 12cm. **(3)**

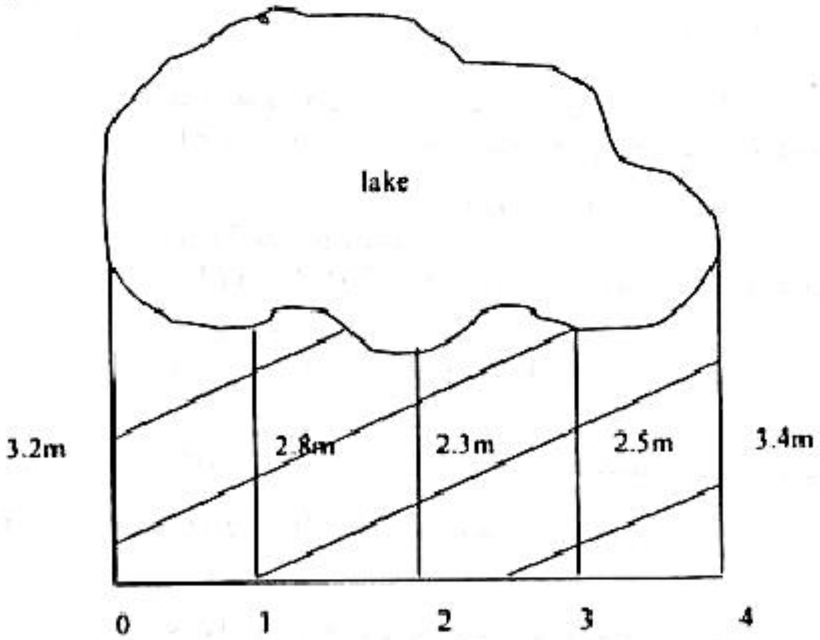
c) i) Show that the gradient of the tangent to the curve $y = \log[x(x+1)]$ is $\frac{3}{2}$ when x = 1. **(3)**

ii) Hence find the equation of the normal to the curve $y = \log[x(x+1)]$ at the point when x = 1. **(2)**

QUESTION 5

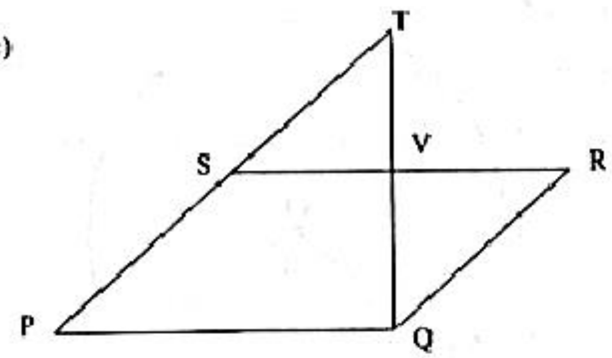
a) Find the volume generated when the area between the curve $y = 9 - x^2$ and the x axis is rotated about the y axis. **(3)**

b)



The shaded area on the side of a lake is to be grassed. The above plan was drawn showing the measurements.
Using Simpson's Rule, find the approximate area of the grassed section. **(3)**

c)



PQRS is a parallelogram and PT is a straight line though S.
TQ is perpendicular to PQ.

i) Prove $\Delta VRQ /// \Delta QPT$

ii) If $QR = 5$, $VR = 4$, $ST = 8$ find TP and QP and hence SV.

(6)

QUESTION 6

a) Find the equation of the parabola with focus (-2,-1) and directrix y = 5. **(2)**

b) For the curve $y = x(x-3)^2$

i) Find any stationary points and describe their nature. **(4)**

ii) Find the point of inflexion. **(2)**

iii) Draw the graph of $y = x(x-3)^2$ for $-1 \le x \le 5$, showing significant points **(2)**

iv) What is the maximum value of $x(x-3)^2$ for $-1 \le x \le 5$? **(1)**

v) For what values of x is $y = x(x-3)^2$ concave down for $-1 \le x \le 5$? **(1)**

QUESTION 7

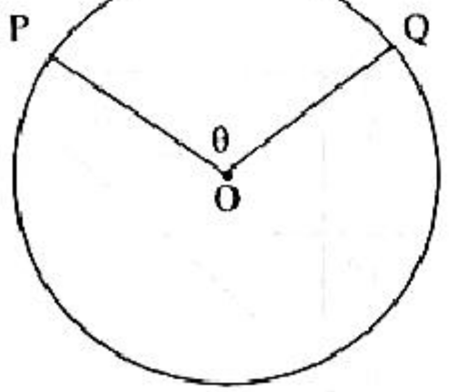
a) i) Draw the graph of y = 3 sin 2x for $0 \le x \le \pi$ **(2)**

ii) By adding another line to your graph, find the number of solutions of the equation 3 sin 2x - x = 0 in the interval $0 \le x \le \pi$ **(1)**

iii) Find the area bounded by the curve y = 3 sin 2x, the x axis, $x = \frac{\pi}{4}$ and $x = \frac{7\pi}{8}$. Give your answer in exact form. **(5)**

b) If $\frac{d^2y}{dx^2} = 6x + 2$, find the equation of the curve given that it has a minimum turning point at (1,3). **(4)**

QUESTION 8



a) Arc PQ subtends an angle of θ radians at the centre.
The radius is r cm and the perimeter of the minor sector is 10 cm.

i) Show that the area of the sector is $A = 5r - r^2$ **(3)**

ii) Hence find the radius of the sector of maximum area when the perimeter of the sector is 10 cm. **(2)**

b) The amount M grams of a chemical is given by

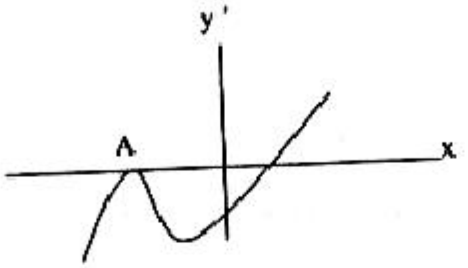
$M = M_0 e^{-kt}$ where M_0 and k are positive constants and time t is measured in years.

i) Show that M satisfies the equation $\frac{dM}{dt} = -kM$ **(1)**

ii) Find k (in exact form) if 200 grams of the chemical decomposes to 150 grams at the end of 2 years. **(2)**

iii) Find the amount of the chemical which has decomposed by the end of 10 years (to the nearest gram) **(2)**

c)

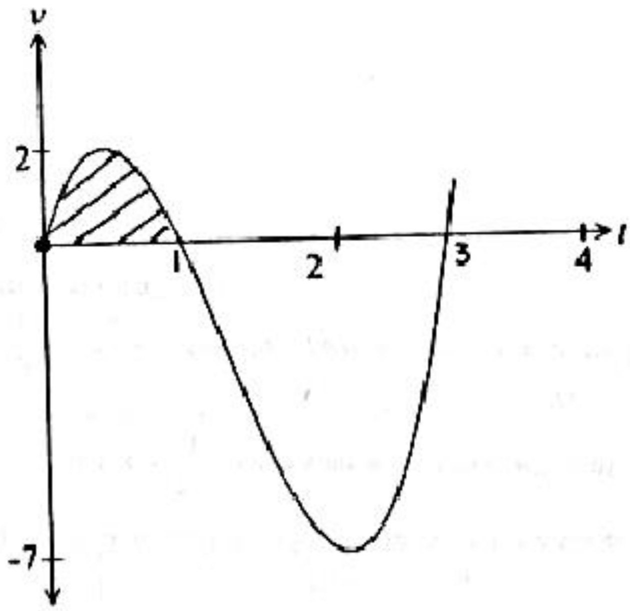


The graph shows the gradient function $y = f'(x)$.

Find the nature of the stationary point of $y = f(x)$ corresponding to A.
Explain. **(2)**

QUESTION 9

a)



not drawn to scale

The graph represents the velocity (in m/s) of a particle after t seconds.
The particle is moving in a straight line.

i) What is the velocity of the particle after 0.5 seconds? **(1)**

ii) What is the acceleration of the particle after 2 seconds? **(1)**

iii) At what time(s) does the particle change direction? **(1)**

iv) What does the shaded area on the graph represent? **(1)**

v) When is the speed of the particle maximum? $(0 \le t \le 3)$ **(1)**

b) Find the values of m for which the equation

$mx^2 + (m+1)x + 1 = 0$ has real and different roots? **(2)**

c) Find the maximum and minimum values of $\dfrac{\cos x}{1 + \sin x}$ for $0 \le x \le \pi$. **(5)**

QUESTION 10

a) Jacqui borrowed $400 000 from the bank on 1 January 1998. She was not charged interest for the first 6 months. Thereafter she was charged interest of 15% per annum compounded monthly.

She agreed to repay the loan by equal annual instalments of M dollars on 31 December of each year. The loan was to be repaid by the end of 2001.

i) How much did she owe on 31 December 1998 after making her first payment? **(1)**

ii) How much did she owe on 31 December 1999 after making her second payment? **(1)**

iii) Show that $M = \dfrac{400000(1.0125)^{42}(1.0125^{12} - 1)}{1.0125^{48} - 1}$ **(2)**

iv) Hence find her annual instalment. **(1)**

b) A particle P is moving in a straight line along the x axis. Its position at time t seconds is given by the equation $x = e^{\frac{t}{2}} - t$.

i) Find an expression for the velocity of the particle at time t. **(1)**

ii) In what direction is the particle moving initially? **(1)**

iii) When does the particle come to rest? **(1)**

iv) Find the acceleration of the particle at time t. **(1)**

v) What comment can you make about the acceleration and how does it affect the future motion of the particle. **(1)**

vi) Find the distance travelled by the particle in the first 3 seconds. **(2)**

END OF EXAM