Ascham School

Mathematics Trial Examination 2011

Time Allowed:

3 hours plus 5 minutes' reading time

Total marks - 120

Instructions

- All questions may be attempted.
- All questions are of equal value (12 marks).
- All necessary working should be shown in every question.
- Write using black or blue pen.
- Board -approved calculators and templates may be used.
- A table of standard integrals is provided at the back of this paper.
- Start each question in a new booklet.
- If you use a second booklet for a question, place it inside the first and indicate on your first booklet that it is the first of two booklets.
- Write your name/number, teacher's name and question number on each booklet.

QUESTION 1

a) Find the exact value of $\cos(\frac{5\pi}{6})$. **(1)**

b) Solve for x:

i) $|x-3|<4$. **(2)**

ii) $x^2=6x$. **(2)**

iii) $2-\frac{x-1}{10}=\frac{x}{5}$. **(2)**

iv) $2\ln x=\ln(2x-1)$. **(2)**

c) Find the focus of the parabola $(x-3)^2=8y$. **(2)**

d) Write down the equation of a circle with centre (-1, 2) and radius 5 units. **(1)**

QUESTION 2

a) Differentiate with respect to x:

i) $x^2\cos x$ **(2)**

ii) $\frac{3x^3-1}{x}$ **(2)**

iii) ln 2. **(1)**

b) Integrate with respect to x:

i) $\sqrt[3]{e^{2x}}$ **(2)**

ii) $\frac{3x}{x^2-4}$. **(2)**

c) Evaluate to 2 significant figures: $\sin\frac{1}{\sqrt{2}}$. **(1)**

d) Evaluate $\int_1^4 |x-2|\,dx$. **(2)**

QUESTION 3

a) A is the point (6,3), B is (3,4) and C is (4,-3)
Draw a set of axes and put in the above points.
Your diagram need not be to scale.

i) Show that AB and AC are perpendicular. **(3)**

ii) Without using a protractor, find the size of angle ACB to the nearest minute **(3)**

iii) If D is the point (7, - 4), show that ABCD is a parallelogram. **(2)**

iv) Find the area of the parallelogram ABCD. **(1)**

v) If AC and BD intersect at E, find the coordinates of E. **(1)**

b) The gradient function of a curve is $6\sin 2x$. Find the equation of the curve if it passes through the point $(\frac{\pi}{2}, 3)$. **(2)**

QUESTION 4

a) Evaluate $\sum_{r=4}^{24}(4r-7)$. **(3)**

b) On a set of axes, shade the region represented by $2x - y + 6 \geq 0$. **(2)**

c) A circle has a circumference of 12π cm.
If an angle of $\frac{\pi}{3}$ is subtended at the centre of the circle, find the exact area of the minor segment. **(3)**

d) The third term of a geometric series is 36 and the eighth term is $\frac{4}{27}$. Find the first term of the series. **(4)**

QUESTION 5

a) For the curve $y = x^3 - 3x^2 - 9x + 2$,

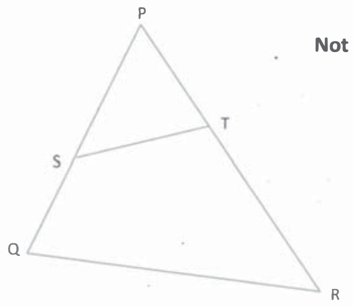
i) find any stationary points and determine their nature, **(4)**

ii) find any points of inflexion, **(2)**

iii) sketch the graph showing all significant points, **(2)**

iv) for what values of x is the curve decreasing with downward concavity? **(1)**

b)



Not to scale

In the diagram above,

PS = 1.2, SQ = 0.8, PT = 0.8 and TR = 2.2.

Draw the diagram in your answer booklet.

i) Prove that $\triangle$PST is similar to $\triangle$PRQ . **(2)**

ii) If ST = 1.7, find the length of QR. **(1)**

QUESTION 6

a) Use Simpson's Rule with 5 function values, to approximate to 3 decimal places:
$\int_5^7 \log_e(x-3)\,dx$. **(3)**

b) A substance has a half-life of 8 days. Its rate of decay is given by the differential equation $\frac{dM}{dt} = -kM$.

i) Show that $M = M_0\, e^{-kt}$ is a solution to the differential equation above, where M is the mass of the substance present at time t days. **(1)**

ii) Show that $k = \frac{\ln 2}{8}$. **(2)**

iii) How long will it take for 75% of the substance to have decayed? **(2)**

c) The equation $x^2 + (m-3)x + (m-1) = 0$ has one root that is half the other. Find the values of m and the values of the two roots. **(4)**

QUESTION 7

a) i) Draw the graph of $y = \tan\frac{x}{2}$ for $-\pi \le x \le 3\pi$. **(2)**

ii) Find the equation of the normal to the graph $y = \tan\frac{x}{2}$ when $x = \frac{\pi}{2}$. **(3)**

iii) As accurately as possible, sketch this normal on to the same set of axes as the graph in i), showing the co-ordinates of the points of intersection with the axes. **(2)**

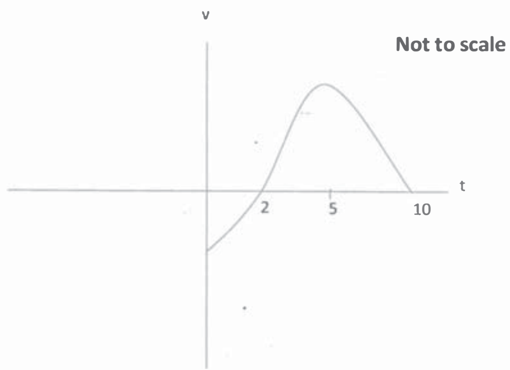
b) The position of a particle on a number line is given by $x = 2\sin t + t\sqrt{3}$ where t is measured in seconds and x in metres..

i) When is the particle at rest for $0 \le t \le 2\pi$ **(2)**

ii) How far has the particle travelled in the first π seconds? Give an exact answer in simplest form. **(3)**

QUESTION 8

a) The graph of velocity vs time for a particle is shown below.



i) When is the particle at rest? **(1)**

ii) When is the particle moving to the right? **(1)**

iii) When is the acceleration of the particle equal to zero? **(1)**

iv) Is the particle speeding up or slowing down when t = 6. Explain. **(1)**

b) i) By solving an equation, show that the graphs $y = e^x$ and $y = e^{2x} - 2$ intersect when x = ln2. **(2)**

ii) Draw the two graphs in part i) on the same set of axes, showing intercepts on axes. **(2)**

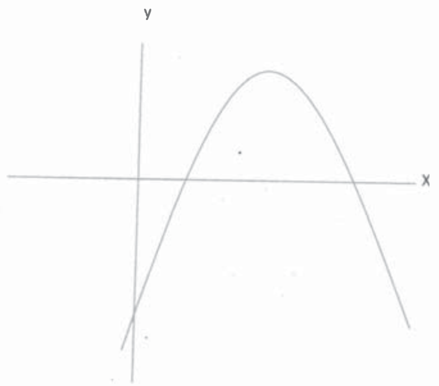
iii) Find the exact area bounded by the graphs $y = e^x$ and $y = e^{2x} - 2$ and the line x = 0. **(4)**

QUESTION 9

a) The curve $y=-x^2+6x-8$ is given below.

i) Find the points of intersection with the x axis **(1)**

ii) The equation of the parabola can be rearranged as $x=3\pm\sqrt{1-y}$.
Find the exact volume created when the area between the curve the x axis, x = 3 and x = 4 is rotated about the y axis. **(4)**



b) Charlotte decides she would like to buy a car.
She opens a savings account on 1 January 2010 by investing \$2000 in the Bell Bank. She continues to invest \$2000 at the start of each year thereafter. She receives interest at the rate of 6% per annum compounded annually.

i) Show that the value of her account at the end of 2013 is \$9274.19. **(3)**

ii) On 1 January 2014, the interest rate goes up to 8% per annum and she receives this new interest rate on all money in her account.

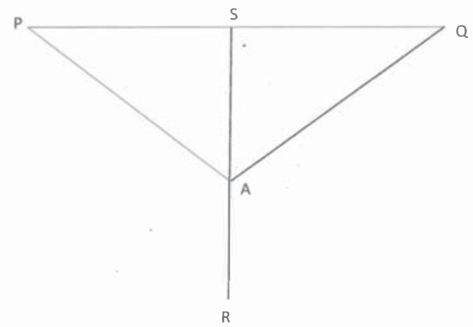
How much does she have in her account at the end of 2020 if she continues her original plan of investing \$2000 at the start of each year? **(4)**

QUESTION 10

a) For what values of m will the curve $y=\frac{mx^3}{3}+\frac{3x^2}{2}-4x+7$ be decreasing for all x? **(3)**

b) Towns P, Q and R are located 6km due west, 6km due east and 10 km due south respectively, of a town S.
A road is to run due north from R to a point A which is x km from R.
From A, a branch road is to run to P and another branch road to Q.

Copy the diagram below into your answer booklet and fill in the data.



i) Show that the total length L of roads AP, AQ and AR is given by

$L=x+2\sqrt{x^2-20x+136}$ **(2)**

ii) Find the exact value of x such that L is a minimum. **(5)**

iii) Show that the minimum total length for the three roads AP, AQ and AR is $(10+6\sqrt{3})$ km. **(2)**

END OF EXAM

Standard Integrals

$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \; x \neq 0, \text{ if } n < 0$

$\int \frac{1}{x} dx = \ln x, \quad x > 0$

$\int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$

$\int \cos ax \, dx = \frac{1}{a}\sin ax, \quad a \neq 0$

$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \quad a \neq 0$

$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \quad a \neq 0$

$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \quad a \neq 0$

$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$

$\int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$

$\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left(x + \sqrt{x^2-a^2}\right), \quad x > a > 0$

$\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x + \sqrt{x^2+a^2}\right)$

NOTE: $\ln x = \log_e x, \quad x > 0$

YEAR 12 MATHEMATICS TRIAL 2011

QUESTION 1

a) $\cos\frac{5\pi}{6} = \cos 150^\circ = -\frac{\sqrt{3}}{2}$ (1)

b) i) $|x-3| < 4$

$-4 < x-3 < 4$

$-1 < x < 7$ ✓✓ (2)

-1 mark if $x > -1$ or $x < 7$

ii) $x^2 = 6x$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0$ ✓ or 6 ✓ (2)

iii) $2 - \frac{x-1}{10} = \frac{x}{5}$

$20 - x + 1 = 2x$ ✓

$-3x = -21$

$x = 7$ ✓ (2)

iv) $2\ln x = \ln(2x-1)$

$x^2 = 2x - 1$ ✓

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$ ✓ (2)

c) $(x-3)^2 = 8y$

Vertex $= (3, 0)$ ✓

$4a = 8 \quad a = 2$

Focus $= (3, 2)$ ✓ (2)

d) $(x+1)^2 + (y-2)^2 = 25$ (1)

QUESTION 2

a) i) $\frac{d}{dx} x^2\cos x = \cos x \times 2x + x^2 \times -\sin x$

$= 2x\cos x - x^2 \sin x$ (2)

ii) $\frac{d}{dx}\frac{3x^3-1}{x} = \frac{d}{dx} 3x^2 - x^{-1}$

$= 6x + x^{-2}$

$= 6x + \frac{1}{x^2}$ (2)

iii) $\frac{d}{dx}\ln 2 = 0$ (1)

b) i) $\int \sqrt[3]{e^{2x}} dx = \int e^{\frac{2x}{3}} dx$

$= \frac{e^{\frac{2x}{3}}}{\frac{2}{3}} + c$ ✓

$= \frac{3}{2} e^{\frac{2x}{3}} + c$ ✓

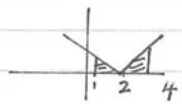
last line not necessary $\rightarrow$ $\left(= \frac{3}{2}\sqrt[3]{e^{2x}} + c\right)$

ii) $\int \frac{3x}{x^2-4} dx = \frac{3}{2}\int\frac{2x}{x^2-4} dx$

$= \frac{3}{2}$ ✓ $\log_e(x^2-4)$ ✓ $+ c$ (2)

c) $\sin\frac{1}{\sqrt{2}} \doteq 0.65 \; (2sf)$ (1)

d) $\int_1^4 |x-2| \, dx$



$= \left(\frac{1}{2}\times 1 \times 1\right) + \frac{1}{2}(2\times 2)$ ✓ ✓

$= \frac{1}{2} + 2$

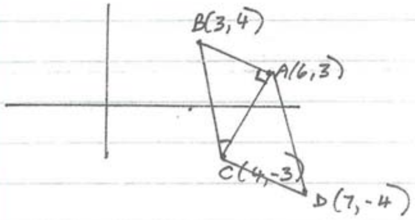
$= 2\frac{1}{2}$ (2)

no marks if use integration unless 2 lines used.

QUESTION 3

or other methods

a)



i) $m_{AB} = \frac{1}{-3}$ ✓

$m_{AC} = \frac{6}{2}$
$= 3$ ✓

$m_{AB} \times m_{AC} = -\frac{1}{3} \times 3$
$= -1$ ✓ (3)

$\therefore AB \perp AC$

ii) $AB = \sqrt{3^2 + 1^2}$
$= \sqrt{10}$ ✓

$AC = \sqrt{2^2 + 6^2}$
$= \sqrt{40}$ ✓
$= (2\sqrt{10})$

$\frac{AB}{AC} = \tan \angle ACB$

$\frac{\sqrt{10}}{2\sqrt{10}} = \tan \angle ACB$

$\tan \angle ACB = \frac{1}{2}$ ✓ (3)

$\angle ACB = 26^\circ 34'$ (to n. min)

iii) $m_{AB} = -\frac{1}{3}$

$m_{CD} = \frac{-1}{3}$

$\therefore AB \parallel CD$ ✓

$m_{BC} = \frac{7}{-1}$

$m_{AD} = \frac{7}{-1}$

$\therefore BC \parallel AD$ ✓

$\therefore ABCD$ is a parallelogram (2 pairs opposite sides are parallel) (2)

iv) Area of parallelogram ABCD
$= 2 \times$ Area of $\triangle ABC$
$= 2 \times \frac{1}{2} AB \times AC$
$= \sqrt{10} \times \sqrt{40}$
$= \sqrt{400}$
$= 20u^2$ (1)

v) E is midpoint of AC (diagonals bisect each other)
$\therefore E = (5, 0)$ (1)

b) $y' = 6\sin 2x$

$y = \frac{-6\cos 2x}{2} + c$

$y = -3\cos 2x + c$ ✓

Sub $(\frac{\pi}{2}, 3)$

$3 = -3\cos\pi + c$
$3 = -3 \times -1 + c$
$3 = 3 + c$
$c = 0$ ✓ (2)

$\therefore y = -3\cos 2x$

must show $c = 0$

QUESTION 4

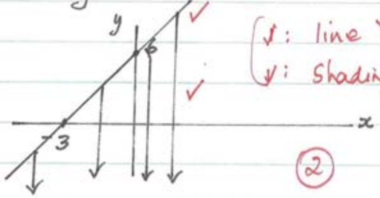
a) $\sum_{r=4}^{24} (4r - 7)$

$T_1 = 9$
$T_2 = 13$
$T_3 = 17$ ✓

$\therefore a = 9 \quad d = 4 \quad n = 21$

$S_{21} = \frac{21}{2}\{2 \times 9 + 20 \times 4\}$ ✓
$= \frac{21}{2}\{18 + 80\}$
$= \frac{21}{2} \times 98$
$= 1029$ ✓ (3)

b) $2x - y + 6 \geq 0$ ✓



✓: line
✓: shading (2)

c) $C = 12\pi$
$2\pi r = 12\pi$
$2r = 12$
$r = 6$ ✓

Area of minor segment
$= \frac{1}{2}r^2(\theta - \sin\theta)$
$= \frac{1}{2} 36 (\frac{\pi}{3} - \sin\frac{\pi}{3})$ ✓
$= 18(\frac{\pi}{3} - \frac{\sqrt{3}}{2})$
$= 6\pi - 9\sqrt{3}$ cm^2 ✓ (3)

d) $T_3 = 36 \quad T_8 = \frac{4}{27}$

$ar^2 = 36$ — (1) ✓
$ar^7 = \frac{4}{27}$ — (2) ✓

(2) ÷ (1) $r^5 = \frac{1}{243}$
$r = \frac{1}{3}$ ✓

Sub in (1) $a \times \frac{1}{9} = 36$
$a = 324$

$\therefore$ First term is 324 ✓ (4)

QUESTION 5

a) $y = x^3 - 3x^2 - 9x + 2$

i) $y' = 3x^2 - 6x - 9$
$y'' = 6x - 6$

For stationary points $y' = 0$
$3(x^2 - 2x - 3) = 0$
$(x - 3)(x + 1) = 0$
$x = 3, -1$
$y = -25, 7$

$\therefore$ stationary points are $(3, -25)$, $(-1, 7)$ ✓ ✓

Nature of stationary points (4)

If $x = 3 \quad y'' = 12 > 0$
$\therefore (3, -25)$ is a minimum turning pt ✓

If $x = -1 \quad y'' = -12 < 0$
$\therefore (-1, 7)$ is a maximum turning pt ✓

ii) For possible points of inflexion

$y''=0$

$6x-6=0$

$x=1$

$y=-9$ ✓

$\therefore (1,-9)$ is a possible point of inflexion

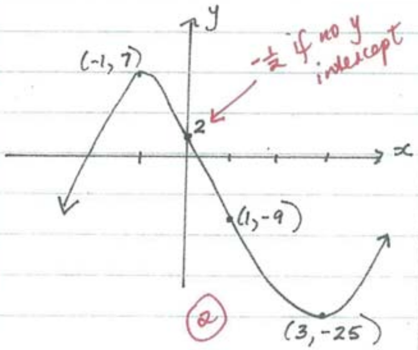
Test for change in concavity

x	0	1	2
y''	-6	0	6
	∩		∪

$\therefore$ there is a change in concavity (2)

$\therefore (1,-9)$ is a point of inflexion ✓

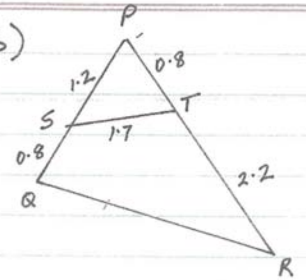
iii)



(2)

iv) $-1<x<1$ (1) no part marks

b)



i) In Δs PST, PRQ

$\angle P$ is common

$\frac{PS}{PR}=\frac{1.2}{3}=\frac{2}{5}$

$\frac{PT}{PQ}=\frac{0.8}{2}=\frac{2}{5}$

$\therefore \Delta PST \parallel\!| \Delta PRQ$ (sides around equal $\angle$ in same ratio) (2)

ii) $\frac{ST}{QR}=\frac{2}{5}$ (matching sides of similar Δs)

$\frac{1.7}{QR}=\frac{2}{5}$

$2QR=8.5$

$QR=4.25$ ✓ (1)

QUESTION 6

a) $\int_5^7 \log_e(x-3)\,dx$

x	5	5.5	6	6.5	7
$\log_e(x-3)$	ln 2	ln 2.5	ln 3	ln 3.5	ln 4
	y_0	y_1	y_2	y_3	y_4

✓

$\int_5^7 \log_e(x-3)\,dx$

$\doteq \frac{0.5}{3}\{\ln 2+\ln 4+4(\ln 2.5+\ln 3.5)+2\ln 3\}$ ✓

$\doteq 2.1588\ldots$

$\doteq 2.159$ (3dp) ✓ (3)

b) $\frac{dM}{dt}=-kM$

i) $M=M_0e^{-kt}$

$\frac{dM}{dt}=M_0e^{-kt}\times -k$

$=M\times -k$

$=-kM$ (1)

ii) $\frac{1}{2}M_0=M_0e^{-k\times 8}$ ✓

must be M_0 not M

$\frac{1}{2}=e^{-8k}$

$\ln\frac{1}{2}=-8k$

$k=\frac{\ln\frac{1}{2}}{-8}$

$=\frac{-\ln 2}{-8}$ show steps

$=\frac{\ln 2}{8}$ ✓ (2)

iii) $\frac{3}{4}$ decayed.

$\therefore \frac{1}{4}$ left

$\frac{1}{4}M_0=M_0e^{-\frac{\ln 2}{8}\times t}$

✓ $\frac{1}{4}=e^{-\frac{\ln 2}{8}\times t}$

$\ln\frac{1}{4}=-\frac{\ln 2}{8}\times t$

$t=\ln\frac{1}{4}\div\left(-\frac{\ln 2}{8}\right)$

$=16$ ✓ (2)

$\therefore$ it will take 16 days for 75% to have decayed.

c) $x^2+(m-3)x+(m-1)=0$

Let roots be α and 2α

$3\alpha=-m+3$ — ①

$2\alpha^2=m-1$ — ② ✓

From ① $m=-3\alpha+3$

Sub in ②: $2\alpha^2=-3\alpha+3-1$

$2\alpha^2+3\alpha-2=0$

$(2\alpha-1)(\alpha+2)=0$

$\alpha=\frac{1}{2}$ or -2 ✓

Sub in ① $m=-3\times\frac{1}{2}+3$ or $-3\times -2+3$

$=1\frac{1}{2},\ 9$ ✓

$\therefore m=1\frac{1}{2}$ or 9

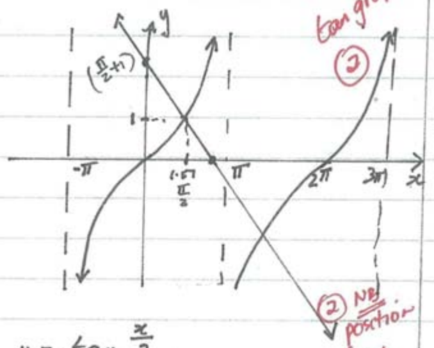
roots are $\frac{1}{2}$ & 1 or -2 and -4 ✓ (4)

QUESTION 7

a) i) $y = \tan\frac{x}{2}, \ -\pi \le x \le 3\pi$

Period $= \frac{\pi}{n} = \frac{\pi}{\frac{1}{2}} = 2\pi$

tan graph ②



ii) $y = \tan\frac{x}{2}$

$y' = \sec^2\frac{x}{2} \times \frac{1}{2}$

$= \frac{1}{2}\sec^2\frac{x}{2}$ ✓

Sub $x = \frac{\pi}{2}$

$y' = \frac{1}{2}\left(\sec\frac{\pi}{4}\right)^2$

$= \frac{1}{2}(\sqrt{2})^2$

$= 1$

$\therefore$ gradient of normal is -1 ✓

$x = \frac{\pi}{2} \quad y = \tan\frac{\pi}{4}$

$= 1$

$y - 1 = -1\left(x - \frac{\pi}{2}\right)$

$y - 1 = -x + \frac{\pi}{2}$

$x + y - 1 - \frac{\pi}{2} = 0$

is eqn of normal ✓

x int: $1 + \frac{\pi}{2} \doteqdot 2.57$

y int $= 1 + \frac{\pi}{2} \doteqdot 2.57$ ③

b) $x = 2\sin t + t\sqrt{3}$

i) $v = 2\cos t + \sqrt{3}$

Particle at rest when $v = 0$

$2\cos t = -\sqrt{3}$

$\cos t = -\frac{\sqrt{3}}{2}$

(relt $= 30°: \frac{\pi}{6}$) II, III

$t = \frac{5\pi}{6}, \frac{7\pi}{6}$ ✓ ✓ ②

$\therefore$ particle at rest at $\frac{5\pi}{6}$ + $\frac{7\pi}{6}$ seconds

ii) $t = 0 \quad x = 2\sin 0 + 0$

$= 0$ ½

$t = \frac{5\pi}{6} \quad x = 2\sin\frac{5\pi}{6} + \frac{5\pi}{6}\sqrt{3}$

$= 2 \times \frac{1}{2} + \frac{5\sqrt{3}\pi}{6}$

$= 1 + \frac{5\sqrt{3}\pi}{6}$ ✓

$t = \pi \quad x = 2\sin\pi + \pi\sqrt{3}$

$= 0 + \pi\sqrt{3}$

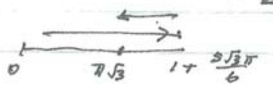
$= \pi\sqrt{3}$ ½

$\therefore$ Distance $= 2\left(1 + \frac{5\sqrt{3}}{6}\pi\right) - \pi\sqrt{3}$

$= 2 + \frac{5\sqrt{3}}{3}\pi - \pi\sqrt{3}$

$= \frac{6 + 5\sqrt{3}\pi - 3\sqrt{3}\pi}{3}$

$= \frac{6 + 2\sqrt{3}\pi}{3}$ ✓



QUESTION 8

a) i) $t = 2, 10$ ①

ii) $2 < t < 10$ ①

iii) $t = 5$ ①

iv) $t = 6 \quad v > 0$

$a < 0$ ①

$\therefore$ particle slowing down

b) i) $y = e^x$ — ①

$y = e^{2x} - 2$ — ②

$e^{2x} - 2 = e^x$

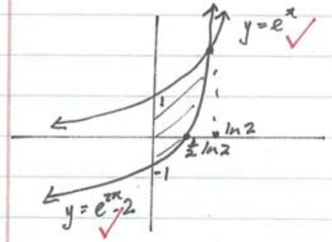
$e^{2x} - e^x - 2 = 0$ ✓

$(e^x + 1)(e^x - 2) = 0$ ②

$e^x = -1$ or $e^x = 2$ ✓

no solution $\quad x = \ln 2$

ii) ②



$e^{2x} = 2$

$\ln 2 = 2x$

$x = \frac{1}{2}\ln 2$

iii) Area $= \int_0^{\ln 2} e^x - (e^{2x} - 2)\,dx$

$= \int_0^{\ln 2} e^x - e^{2x} + 2\,dx$ ✓

$= \left[e^x - \frac{e^{2x}}{2} + 2x\right]_0^{\ln 2}$ ✓

$= e^{\ln 2} - \frac{e^{2\ln 2}}{2} + 2\ln 2 - \left(e^0 - \frac{e^0}{2}\right)$ ✓

$= 2 - \frac{4}{2} + 2\ln 2 - 1 + \frac{1}{2}$

$= -\frac{1}{2} + 2\ln 2$ ✓ ④

$= \left(2\ln 2 - \frac{1}{2}\right)u^2$

QUESTION 9

a) i) $y = -x^2 + 6x - 8$

$x^2 - 6x + 8 = 0$

$(x - 2)(x - 4) = 0$

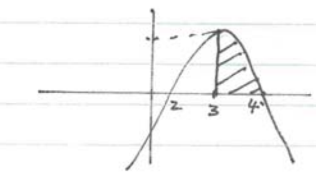
$x = 2, 4$ ✓ ①

ii) $x = 3 \pm \sqrt{1 - y}$

$x = 3$

$y = -9 + 18 - 8$

$= 1$



Volume $= \pi\int_0^1 (3+\sqrt{1-y})^2 - 3^2 \, dy$

$= \pi\int_0^1 9+6\sqrt{1-y}+1-y-9 \, dy$

$= \pi\int_0^1 6\sqrt{1-y} + 1 - y \, dy$

$= \pi\left[\frac{6(1-y)^{1\frac12}}{1\frac12 \times -1} + y - \frac{y^2}{2}\right]_0^1$

$= \pi\left[0+1-\frac12-\left(\frac{6}{-1\frac12}+0-0\right)\right]$

$= \left(\frac12 + 4\right)\pi$ (4)

$= 4\frac12\pi u^3 = \frac92\pi u^3 = \frac{9\pi}{2}u^3$

b) Let A_n be value of nth investment

$A_1 = 2000(1.06)^4$

$A_2 = 2000(1.06)^3$

$\vdots$

$A_4 = 2000(1.06)^1$

Total at end of 2013

$= 2000(1.06 + ... 1.06^4)$

$= 2000 \times 1.06 \frac{(1.06^4 - 1)}{1.06 - 1}$

$= \$9274.19$ (3)

$A_5 = 2000(1.08)^7$

$A_6 = 2000(1.08)^6$

$\vdots$

$A_{11} = 2000(1.08)^1$

∴ Total =

$9274.19(1.08)^7 +$

$2000(1.08 + 1.08^2 + .. 1.08^7)$

$= 9274.19(1.08)^7 + 2000 \times 1.08 \times \frac{(1.08^7 - }{1.08 - }$

$= 15894.33 + 19273.26$

$= \$35167.59$ (4)

QUESTION 10

a) $y = \frac{mx^3}{3} + \frac{3x^2}{2} - 4x + 7$

$y' = \frac{3mx^2}{3} + \frac{6x}{2} - 4$

$= mx^2 + 3x - 4$ ½

∴ curve to be decreasing

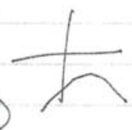
$y' < 0$ ½ Also $m < 0$

ie. $mx^2 + 3x - 4 < 0$ for all x

$\Delta = 9 - 4 \times m \times -4$

$= 9 + 16m$

$9 + 16m < 0$ (no roots)



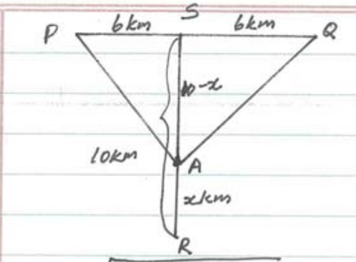
$16m < -9$

$m < -\frac{9}{16}$ (3)

m is < 0

∴ $m < -\frac{9}{16}$.

b)



i) $AP = \sqrt{(10-x)^2 + 6^2}$

$= \sqrt{100 - 20x + x^2 + 36}$

$= \sqrt{x^2 - 20x + 136}$

$AQ = \sqrt{x^2 - 20x + 136}$

∴ $L = 2\sqrt{x^2-20x+136} + x$ (2)

ii) For min L, $L' = 0$ & $L'' > 0$

$L' = 2 \times \frac12 (x^2 - 20x + 136)^{-\frac12}(2x - 20) + 1$

$= \frac{2(x-10)}{\sqrt{x^2-20x+136}} + 1$

$L' = 0$

$2x - 20 = -\sqrt{x^2 - 20x + 136}$

$4x^2 - 80x + 400 = x^2 - 20x + 136$

$3x^2 - 60x + 264 = 0$

$x^2 - 20x + 88 = 0$

$x = \frac{20 \pm \sqrt{400 - 4\times 88}}{2}$

$= \frac{20 \pm \sqrt{48}}{2}$

$= \frac{20 \pm 4\sqrt3}{2}$

$= 10 \pm 2\sqrt3$

But $x < 10$

∴ $x = 10 - 2\sqrt3$ (5)

iii) $L = 2\sqrt{x^2 - 20x + 136} + x$

$= 2\sqrt{(10-2\sqrt3)^2 - 20(10-2\sqrt3) + 136} + (10 - 2\sqrt{\ }$

$= 2\sqrt{100 - 40\sqrt3 + 12 - 200 + 40\sqrt3 + 136} + 10 - 2\sqrt3$

$= 2\sqrt{48} + 10 - 2\sqrt3$

$= 8\sqrt3 + 10 - 2\sqrt3$

$= (10 + 6\sqrt3)$ km (2)

must be numbers here no just + &

Check $x = 10 - 2\sqrt3$

(6.5)

x	5	$10-2\sqrt3$	7
L'	-0.8	0	0.11

$x=5$ $L' = \frac{-10}{\sqrt{31}} + 1 = -0.8$

$x=7$ $L' = \frac{-6}{\sqrt{45}} + 1 = 0.11$

∴ MIN