**ASCHAM SCHOOL**

**MATHEMATICS TRIAL EXAMINATION 2012**

### General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen. Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 11–16

**Total marks – 100**

**Section I** Pages i–ii **10 marks**

- Attempt Questions 1–10 using Multiple Choice sheet
- Allow about 15 minutes for this section

**Section II** Pages iii–viii **90 marks**

- Attempt Questions 11–16
- Allow about 2 hours 45 minutes for this section
- Do each question in a separate booklet.
- Write your name and your teacher's name on each booklet.
- Clearly label the front of each booklet with the number of the question.

**Collection**

- Start each question of Section II in a new booklet.
- If you use a second booklet for a question, place it inside the first.
- Write your name/number, teacher's name and question number on each booklet.

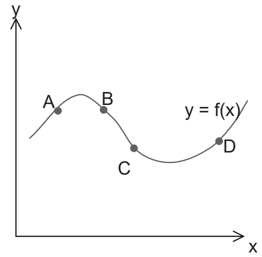
## Section 1 (10 marks)

Objective response questions to be completed on the given sheet.

**1.** Each interior angle of a regular decagon is:

(A) 140° (B) 144° (C) 148° (D) 145°

**2.**




For the graph of $y = f(x)$ shown, state the point at which $f'(x) < 0$ and $f''(x) > 0$.

(A) A

(B) B

(C) C

(D) D

**3.** A parabola has its focus at (0, 4). The equation of its directrix is $x = -4$.
Which of the following is the equation of the parabola?

(A) $(y+2)^2 = 8(x-4)$

(B) $(y-4)^2 = 8(x+2)$

(C) $x^2 = 16y$

(D) $(x+2)^2 = 8(y-4)$

**4.** The roots of the equation $2x^2 + 4x - 7 = 0$ are $\alpha$ and $\beta$. The value of $\alpha + \beta + 2\alpha\beta$ is:

(A) 9 (B) -18 (C) -9 (D) 18

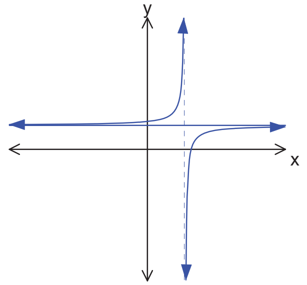
**5.** The sum of the first eleven terms of the geometric series which begins $2 - 1 + \frac{1}{2} - \ldots\ldots\ldots\ldots$ is:

(A) $\frac{1023}{256}$ (B) $\frac{341}{512}$ (C) $\frac{2047}{1536}$ (D) $\frac{683}{512}$

**6.** Interest of 10% p.a., compounded annually, is paid on an investment. After 10 years $100 will have grown to about:

(A) $260 (B) $250 (C) $235 (D) $200

**7.**




The graph could be represented by the equation:

(A) $y = -\dfrac{1}{x-3} - 2$

(B) $y = -\dfrac{1}{x-3} + 2$

(C) $y = -\dfrac{1}{x+3} + 2$

(D) $y = -\dfrac{1}{x+3} - 2$

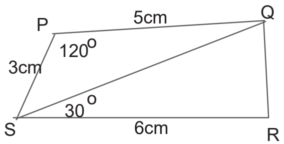
**8.** The line through (-1,2) which is perpendicular to $5x - 3y - 1 = 0$ is:

(A) $3x + 5y = 7$ (B) $5x + 3y = 1$ (C) $3x - 5y = -13$ (D) $5y + 3x = 1$

**9.** The minimum value of $x^2 - 7x + 10$ is:

(A) 2 (B) $3\frac{1}{2}$ (C) $-2\frac{1}{4}$ (D) $2\frac{1}{4}$

**10.**




In the quadrilateral, $\angle SPQ = 120°$, $\angle QSR = 30°$, PQ = 5cm, PS = 3cm and SR = 6cm. Area of $\Delta QSR$ is:

(A) 73.5 $cm^2$

(B) 10.4$cm^2$

(C) 10.5$cm^2$

(D) 21$cm^2$



## Section II (90 marks)

**Answer each whole question in separate booklets**

**Question 11 (15 marks)** **<u>Start this question in a new booklet</u>**

a) Write $\sin 4$ correct to 3 significant figures. **[1]**

b) Solve $|2x - 1| < 5$ **[2]**

c) Find a primitive of $\dfrac{3}{x} - x^2$ **[2]**

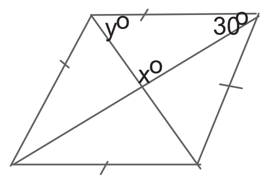
d) Simplify $8^n \times 2^{2n}$ **[2]**

e) Find $a$ and $b$ if $6\sqrt{5} - \dfrac{1}{\sqrt{5}-2} = a + b\sqrt{5}$ **[2]**

f) Find $x$ if $\log_x(3x+4) = 2$ **[2]**

g) Simplify $sec^2\theta\left(1-\cos^2\theta\right)$ **[2]**

h)




Find the size of $x$ and $y$ with reasons.

**[2]**

**Question 12 (15 marks)** **Start this question in a new booklet**

a) For the sequence 243, 81, 27, ……. find the least value of $n$ such that $T_n < \frac{1}{1000}$? **[2]**

b) Differentiate with respect to $x$:

(i) $\frac{1}{2x^4}$ **[1]**

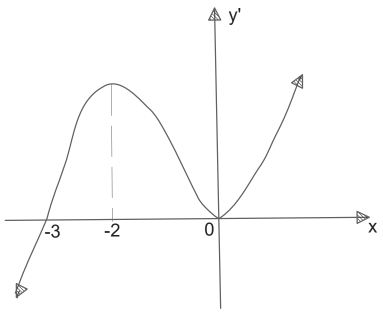
(ii) $\frac{\log x}{x}$ **[2]**

(iii) $\cos^2 3x$ **[2]**

c) Find $\int \sec^2 4x \, dx$ **[1]**

d) Evaluate exactly $\int_{-2}^{0} \frac{x^2}{5-x^3} dx$ **[2]**

e)




The graph shows the **derivative,** $y'$, of a function $y = f(x)$.

(i) State the value(s)of $x$ where $y = f(x)$ is increasing. **[1]**

(ii) State the value(s) of $x$ where $y = f(x)$ has a point of inflexion. **[1]**

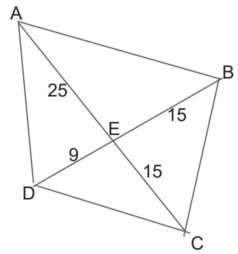
(iii) State the value(s) of $x$ where $y = f(x)$ has a minimum turning point. **[1]**

(iv) If $f(0) = 1$ sketch the graph of $y = f(x)$. **[2]**



**Question 13 (15 marks)** **Start this question in a new booklet**

a)




The diagonals of quadrilateral ABCD intersect at E. Sketch the diagram showing the measurements given.

(i) Prove ΔAEB│││ΔCED. **[2]**

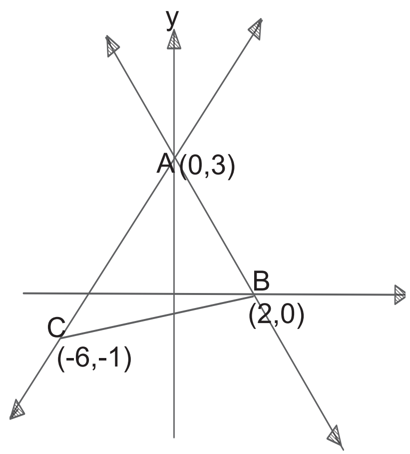
(iii) Hence prove AB││DC **[1]**

b) The radius of a circle is 3cm. What angle at the centre is subtended by a sector of area $18\text{cm}^2$. Give your answer correct to the nearest degree. **[2]**

c) (i) Find, in terms of $m$, an expression for the discriminant $x^2 + 2mx + (3m-2)$. **[1]**

(ii) Hence find the values of $m$ if $x^2 + 2mx + (3m-2) = 0$ has real roots. **[1]**

d)




The diagram shows the points A(0,3), B(2,0) and C(-6,-1) on a number plane.

(i) Find the distance AB. **[1]**

(ii) Show that the equation of AB is 3x + 2y – 6 = 0. **[1]**

(iii) Find the perpendicular distance of C from AB. **[2]**

(iv) Given that AC⊥AB find the size of ∠ABC correct to the nearest minute. **[2]**

(v) Given M is the midpoint of AC and N is the midpoint of BC state, with reason, the length of MN. **[2]**

## Question 14 (15 marks) <u>Start this question in a new booklet</u>

a) Given the table of values for $f(x)$ below, use Simpson's Rule with 5 function values to find $\int_0^{16} f(x)\,dx$.

| $x$ | 0 | 4 | 8 | 12 | 16 |
|---|---|---|---|---|---|
| $f(x)$ | 1 | 11 | 34 | 26 | 17 |

**[2]**

b) Find the equation of the normal to $y = \frac{e^{3x}}{4} - 2$ at the point where $x = \ln 2$, giving your answer in general form. **[3]**

c) Given that $y = (1 - x)e^x$:

(i) Show that $y' = -xe^x$ and find $y''$. **[3]**

(ii) Find the turning point and show that it is a maximum. **[2]**

(iii) Show that there is a point of inflexion at $\left(-1, \frac{2}{e}\right)$. **[2]**

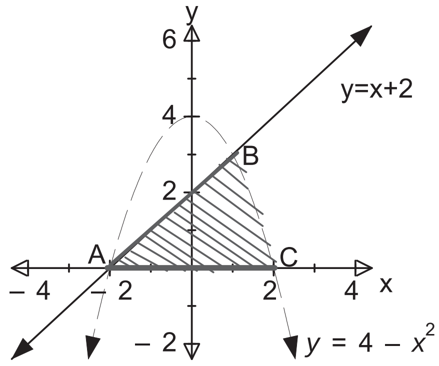
(iv) Determine y as $x \to -\infty$ and as $x \to \infty$. **[1]**

(v) Hence sketch $y = (1 - x)e^x$ and write its range. **[2]**



## Question 15 (15 marks) <u>Start this question in a new booklet</u>

a) The graphs of $y = 4 - x^2$ and $y = x + 2$ intersect at the points (-2,0) and (1,3).




(i) State the 3 inequalities required to indicate the shaded region ABC. **[1]**

(ii) Calculate the volume of the solid formed when the shaded region ABC is rotated about the $x$-axis. **[4]**

b) The population P, of the seal population on Macquarie Island has been found to increase each year since the end of 1982, when there were only 30 seals. By the end of 1992 the population had grown to 180.

Given that $\frac{dP}{dt} = kP$ after $t$ years:

(i) Show that $P = P_0 e^{kt}$ satisfies $\frac{dP}{dt} = kP$. **[1]**

(ii) Find the values of $P_0$ and $k$. **[2]**

(iii) Determine the expected population at the end of 2012. **[1]**

(iv) Find the rate at which the population will be expected to increase by the end of 2012. **[1]**

See next page for (c)

c) A company borrows \$200 000 from a bank to be paid back in 20 equal annual instalments. The bank charges 6% p.a. interest compounded half-yearly. Let \$ $A_n$ be the amount owing after $n$ years and \$$Y$ be the value of each yearly instalment.

(i) Show that $A_1 = 200\,000(1.03)^2 - Y$ . **[1]**

(ii) Find an expression for $A_{20}$ . **[2]**

(iii) Find the value of each yearly instalment. **[2]**

**Question 16 on next page**



**Question 16 (15 marks)** **Start this question in a new booklet**

a) The displacement, $x$ cm of a particle traveling in a straight line at time t seconds is given by $x = 5e^{-2t} - 3 + 8t$ .

(i) Find expressions for the velocity, $v$ and acceleration, $a$ in terms of $t$. **[2]**

(ii) Describe the motion initially. **[2]**

(iii) Determine the limiting velocity as $t \to \infty$ . **[1]**

b) (i) Sketch $y = -4\cos 2x$ for $0 \le x \le 2\pi$ . **[2]**

(ii) Hence find (α) $\int_0^{2\pi} -4\cos 2x\,dx$ **[1]**

(β) The area bounded by $y = -4\cos 2x$ and the $x$-axis from $x = 0$ to $x = 2\pi$ . **[2]**

c) A designer T-shirt manufacturer finds that the total cost, \$C to make $x$ T-shirts is $C = 4000 + \frac{3x}{20} + \frac{x^2}{1000}$ and the selling price \$E for each T-shirt sold is $E = 30 - \frac{x}{200}$ .

Assuming all but 30 T-shirts are sold:

(i) Show that the total sales, \$S is given by $S = -\frac{x^2}{200} + \frac{603x}{20} - 900$ **[1]**

(ii) Show that the profit, \$P is given by $P = -\frac{3}{500}x^2 + 30x - 4900$ . **[1]**

(iii) Find the maximum profit and show that the price for selling each T-shirt must be \$17.50 to achieve this maximum profit. **[3]**

**END OF EXAMINATION**

## Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$



**Student Number:__________________**

**Name:________________**

**SECTION I** **Mathematics Multiple Choice Answer Sheet** **10 Marks**

**This sheet must handed in separately**

**Shade the correct answer:**

| | | | | |
|---|---|---|---|---|
| 1. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 2. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 3. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 4. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 5. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 6. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 7. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 8. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 9. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |
| 10. | A ⬭ | B ⬭ | C ⬭ | D ⬭ |

# Ascham Trial Yr 12 Mathematics 2012 Solutions

**Student Number:**__________________

**Name:** Marking Copy

**SECTION I** **Mathematics Multiple Choice Answer Sheet** **10 Marks**

**This sheet must handed in separately**

**Shade the correct answer:**

| | A | B | C | D |
|---|---|---|---|---|
| 1. | ○ | ● | ○ | ○ |
| 2. | ○ | ○ | ● | ○ |
| 3. | ○ | ● | ○ | ○ |
| 4. | ○ | ○ | ● | ○ |
| 5. | ○ | ○ | ○ | ● |
| 6. | ● | ○ | ○ | ○ |
| 7. | ○ | ● | ○ | ○ |
| 8. | ● | ○ | ○ | ○ |
| 9. | ○ | ○ | ● | ○ |
| 10. | ○ | ○ | ● | ○ |

## Question 11

a) $\sin 4 = -0.7568\ldots$

$= -0.757$ (to 3 sig figs)

b) $|2x-1| < 5$

$-5 < 2x - 1 < 5$

$-4 < 2x < 6$

$-2 < x < 3$

c) Primitive of $\frac{3}{x} - x^2$

$= 3\log x - \frac{x^3}{3}\ (+C)$

d) $8^n \times 2^{2n} = 2^{3n} \times 2^{2n}$

$= 2^{5n}$

e) $6\sqrt{5} - \frac{1}{\sqrt{5}-2} = a + b\sqrt{5}$

$LHS = 6\sqrt{5} - \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$

$= 6\sqrt{5} - \left(\frac{\sqrt{5}+2}{5-4}\right)$

$= 6\sqrt{5} - \sqrt{5} - 2$

$= 5\sqrt{5} - 2$

$a = -2 \qquad b = 5$

f) $\log_x(3x+4) = 2$

$x^2 = 3x + 4$

$x^2 - 3x - 4 = 0$

$(x+1)(x-4) = 0$

$x = -1$ or $x = 4$

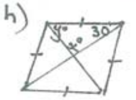
But $x > 0 \quad \therefore x = 4$

g) $\sec^2\theta\,(1 - \cos^2\theta)$

$= \frac{1}{\cos^2\theta} \times \sin^2\theta$

$= \tan^2\theta$

h)




$x° = 90°$ (diagonals of a rhombus bisect at right ∠'s)

$y° = 180° - 30° - 90°$

$= 60°$

(∠ sum of Δ)

## Question 12

a) $243, 81, 27, \ldots\ldots$

$a = 243 \quad r = \frac{1}{3}$

$T_n < \frac{1}{1000}$

$243\left(\frac{1}{3}\right)^{n-1} < \frac{1}{1000}$

$\left(\frac{1}{3}\right)^{n-1} < \frac{1}{243000}$

$(n-1)\log\left(\frac{1}{3}\right) < \log\left(\frac{1}{243000}\right)$

$n - 1 > \frac{\log\frac{1}{243000}}{\log\frac{1}{3}}$

$n > \frac{\log\frac{1}{243000}}{\log\frac{1}{3}} + 1$

$> 12.2877\ldots$

∴ Least value of $n$ is 13

b) (i) $y = \frac{1}{2x^4}$

$= \frac{1}{2}x^{-4}$

$y' = \frac{-2}{x^5}$

(ii) $y = \frac{\log x}{x}$ $\qquad$ $u = \log x \quad v = x$; $u' = \frac{1}{x} \quad v' = 1$

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{x \times \frac{1}{x} - \log x \times 1}{x^2}$

$= \frac{1 - \log x}{x^2}$

(iii) $y = \cos^2 3x$

$= (\cos 3x)^2$

$y' = 2(\cos 3x)^1 \times -3\sin 3x$

$= -6\cos 3x \sin 3x$

c) $\int \sec^2 4x\,dx = \frac{1}{4}\tan 4x + C$

d) $\int_{-2}^{0} \frac{x^2}{5-x^3}\,dx = -\frac{1}{3}\int_{-2}^{0} \frac{-3x^2}{5-x^3}\,dx$

$= -\frac{1}{3}\left[\ln(5-x^3)\right]_{-2}^{0}$

$= -\frac{1}{3}\left[\ln 5 - \ln 13\right]$

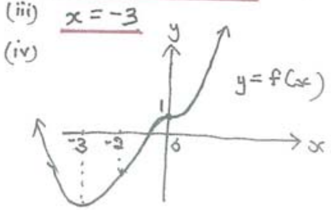
$= \frac{1}{3}\ln\frac{13}{5}$

e) (i) $x > -3,\ x \neq 0$

(ii) $x = -2$ and $x = 0$

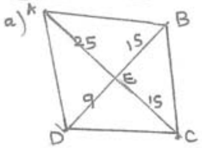
(iii) $x = -3$

(iv)



## Question 13

a)




(i) In $\Delta AEB$ and $\Delta CED$

$\frac{AE}{CE} = \frac{25}{15} = \frac{5}{3}$

$\frac{EB}{ED} = \frac{15}{9} = \frac{5}{3}$

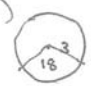
$\angle AEB = \angle CED$ (vertically opposite $\angle$'s)

$\therefore \Delta AEB \;|||\; \Delta CED$ (sides about equal angles are in proportion.

(ii) $\angle BAE = \angle DCE$ (matching $\angle$'s in similar $\Delta$'s)

$AB \parallel DC$ (a pair of alternate $\angle$'s are equal)

b)




Area $= \frac{1}{2} r^2 \theta$

$18 = \frac{1}{2} \times 3^2 \times \theta$

$\theta = 4^c$

$= 4 \times \frac{180}{\pi}^\circ$

$= 229.183\ldots^\circ$

$= 229^\circ$ (nearest degree)

c) (i) $x^2 + 2mx + (3m-2)$

$\Delta = b^2 - 4ac$

$= (2m)^2 - 4 \times 1 \times (3m-2)$

$= 4m^2 - 12m + 8$

(ii) $x^2 + 2mx + (3m-2) = 0$ has real roots if $\Delta \geq 0$

$4m^2 - 12m + 8 \geq 0$

$m^2 - 3m + 2 \geq 0$

$(m-2)(m-1) \geq 0$


$m \leq 1$ or $m \geq 2$

d)

(i) $AB = \sqrt{(\;\;-0)^2 + (0-3)^2}$

$= \sqrt{4+9}$

$= \sqrt{13}$ units

(ii) $m_{AB} = \frac{0-3}{2-0}$

$= -\frac{3}{2}$

Equation of AB $y = -\frac{3}{2}x + 3$

$2y = -3x + 6$

as required $3x + 2y - 6 = 0$

(iii)

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$ $(x_1, y_1) = (-6, -1)$

$= \left|\frac{3 \times -6 + 2 \times -1 - 6}{\sqrt{3^2+2^2}}\right|$

$= \left|\frac{-26}{\sqrt{13}}\right|$

$= \frac{26}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$

$= 2\sqrt{13}$ units

(iv) In $\Delta ABC$ $\tan\angle ABC = \frac{AC}{AB}$

$= \frac{2\sqrt{13}}{\sqrt{13}}$

$= 2$

$\angle ABC = 63^\circ 26'$ (nearest min)

(v) $MN = \frac{1}{2} AB$ (the line joining the midpoints of 2 sides of $\Delta ABC$ is $\parallel$ to third side (AB) and equal to half of it)

$= \frac{1}{2}\sqrt{13}$

## Question 14

a) $\int_0^{16} f(x)\,dx \doteqdot \frac{4}{3}[1 + 4 \times 11 + 2 \times 34 + 4 \times 26 + 17]$

$\doteqdot \frac{4}{3} \times 234$

$\doteqdot 312$

b) $y = \frac{e^{3x}}{4} - 2$

$y' = \frac{3}{4} e^{3x}$

At $x = \ln 2$ $y = \frac{e^{3\ln 2}}{4} - 2$

$= \frac{e^{\ln 8}}{4} - 2$

$= \frac{8}{4} - 2$

$= 0$

$y' = \frac{3}{4} e^{3\ln 2}$

$= \frac{3}{4} \times 8$

$= 6$

Equation of normal

$y - 0 = -\frac{1}{6}(x - \ln 2)$

$6y = -x + \ln 2$

$x + 6y - \ln 2 = 0$

c) (i) $y = (1-x)e^x$

$u = 1-x$ $v = e^x$

$u' = -1$ $v' = e^x$

$y' = vu' + uv'$

$= e^x \times -1 + (1-x)e^x$

$= e^x[-1 + 1 - x]$

$= -xe^x$ as required

$y'' = vu' + uv'$ $u = -x$ $v = e^x$ $u' = -1$ $v' = e^x$

$= e^x \times -1 + -x \times e^x$

$= e^x[-1 - x]$

$= -e^x(1+x)$

(ii) When $y' = 0$ $-xe^x = 0$

$x = 0$

When $x = 0$ $y = (1-0)e^0$ $= 1$ $y'' = -e^0(1+0)$ $= -1 < 0$

$\therefore$ Turning point is $(0,1)$ and it is a maximum turning point

(iii) When $y'' = 0$ $-e^x(1+x) = 0$

$x = -1$

When $x = -1$ $y = (1+1)e^{-1}$

$= \frac{2}{e}$

| $x$ | $-2$ | $-1$ | $0$ |
|---|---|---|---|
| $y''$ | $\frac{1}{e^2}$ | $0$ | $-1$ |
| | ∪ | | ∩ |

Change in concavity

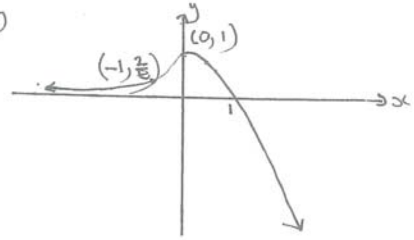
$\therefore$ Point of inflexion is $(-1, \frac{2}{e})$ as required

(iv) $y = (1-x)e^x$

As $x \to -\infty$ $y \to 0^+$

As $x \to \infty$ $y \to -\infty$

(v)




Range is $y \leq 1$

## Question 15

a) (i) $y \geq 0$

$y \leq 4 - x^2$

$y \leq x + 2$

(ii)

Volume $= \pi\int_{-2}^{1}(x+2)^2\,dx + \pi\int_{1}^{2}(4-x^2)^2\,dx$

$= \pi\int_{-2}^{1} x^2+4x+4\,dx + \pi\int_{1}^{2} 16-8x^2+x^4\,dx$

$= \pi\left[\frac{x^3}{3}+2x^2+4x\right]_{-2}^{1} + \pi\left[16x-\frac{8}{3}x^3+\frac{x^5}{5}\right]_{1}^{2}$

$= \pi\left[\left(\frac{1}{3}+2+4\right)-\left(\frac{-8}{3}+8-8\right)\right] + \pi\left[\left(32-\frac{64}{3}+\frac{32}{5}\right)-\left(16-\frac{8}{3}+\frac{1}{5}\right)\right]$

$= \pi\left[6\tfrac{1}{3}+2\tfrac{2}{3}\right]+\pi\left[17\tfrac{1}{5}-13\tfrac{8}{15}\right]$

$= 9\pi + \frac{53}{15}\pi$

$= \frac{188}{15}\pi$ units$^3$

b) (i) $P = P_0e^{kt}$

$\frac{dP}{dt} = kP_0e^{kt}$

$= kP$

$\therefore P = P_0e^{kt}$ satisfies $\frac{dP}{dt} = kP$

(ii) $P = P_0e^{kt}$

When $t = 0$ $P = 30$

$30 = P_0e^0$

$\therefore P_0 = 30$

When $t = 10$ $P = 180$

$180 = 30e^{10k}$

$e^{10k} = 6$

$10k = \ln 6$

$k = \frac{1}{10}\ln 6$

$(= 0.17917\ldots)$

(iii) $P = 30e^{(\frac{1}{10}\ln 6)t}$

When $t = 30$

$P = 30e^{(\frac{1}{10}\ln 6)\times 30}$

$= 30e^{3\ln 6}$

$= 30\times 6^3$

$= 6480$

$\therefore$ Expected population is 6480

(iv) $\frac{dP}{dt} = kP$

$= \left(\frac{1}{10}\ln 6\right)\times 6480$

$= 1151.060\ldots$

$\therefore$ Rate of increase is 1151 /yea

c) Let \$$A_n$ be amount owing after n years

(i) Let \$Y be each yearly instalment

$= 200000 + 200000(.03)^2 - Y$

$A_1 = 200000(1+.03)^2 - Y$

$= 200000(1.03)^2 - Y$

as required

(ii) $A_2 = 200000(1.03)^4 - Y(1.03)^2 - Y$

$\vdots$

$A_{20} = 200000(1.03)^{40} - Y[1 + 1.03^2 + \ldots \ldots 1.03$

(iii) $A_{20} = 0$

$\therefore 200000(1.03)^{40} = Y[1 + 1.03^2 + \ldots + 1.$

$a = 1$ $r = 1.03^2$ $n =$

$200000(1.03)^{40} = Y \times \frac{1(1.03^{40}-1)}{(1.03^2-1)}$

$Y = \frac{200000(1.03)^{40}\times(1.03^2-1)}{(1.03^{40}-1)}$

$= 17564.525\ldots$

$\therefore$ Each yearly instalment is \$17 564.5 (nearest cent)

## Question 16

a) $x = 5e^{-2t} - 3 + 8t$

(i) $v = -10e^{-2t} + 8$

$a = 20e^{-2t}$

(ii) When $t = 0$ $x = 5 - 3 = 2$

$v = -10 + 8 = -2$

$a = 20$

Initially particle is 2 cm on the positive side of the origin, moving at 2cm/s in the negative direction and undergoing acceleration of 20 cm/s$^2$ in the positive direction, so it is slowing down.

(iii) $v = -10e^{-2t} + 8$

As $t \to \infty$ $v \to -10\times 0 + 8$

$\to 8$

$\therefore$ Limiting velocity is 8cm/s in the positive direction

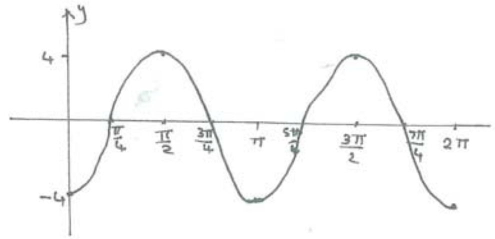
b) (i) $y = -4\cos 2x$

D: $0 \leq x \leq 2\pi$

R: $-4 \leq y \leq 4$

P: $\frac{2\pi}{2} = \pi$

A: 4




(ii) (α) $\int_0^{2\pi} -4\cos 2x\,dx = 0$ (by symmetry)

(β) Area $= -8\int_0^{\frac{\pi}{4}} -4\cos 2x\,dx$

$= -8\left[-2\sin 2x\right]_0^{\frac{\pi}{4}}$

$= 16\left[\sin\frac{\pi}{2} - \sin 0\right]$

$= 16[1 - 0]$

$= 16$ units$^2$

c) $C = 4000 + \frac{3}{20}x + \frac{x^2}{1000}$

$E = 30 - \frac{x}{200}$ for $(x - 30)$ T-shirts

(i) $S = E \times (x - 30)$

$= \left(30 - \frac{x}{200}\right)(x - 30)$

$= 30x - 900 - \frac{x^2}{200} + \frac{3x}{20}$

$= -\frac{x^2}{200} + \frac{603}{20}x - 900$

as required.

(ii) $P = S - C$

$= \left(-\frac{x^2}{200} + \frac{603}{20}x - 900\right) - \left(4000 + \frac{3}{20}x + \frac{x^2}{1000}\right)$

$= -\frac{3}{500}x^2 + 30x - 4900$

as required

(iii) $\frac{dP}{dx} = -\frac{3}{250}x + 30$

$\frac{d^2P}{dx^2} = \frac{-3}{250} < 0$

$\therefore$ Maximum profit occurs when $\frac{dP}{dx} = 0$ $\quad -\frac{3}{250}x + 30 = 0$

$\frac{3x}{250} = 30$

$x = 2500$

$\therefore$ Maximum profit occurs when $x = 2500$

When $x = 2500$

$E = 30 - \frac{2500}{200}$

$= 17.5$

$\therefore$ Cost of each T-shirt to achieve maximum profit is \$17.50