Name_______________

ASCHAM SCHOOL

MATHEMATICS TRIAL EXAMINATION 2013

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen. Black pen is preferred.
- Board-approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- Show all necessary working in Questions 11–16.

Total marks – 100

Section I 10 marks

- Attempt Questions 1–10 using the Multiple Choice sheet.
- Allow about 15 minutes for this section.

Section II 90 marks

- Attempt Questions 11–16.
- Allow about 2 hours 45 minutes for this section.
- Do each question in a separate booklet.
- Write your name/number and your teacher's name on each booklet.
- Clearly label the front of each booklet with the number of the question.

Collection

- Start each question of Section II in a new booklet.
- If you use a second booklet for a question, place it inside the first.

  Indicate on the outside of the first booklet that you have used two booklets for that question.
- Write your name/number, teacher's name and question number on each booklet.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet at the back of this exam paper for Questions 1 – 10

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1 Evaluate $\sqrt[3]{3\frac{3}{7}}$ to three significant figures.

(A) 1.087

(B) 1.09

(C) 1.508

(D) 1.51

2 The first and last terms of an arithmetic series are 10 and 60.
If the sum of the series is 3535, how many terms are there in the series?

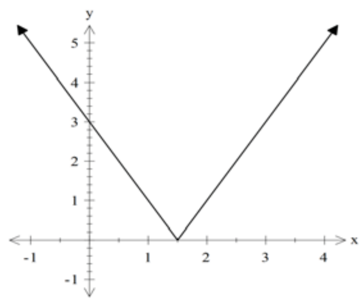
(A) 11

(B) 101

(C) 110

(D) 51

3 What is the equation of the graph drawn below?



(A) $y=|2x+3|$

(B) $y=|2x-3|$

(C) $y=|x-1.5|$

(D) $y=|x+1.5|$

4 The perimeter of a sector is 30cm. If the angle at the centre is 3 radians, what is the radius of the circle?

(A) 10cm

(B) 20cm

(C) 6cm

(D) 3cm

5 Find $\int \frac{1}{x^2}dx$

(A) $\log(x^2)+\text{c}$

(B) $\frac{-2}{x^3}+c$

(C) $\frac{-1}{x}+c$

(D) $2x\log(x^2)+\text{c}$

6 What is the equation of a parabola with focus (2, 3) and directrix y = - 5?

(A) $(x+1)^2=16(y-2)$

(B) $(x-2)^2=16(y+1)$

(C) $(x-2)^2=4(y-1)$

(D) $(y+1)^2=4(x-2)$

7 What is the limiting sum of the series $-\frac{1}{27}+\frac{1}{9}-\frac{1}{3}$?

(A) $\frac{-1}{108}$

(B) $\frac{1}{54}$

(C) Can't be found

(D) $\frac{1}{108}$

8 What is the derivative of $\frac{4}{3x^3}$?

(A) $-\frac{4}{x^4}$

(B) $-\frac{2}{3x^2}$

(C) $-\frac{4}{x^2}$

(D) $-\frac{36}{x^4}$

9 $(2\sqrt{3}-5)^2$ is equal to

(A) $1-20\sqrt{3}$

(B) 37

(C) $37-20\sqrt{3}$

(D) $37-10\sqrt{3}$

10 What is the compound interest on \$1000 invested for 5 years at 6% per annum interest, compounded monthly? (to nearest dollar)

(A) \$1338

(B) \$349

(C) \$1349

(D) \$1025

End of Multiple Choice

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use the Question 11 Writing Booklet

a) Factorise $2x^2+3x-20$. **(1)**

b) Find the exact value of $\cos 210^0$. **(1)**

c) Find $\int \frac{2}{e^{2x}}dx$. **(1)**

d) One of the roots of the equation $kx^2-2x-3=0$ is -3.
Find the value of k. **(1)**

e) Differentiate $\sin^2 2x$. **(2)**

f) Solve $|2x-3|\le 5$. **(2)**

g) Find $\int \frac{3x}{x^2-4}dx$. **(2)**

h) Solve for x correct to 2 significant figures:

$(\log_e 2x)^2=16$ **(3)**

i) Is $f(x)=(x-x^5)^2$ odd or even? Show all working. **(2)**

End of question 11

Question 12 (15 marks) Use a new booklet

a) Differentiate with respect to x:

i) $\dfrac{e^x}{\log_e 2x}$ **(3)**

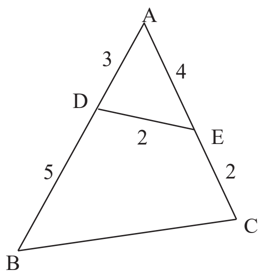
ii) $\log\left[x^4(x-1)^2\right]$ **(2)**

b) Find the primitive of $\cos(\frac{1-x}{5})$. **(1)**

c) Find, in general form, the equation of the tangent to the curve $y = x\ln x$ at the x intercept. **(3)**

d) If α and β are the roots of the equation $2x^2 - 3x + 4 = 0$, find the value of $\alpha^2 + \beta^2$. **(3)**

e) By proving 2 triangles similar, find the length of BC. **(3)**

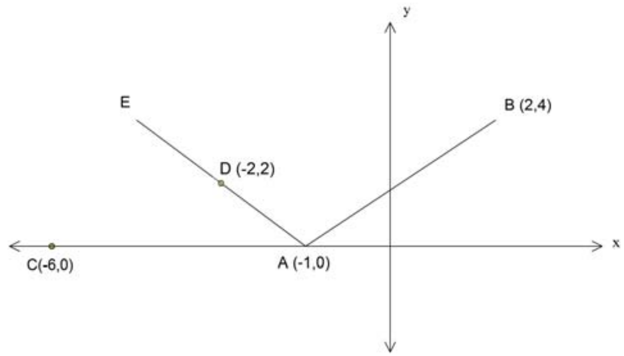


Not to scale

End of question 12

Question 13 (15 marks) Start a new booklet.

a) A, B and C are the points (-1,0), (2,4) and (-6,0) respectively.
D is the point (-2,2) and is the midpoint of AE.



Not to scale

i) Find the length of the interval AB **(2)**

ii) Find the midpoint of BC **(1)**

iii) Find the coordinates of E **(2)**

iv) What type of quadrilateral is ABEC?
Give a clear explanation for your answer. **(2)**

b) Find the equation of the locus of the point $P(x, y)$ which moves so that it is twice the distance from R(-3,4) as it is from S(-1,2). **(4)**

c) The slope at any point on a curve is given by $3\sec^2 2x$.
Find the equation of the curve if it passes through the point $(\frac{\pi}{3}, \frac{\sqrt{3}}{2})$. **(4)**

End of question 13

Question 14 (15 marks) Start a new booklet.

a) For what values of k will the equation $9x^2 - kx + 1 = 0$ have real and different roots? **(3)**

b) Evaluate $\int_0^n (1+\sin 2\pi x)\,dx$. **(3)**

c) i) Sketch the function $y = 4\cos 2x$ for $0 \le x \le \pi$. **(2)**

ii) Find the area between the curve $y = 4\cos 2x$ and the x axis from $x = 0$ to $x = \frac{\pi}{2}$. **(3)**

d) A tank contains 50 litres of water. A tap at the base of the tank allows water to flow out at a rate proportional to the quantity of water still in the tank at that time. After 2 minutes, 10 litres have run out.

Use the equation $W = W_0 e^{-kt}$, where W is the amount of water in the tank and t is time in minutes.

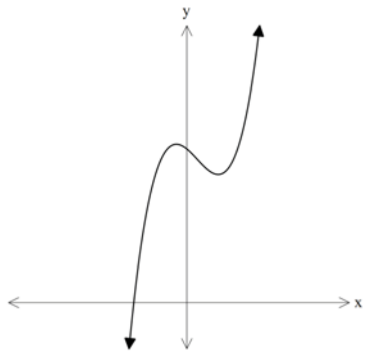
i) Show that $k = -\frac{1}{2}\ln\frac{4}{5}$. **(2)**

ii) How much water has run out after 10 minutes? (To the nearest litre) **(2)**

End of question 14

Question 15 (15 marks) Start a new booklet

a) The graph of $y = x^3 - x^2 - x + 6$ is sketched below.



Not to scale

i) Find the coordinates of the stationary points. **(3)**

ii) Find any point(s) of inflexion **(2)**

iii) For what values of x is the curve decreasing and concave up? **(1)**

iv) For what values of p has the equation

$x^3 - x^2 - x + 6 = p$ exactly two real solutions? **(2)**

b) Use Simpson's Rule with 5 function values to evaluate, to 2 decimal places,

$\int_1^3 \log_e x\,dx.$ **(3)**

c) A particle moves in a straight line such that its position at time t seconds is given by $x = t - \log_e t$.

i) When is the particle at rest? **(1)**

ii) Find the exact distance travelled by the particle between $t = \frac{1}{2}$ and $t = 1\frac{1}{2}$. **(3)**

End of Question 15

Question 16 (15 marks) Start a new booklet

a) Diana borrows \$10 000 and arranges to pay it back with interest in 20 equal instalments every three months over 5 years. She is charged 6% per annum interest compounded monthly.

Let A_n be the amount owing after n months and let M be the instalment.

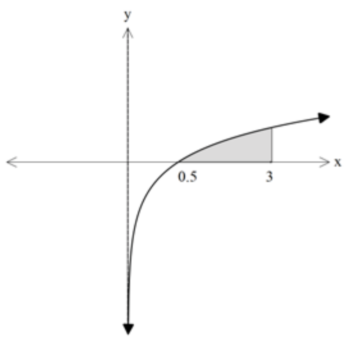
i) Find the amount owing after the first three months, just after she has made her first payment. **(1)**

ii) Show that $M = \dfrac{10000(1.005^{63} - 1.005^{60})}{1.005^{60} - 1}$. **(3)**

iii) Find the size of the instalment she pays. **(1)**

iv) How much does Diana pay in interest? **(1)**

b) The graph of $y = \log_e 2x$ is given below.

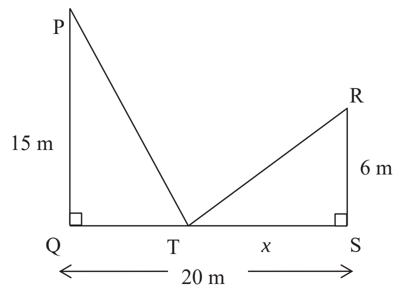


Not to scale

Find the volume when the shaded region between the curve, the x axis and the line $x = 3$ is rotated about the y axis. **(3)**

Question 16c is on the next page

c)



Not to scale

Two poles, PQ and RS are 20 metres apart. PQ is 15 metres high and RS is 6 metres high. A length of wire is attached to the top of each pole and also staked to the ground at T somewhere between the two poles.

Let TS = x metres.

i) Show that the length of wire $L = \sqrt{36 + x^2} + \sqrt{625 - 40x + x^2}$. **(2)**

ii) Find the shortest length of wire that can be used. **(4)**

End of exam

Standard Integrals

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

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Student Number:___________________

Name:_________________

SECTION I **Mathematics Multiple Choice Answer Sheet** **10 Marks**

This sheet must be handed in separately. Detach it from the question paper.

Shade the correct answer:

1.	A ⬭	B ⬭	C ⬭	D ⬭
2.	A ⬭	B ⬭	C ⬭	D ⬭
3.	A ⬭	B ⬭	C ⬭	D ⬭
4.	A ⬭	B ⬭	C ⬭	D ⬭
5.	A ⬭	B ⬭	C ⬭	D ⬭
6.	A ⬭	B ⬭	C ⬭	D ⬭
7.	A ⬭	B ⬭	C ⬭	D ⬭
8.	A ⬭	B ⬭	C ⬭	D ⬭
9.	A ⬭	B ⬭	C ⬭	D ⬭
10.	A ⬭	B ⬭	C ⬭	D ⬭

2013 Y12 MATHEMATICS TRIAL

SECTION I
MULTIPLE CHOICE

1. D
2. B
3. B
4. C
5. C
6. B
7. C
8. A
9. C
10. B

QUESTION 11

a) $2x^2+3x-20=(2x-5)(x+4)$ 1

b) $\cos 210^\circ = -\cos 30$
$= -\frac{\sqrt{3}}{2}$ 1

c) $\int \frac{2}{e^{2x}}\,dx = \int 2e^{-2x}\,dx$
$= \frac{2e^{-2x}}{-2}+c$
$= \frac{-1}{e^{2x}}+c$ 1

d) $kx^2-2x-3=0$
$x=-3$:
$9k+6-3=0$
$9k=-3$
$k=-\frac{1}{3}$ 1

e) $\frac{d}{dx}\sin^2 2x = \frac{d}{dx}(\sin 2x)^2$
$=2\sin 2x\cos 2x\times 2$
$=4\sin 2x\cos 2x$ 2

f) $|2x-3|\le 5$
$-5\le 2x-3\le 5$
$-2\le 2x\le 8$
$-1\le x\le 4$ 2

g) $\int\frac{3x}{x^2-4}\,dx = \frac{3}{2}\int\frac{2x}{x^2-4}\,dx$ 2
$=\frac{3}{2}\log_e(x^2-4)+c$

h) $(\log_e 2x)^2=16$
$\log_e 2x=\pm 4$
$2x=e^4$ or $2x=e^{-4}$
$x=\frac{e^4}{2}$ or $\frac{e^{-4}}{2}=\frac{1}{2e^4}$ 3
$=27$ or 9.2×10^{-3} (2sf)

i) $f(x)=(x-x^5)^2$
$f(-x)=(-x-(-x)^5)^2$
$=(-x+x^5)^2$ 2
$f(x)=f(-x)$ (b/c squared)
$\therefore$ even

QUESTION 12

a) i) $\frac{d}{dx} \frac{e^x}{\log_e 2x}$

$= \frac{(\log_e 2x)(e^x) - e^x \cdot \frac{1}{2x} \times 2}{(\log_e 2x)^2}$

$= \frac{e^x \log_e 2x - \frac{e^x}{x}}{(\log_e 2x)^2}$

$= \frac{e^x (x \log_e 2x - 1)}{x(\log_e 2x)^2}$ (3)

ii) $\frac{d}{dx} \log[x^4(x-1)^2]$

$= \frac{d}{dx}(\log x^4 + \log(x-1)^2)$

$= \frac{d}{dx}(4\log x + 2\log(x-1))$

$= \frac{4}{x} + \frac{2}{x-1}$ (2)

b) $\int \cos\left(\frac{1-x}{5}\right) dx$

$\therefore = \frac{\sin\left(\frac{1-x}{5}\right)}{-\frac{1}{5}} + c$

$= -5\sin\left(\frac{1-x}{5}\right) + c$ (1)

c) $y = x \ln x$

$y' = x \cdot \frac{1}{x} + \ln x$

$= 1 + \ln x$

x int: $y = 0 \quad \therefore x \ln x = 0$

$x = 0$ or $\log_e x = 0$

$x = 0$ not valid $\quad \therefore e^0 = x$

$x = 1$

$y = 0$

$(1, 0)$

grad of tangent $= 1 + \ln 1$

$= 1$

$y - y_1 = m(x - x_1)$

$y - 0 = 1(x - 1)$

$y = x - 1$

$x - y - 1 = 0$ (3)

d) $2x^2 - 3x + 4 = 0$

$\alpha + \beta = \frac{3}{2} \quad \alpha\beta = 2$

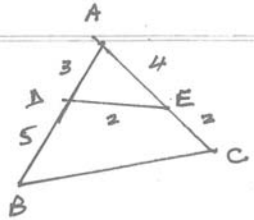
$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{3}{2}\right)^2 - 2 \times 2$

$= \frac{9}{4} - 4$

$= -1\frac{3}{4}$ (2)

e)



In Δs ADE, ABC

$\frac{AD}{AC} = \frac{3}{6} = \frac{1}{2}$

$\frac{AE}{AB} = \frac{4}{8} = \frac{1}{2}$

$\angle A$ is common

$\therefore \Delta ADE ||| \Delta ACB$ (sides around $= \angle$ in same ratio)

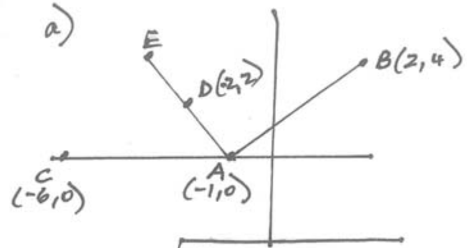
$\frac{AD}{AC} = \frac{DE}{CB} = \frac{AE}{AB}$ (matching sides of similar Δs)

$\frac{2}{BC} = \frac{4}{8}$

$\therefore BC = 4$ (3)

QUESTION 13

a)



i) $AB = \sqrt{3^2 + 4^2}$

$= 5$ (2)

ii) $M = \left(\frac{2-6}{2}, \frac{4-0}{2}\right)$

$= (-2, 2)$

$\therefore$ M is same pt as D. (1)

iii) $(-2, 2) = \left(\frac{x-1}{2}, \frac{y+0}{2}\right)$

$-2 = \frac{x-1}{2} \qquad \frac{y}{2} = 2$

$x - 1 = -4 \qquad y = 4$

$x = -3$

$\therefore E(-3, 4)$ (2)

iv) Diagonals bisect each other: $\therefore$ parallelogram ✓

But $AC = AB = 5$

$\therefore$ rhombus. ✓ (2)

b) $PR = 2PS$

$\sqrt{(x+3)^2 + (y-4)^2} = 2\sqrt{(x+1)^2 + (y-2)^2}$

$x^2 + 6x + 9 + y^2 - 8y + 16 = 4(x^2 + 2x + 1 + y^2 - 4y + 4)$

$x^2 + 6x + y^2 - 8y + 25 = 4x^2 + 8x + 4y^2 - 16y +$

$3x^2 + 2x + 3y^2 - 8y - 5 = 0$ (4)

c) $y' = 3\sec^2 2x$

$y = \frac{3\tan 2x}{2} + c$

Sub $\left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{3}}{2} = \frac{3}{2}\tan\frac{2\pi}{3} + c$

$\frac{\sqrt{3}}{2} = -\frac{3}{2} \times \sqrt{3} + c$

$\frac{\sqrt{3}}{2} = -\frac{3\sqrt{3}}{2} + c$

$c = \frac{\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} = 2\sqrt{3}$

$y = \frac{3}{2}\tan 2x + 2\sqrt{3}$ (4)

QUESTION 14

a) $9x^2 - kx + 1 = 0$

$\Delta = k^2 - 36$

For real & different roots $\Delta > 0$

$k^2 - 36 > 0$

$k < -6$ or $k > 6$ 3

b) $\int_0^n (1 + \sin 2\pi x)\,dx$

$= \left[x - \frac{\cos 2\pi x}{2\pi}\right]_0^n$

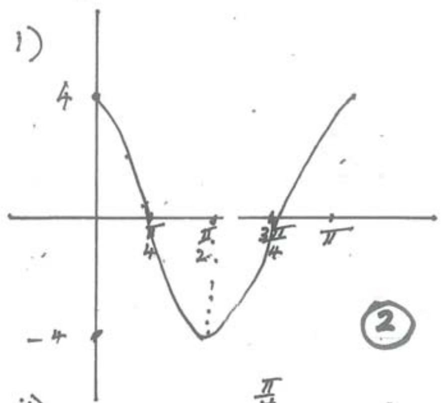
$= n - \frac{\cos 2\pi n}{2\pi} - 0 + \frac{\cos 0}{2\pi}$

$= n - \frac{1}{2\pi} + \frac{1}{2\pi}$

$= n$ 3

c) $y = 4\cos 2x \quad 0 \le x \le \pi$

i)



2

ii) Area $= 2\int_0^{\frac{\pi}{4}} 4\cos 2x\,dx$

$= 2 \times 4\left[\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{4}}$

$= 4\left[\sin\frac{\pi}{2} - \sin 0\right]$

$= 4 \times 1$

$= 4\ u^2$ 3

d)

i) $W = W_0 e^{-kt}$

$W_0 = 50$

$W = 50e^{-kt}$

$t = 2 \quad W = 40$

$40 = 50e^{-k \times 2}$ ✓

$4/5 = e^{-2k}$

$k = \frac{\log\frac{4}{5}}{-2}$ ✓

$= \frac{\left(\log\frac{5}{4}\right)^{-1}}{-2}$

$= \frac{-\log\frac{5}{4}}{-2}$

$= \frac{1}{2}\log\frac{5}{4}$ ✓ 2

ii) $W = 50e^{-kt}$

$t = 10$: $-\frac{1}{2}\ln\frac{5}{4} \times 10$

$W = 50e^{-5\ln\frac{5}{4}}$

$= 50e$

$= 16.384$ ✓

∴ 16.384 litres are in tank

ie. 33.616 litres has run out

$= 34\ l$ (to n litre) ✓ 2

QUESTION 15

a) $y = x^3 - x^2 - x + 6$

i) For stationary points $y' = 0$

$y' = 3x^2 - 2x - 1$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3} \quad x = 1$

$y = 6\frac{5}{27} \quad y = 5$

∴ stationary points are

$(-\frac{1}{3}, 6\frac{5}{27}), \ (1, 5)$ 3

$\frac{167}{27}$

ii) For points of inflexion $y'' = 0$

$6x - 2 = 0$

$x = \frac{1}{3}$

$y = 5\frac{16}{27} = \frac{151}{27}$ ✓

∴ $(\frac{1}{3}, 5\frac{16}{27})$ is a possible point of inflexion

Check concavity

x	0	$\frac{1}{3}$	1
y''	-2	0	4

✓

∴ change in concavity 2

∴ $(\frac{1}{3}, 5\frac{16}{27})$ is a point of inflexion

[if state there is change in concavity on diagram, that is OK for the mark]

iii) $\frac{1}{3} < x < 1$ 1

iv) $p = 5$ and $p = 6\frac{5}{27}$ 2

b)

x	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
$\log x$	$\log 1$	$\log 1\frac{1}{2}$	$\log 2$	$\log 2\frac{1}{2}$	$\log 3$
	y_0				y_4

$\int_1^3 \log x\,dx \doteq \frac{h}{3}\{y_0 + y_4 + 4(y_1 + y_3) + 2y_2\}$

$\doteq \frac{\frac{1}{2}}{3}\{0 + \log 3 + 4(\log 1.5 + \log 2.5) + 2\log 2\}$

$\doteq 1.29532\ldots$

$\doteq 1.30$ (2 dp) 3

c) $x = t - \log_e t$

i) $v = 1 - \frac{1}{t}$

Particle is a rest when $v = 0$

$1 - \frac{1}{t} = 0$

$\frac{1}{t} = 1$

$t = 1$

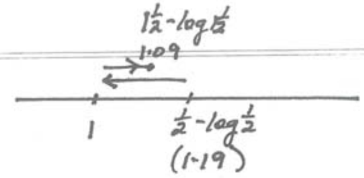
At rest at 1 sec. 1

ii) $t = \frac{1}{2} \quad x = \frac{1}{2} - \log\frac{1}{2}$ ½

$t = 1\frac{1}{2} \quad x = 1\frac{1}{2} - \log 1\frac{1}{2}$ ½

$t = 1 \quad x = 1 - \log 1$

$= 1$ ½



∴ Distance

$= \frac{1}{2} - \log\frac{1}{2} - 1 + 1\frac{1}{2} - \log 1\frac{1}{2} - 1$ ½

$= -\log\frac{1}{2} - \log 1\frac{1}{2}$

$= -(\log\frac{1}{2} + \log 1\frac{1}{2})$ ½

$= -\log\frac{3}{4}$

$= \log\frac{4}{3}$ 3

Question 16

a) $10000

Every 3 months 5 years

$\therefore$ 20 payments

6% pa = 0.005 p/month

i) $A_1 = 10000(1.005)$

$A_3 = 10000(1.005)^3 - M$ (1)

ii) $A_6 = A_3(1.005)^3 - M$

$= [10000(1.005)^3 - M]1.005^3 - M$

$= 10000(1.005^6) - M(1 + 1.005^3)$

$\vdots$

$A_{60} = 10000(1.005)^{60} - M(1 + 1.005^3 + \ldots + 1.005^{57})$

$A_{60} = 10000(1.005)^{60} - \dfrac{M(1.005^{60} - 1)}{1.005^3 - 1}$

$A_{60} = 0$

$10000(1.005)^{60} = \dfrac{M(1.005^{60} - 1)}{1.005^3 - 1}$

$\dfrac{10000(1.005)^{60}(1.005^3 - 1)}{1.005^{60} - 1} = M$

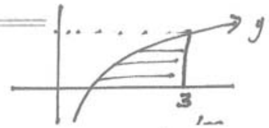
$M = \dfrac{10000(1.005^{63} - 1.005^{60})}{1.005^{60} - 1}$ (3)

iii) $M = \$582.89$ (1)

iv) Interest $= 582.89 \times 20 - 10000$

$= \$1657.80$ (1)

b)



$e^y = 2x$

Volume $= \pi \int_0^{\log 6} 3^2 - \left(\dfrac{e^y}{2}\right)^2 dy$

$= \pi \int_0^{\log 6} 9 - \dfrac{e^{2y}}{4} dy$

$= \pi\left[9y - \dfrac{e^{2y}}{8}\right]_0^{\log 6}$

$= \pi\left[9\log 6 - \dfrac{e^{2\log 6}}{8} - \left(0 - \dfrac{e^0}{8}\right)\right]$

$= \pi\, 9\log 6 - \dfrac{e^{\log 36}}{8} + \dfrac{1}{8}$

$= \pi\, 9\log 6 - \dfrac{36}{8} + \dfrac{1}{8}$

$= \pi\left(9\log 6 - \dfrac{35}{8}\right) u^3$

OR

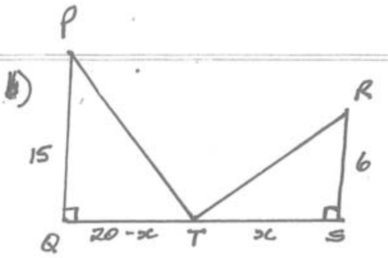
$\cancel{4.37\pi u^3}$ $11.75\pi u^3$

OR

$\cancel{13.73u^3}$ $36.92u^3$ (2dp)

-1 if no subtraction

c)



i) $PT^2 = 15^2 + (20 - x)^2$

$= 225 + 400 - 40x + x^2$

$= 625 - 40x + x^2$

$RS^2 = 6^2 + x^2$

$= 36 + x^2$ (2)

$\therefore L = \sqrt{36 + x^2} + \sqrt{625 - 40x + x^2}$

ii) For min L, $L' = 0$

$L' = \frac{1}{2}(36 + x^2)^{-\frac{1}{2}} \times 2x + \frac{1}{2}(625 - 40x + x^2)^{-\frac{1}{2}}(2x - 40)$

$L' = \dfrac{x}{\sqrt{x^2 + 36}} + \dfrac{x - 20}{\sqrt{625 - 40x + x^2}}$ ✓

$\dfrac{-x}{\sqrt{x^2 + 36}} = \dfrac{x - 20}{\sqrt{625 - 40x + x^2}}$

$\dfrac{x^2}{x^2 + 36} = \dfrac{x^2 - 40x + 400}{625 - 40x + x^2}$ ½

$625x^2 - \cancel{40x^3} + \cancel{x^4} = \cancel{x^4} - \cancel{40x^3} + 400x^2 + 36x^2 - 1440x + 14400$

$625x^2 = 400x^2 + 36x^2 - 1440x + 14400$

$189x^2 + 1440x - 14400 = 0$ ½

$x = \dfrac{-1440 \pm \sqrt{1440^2 - 4(189)(-14400)}}{2 \times 189}$

$= \dfrac{40}{7}$ ✓ or neg ans (not valid)

Check if min L

x	5	$5\frac{5}{7}$	6
L'	-0.06692	0	0.02479

$\therefore$ if $x = \dfrac{40}{7}$, L is min. ½ (4)

$\therefore$ shortest L

$\sqrt{36 + \left(\frac{40}{7}\right)^2} + \sqrt{625 - 40 \times \frac{40}{7} + \left(\frac{40}{7}\right)^2}$

$= 29$ m. ½