Name_________________________

ASCHAM SCHOOL

MATHEMATICS TRIAL EXAMINATION 2016

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black non-erasable pen.
- Board-approved calculators may be used.
- A reference sheet is provided.
- Detach the Multiple Choice answer sheet at the back of this exam paper.
- Show all necessary working in Questions 11–16.

Section I 10 marks

- Attempt Questions 1–10 using the detached Multiple Choice answer sheet at the back of this exam paper.
- Allow about 15 minutes for this section.

Section II 90 marks

- Attempt Questions 11–16.
- Start each question of Section II in a new booklet.
- Write your name/number and your teacher's name on each booklet.
- Clearly label the front of each booklet with the number of the question.
- If you use a second booklet for a question, place it inside the first.
  Indicate on the outside of the first booklet that you have used two booklets for that question.
- Allow about 2 hours 45 minutes for this section.

Collection

- Check that you have written your name/number, teacher's name and question number on each booklet.
- Hand in Section I and Section II Q11-Q16 separately.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet at the back of this exam paper for Questions 1 – 10

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1. The period T of $y = \cos(\frac{\pi x}{2})$ is:

A. $T = 4\pi^2$ **B.** $T = \pi^2$ **C.** $T = 4$ **D.** $T = \frac{1}{4}$

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2. Which of the following expressions is a first step in rationalising the denominator of $\frac{1}{3+\sqrt{5}}$?

A. $\frac{1}{3+\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$ **B.** $\frac{1}{3+\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$ **C.** $\frac{1}{3+\sqrt{5}} \times \frac{1}{3-\sqrt{5}}$ **D.** $\frac{1}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$

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3. The domain of $f(x) = \sqrt{x^2 - 4}$ is:

A. $x \le -2$ or $x \ge 2$ **B.** $x < -2$ or $x > 2$ **C.** $-2 \le x \le 2$ **D.** $-2 < x < 2$

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4. Which of the following will shift the graph of $y = x^2$, 4 units to the left and 2 units up:

A. $y = (x-4)^2 - 2$

B. $y = (x+4)^2 - 2$

C. $y = (x+4)^2 + 2$

D. $y = (x-4)^2 + 2$

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5. Find the exact value of $\sec\frac{7\pi}{6}$.

A. $\frac{\sqrt{3}}{2}$ **B.** $-\frac{\sqrt{3}}{2}$ **C.** $-\frac{2}{\sqrt{3}}$ **D.** $\frac{2}{\sqrt{3}}$

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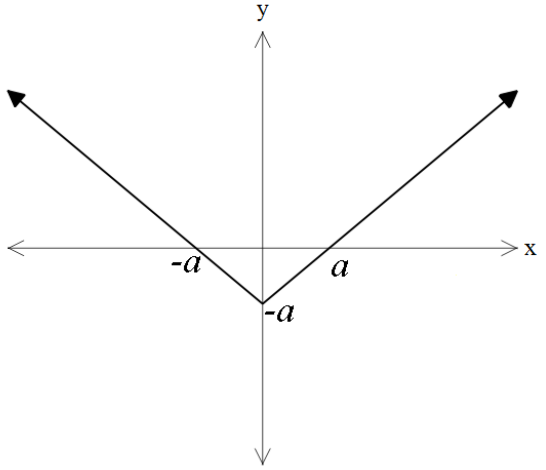
6. The primitive of $e + \frac{1}{x}$ is:

A. $ex - \frac{1}{x^2} + C$ **B.** $e + \ln x + C$ **C.** $ex + \ln x + C$ **D.** $e^x + \ln x + C$

Multiple Choice continues on the next page →

7. Which of the following equations represents the graph below, considering that $a > 0$?



A. $y = a - |x|$

B. $y = |x| - a$

C. $y = |x - a|$

D. $y = |x + a|$

---

8. Which of the following statements is true, if $x = \dfrac{p}{qr}$?

A. $\ln x = \ln p - \ln q + \ln r$ **B.** $\ln x = \ln p - \ln q - \ln r$ **C.** $\ln x = \ln p - \ln(q + r)$ **D.** $\ln x = \dfrac{\ln p}{\ln qr}$

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9. For what values of k is the quadratic $x^2 - kx + 2k$ positive definite?

A. $0 < k < 8$ **B.** $-8 < k < 0$ **C.** $0 \le k \le 8$ **D.** $0 < k < 2$

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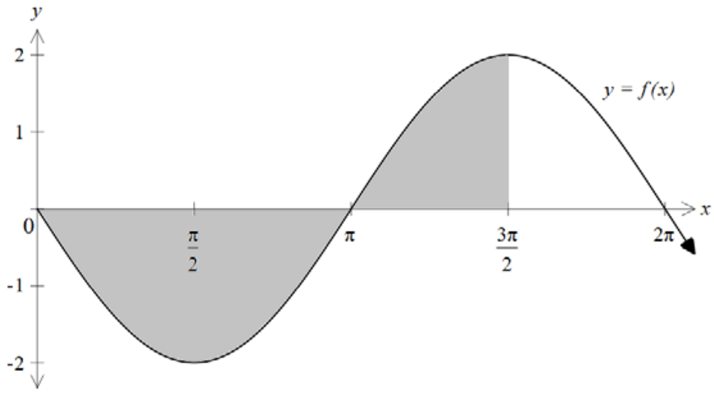
10. Which expression is equivalent to the area of the shaded region?



A. $A = 3\int_0^{\frac{\pi}{2}} f(x)dx$

B. $A = \int_0^{\frac{3\pi}{2}} f(x)dx$

C. $A = \left|\int_0^{\frac{3\pi}{2}} f(x)dx\right|$

D. $A = 3\int_{\pi}^{\frac{3\pi}{2}} f(x)dx$

End of Multiple Choice.

Section II

90 marks
Attempt Questions 11 – 16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 **(15 marks)** Start a new booklet.

a) Solve the equation $14u = 28u^2$. **[2]**

b) Simplify the expression $\dfrac{a+b}{\frac{1}{a}+\frac{1}{b}}$. **[2]**

c) If $\alpha+\beta=-3$ and $\alpha\beta=6$, where α and β are the roots to the monic quadratic equation $ax^2+bx+c=0$, find the coefficients a, b and c. **[2]**

d) Solve $|x-7|\le 3$. **[2]**

e) If $\tan\theta=\dfrac{\sqrt{11}}{5}$ and $\operatorname{cosec}\theta<0$, find the exact value of $\cos\theta$. **[2]**

f) Evaluate in exact form $\int_{\ln 3}^{\ln 9}(e^{3x}-3)dx$. **[3]**

g) Find the limiting sum of $(3-\sqrt{7})+(3-\sqrt{7})^2+(3-\sqrt{7})^3+\ldots$. **[2]**

End of question 11.

Question 12 (15 marks) Start a new booklet

a) Differentiate:

i) $y = \dfrac{x - e^x}{2x}$ **[2]**

ii) $y = \ln(x^2 - 4)$ **[2]**

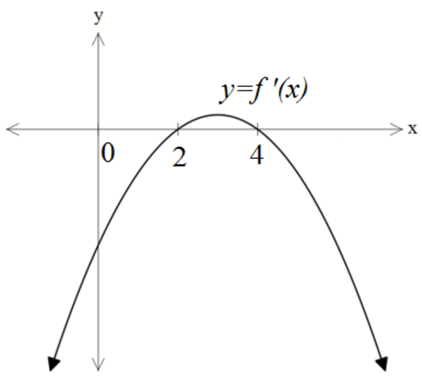
b) Evaluate:

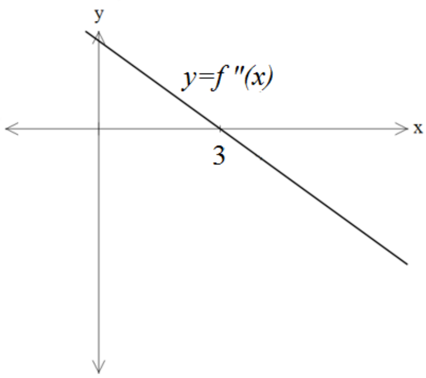
i) $\int_0^{\frac{\pi}{6}} \tan^2 x\, dx$. **[2]**

ii) $\int \dfrac{\cos 3x}{1 + \sin 3x} dx$ **[2]**

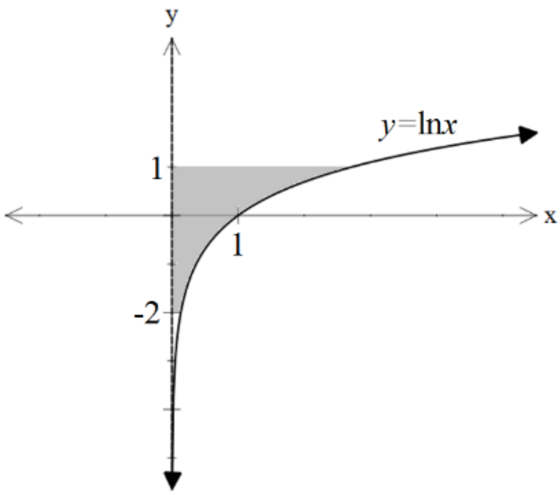
c) The Potter Family deposited a sum of \$15000 in their saving account. How long it will take for them to reach their savings goal of \$25000 if the bank pays interest of 9% per annum, compounded monthly? Round your answer to the nearest month. **[2]**

d) Sketch a possible curve of $f(x)$ given the graphs of $f'(x)$ and $f''(x)$ below: **[2]**





e) Find the exact volume of the solid of revolution if the shaded area to the right, for $-2 \le y \le 1$, is rotated about the y-axis. **[3]**



End of question 12.

Question 13 **(15 marks)** Start a new booklet

a) Solve the equation $3^x + \dfrac{27}{3^x} = 28$ **[3]**

b) Find the equation of the tangent to the curve $y = \dfrac{x^2}{4}$ with the gradient $m = -2$. **[2]**

c) Find k such that lines $l_1 : kx - 3y = 4$ and $l_2 : (k+1)x - 4y = 7$ are parallel. **[2]**

d) Consider the function $f(x) = 2 - \sin x$.

i) Determine the range of $f(x)$. **[1]**

ii) Sketch the graph of $f(x)$ for $0 \le x \le 2\pi$ showing all relevant information. **[2]**

e) Use Simpson's rule with five function values to evaluate exactly $\int_0^{\pi} x\cos x\,dx$. **[3]**

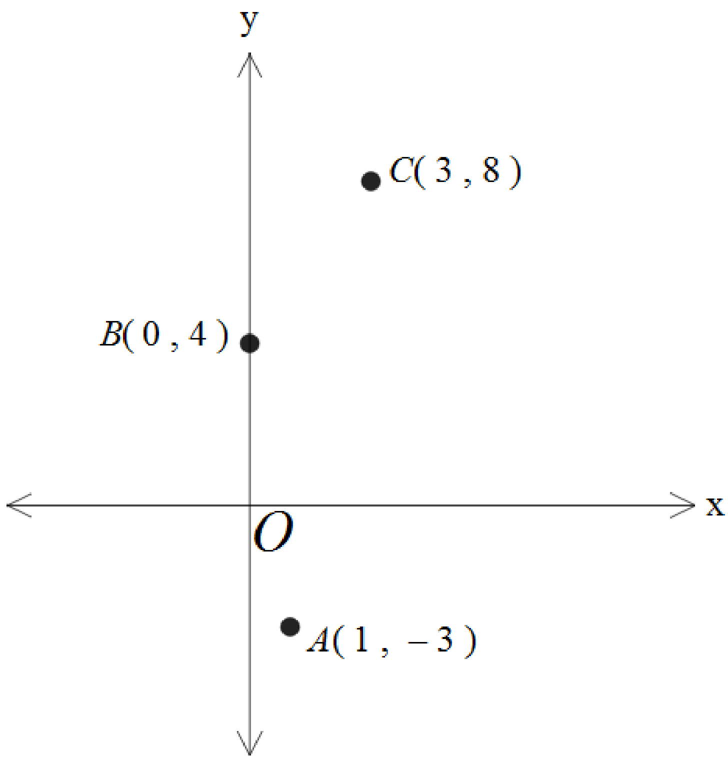
f) Mary decides to train for the marathon by running each day d minutes longer than the previous day, until she is ready for the competition. On the third day she is running for 21 minutes and on the tenth day is running for 49 minutes.

Find how long she is running on the first day and by how much she is increasing the running time each day. **[2]**

End of question 13.

Question 14 **(15 marks)** Start a new booklet

a) Let $A(1,-3)$, $B(0,4)$ and $C(3,8)$ be three points on a number plane.



i) Show that the line BC has an equation $4x-3y+12=0$. **[2]**

ii) Show that the perpendicular distance from A to BC equals the distance from B to C. **[3]**

iii) Hence find the area of the triangle ABC. **[1]**

b) Consider the curve $y=(x+2)e^{-x}$.

i) Find the x- and y- intercepts. **[2]**

ii) Find the stationary point and test its nature. **[2]**

iii) If $y''=xe^{-x}$, show that there is a point of inflexion at $x=0$ and find its y- coordinate. **[2]**

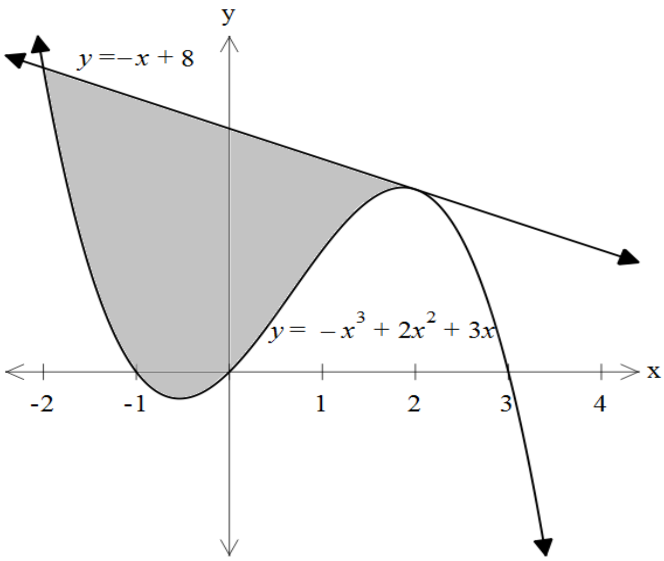
iv) Describe the behaviour of y as $x\to\infty$ and $x\to-\infty$. **[2]**

v) Sketch the graph showing all the relevant information. **[1]**

End of question 14.

Question 15 **(15 marks)** Start a new booklet

a) Consider the curve $y = -x^3 + 2x^2 + 3x$ and a line $y = -x + 8$.



i) Show that they intersect at $x = 2$ and $x = -2$. **[1]**

ii) Hence, find the area of the shaded region, correct to two decimal places. **[2]**

b) The fish population F in the pond is increasing according to the law of natural growth, $\frac{dF}{dt} = kF$, for some constant k.

i) If F_0 is the fish population for $t = 0$, show that $F = F_0 e^{kt}$ satisfies the given differential equation. **[1]**

ii) Find k if the population of the fish triples every two years. **[2]**

iii) Find the number of fish when $t = 0$ if the number of fish at the end of 6 years was 21000. **[2]**

c) i) Show that $\frac{d}{dx}(x^2 \ln x) = 2x \ln x + x$. **[1]**

ii) Hence, find $\int x \ln x \, dx$. **[2]**

Question 15 continues on the next page.

Question 15 continued

d) A charter company determines the cost of plane flights at a rate R cents / hour,

$R = \dfrac{s^2}{100} + 40s + 5476$, and s km / h is the speed of the plane.

A plane has been hired to fly 300 km from Sydney to Canberra, at a speed of s km / h.

i) Show that the total cost of the flight, C cents, is given by

$$C = 300\left(\frac{s}{100} + 40 + \frac{5476}{s}\right).$$ **[1]**

ii) Show that the speed which will minimise the total cost of the flight C is

$s = 740$ km/h. **[2]**

iii) Find the minimum cost of the flight. **[1]**

End of question 15.

<u>Question 16</u> **(15 marks)** Start a new booklet

a) Sally is saving for a trip around the world. The minimum cost of such a trip is \$15000 but she would like to depart with more than that amount, in case of unexpected expenses. At the beginning of each month, starting on 1st of January 2016, she plans to deposit \$200 in her savings account which pays interest at the rate of 3% per annum, compounded monthly. Let A_n be the amount accumulated at the end of n time periods.

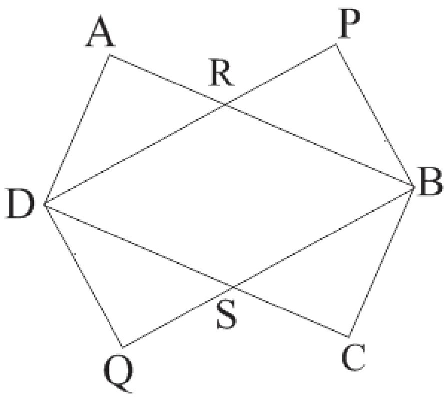
i) Show that an expression for the total value of her savings at the end of n months is given by $A_n = 80200 \times (1.0025^n - 1)$. **[2]**

ii) How much will she have saved by the 1st of January 2025, before the next payment is made that day? **[1]**

iii) If Sally continues with her savings plan, how long will it take for her to have at least \$15000 on her account? **[2]**

iv) Sally decided that she would like to depart on 1st of January 2020 after she has saved \$20000, by making larger deposits each month, from the beginning. Calculate her new monthly instalments, correct to the nearest cent. **[2]**

b) $ABCD$ and $DPBQ$ are two congruent rectangles with shorter side $AD = PB = 3$.



Copy the diagram in your booklet.

i) Prove that $\angle ARD = \angle CSB$. **[2]**

ii) Hence, prove $\Delta ADR \equiv \Delta CBS$. **[2]**

iii) If $AR : RB = 1 : 2$, show that $AR = \sqrt{3}$. **[2]**

iv) Evaluate the area of the octagon $ARPBCSQD$. **[2]**

END OF EXAM.

THEMATICS TRIAL EXAM 2016

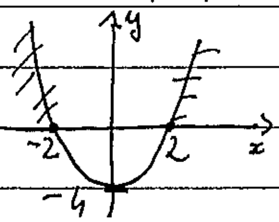
SOLUTIONS

SECTION 1:

1. $T = \frac{2\pi}{\frac{\pi}{2}}$ $= 4$ $\therefore$ C

2. D

3. $x^2 - 4 \geq 0$

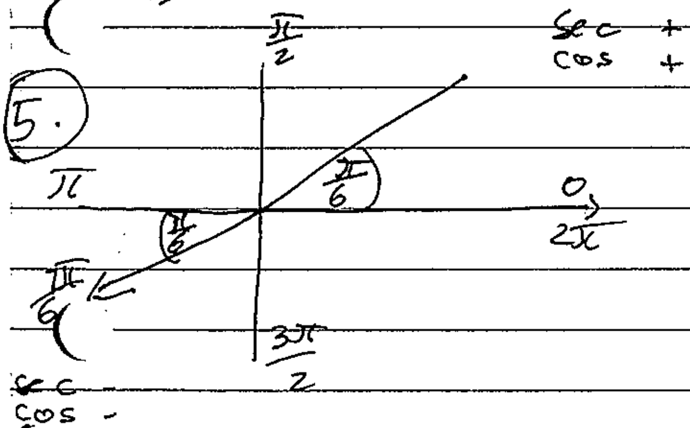


$\therefore x \leq -2$ or $x \geq 2$

$\therefore$ A

4. C

5.



$\sec\frac{7\pi}{6} = -\sec\frac{\pi}{6}$

$= -\frac{1}{\cos\frac{\pi}{6}}$

$= -\frac{1}{\frac{\sqrt{3}}{2}}$

$= -\frac{2}{\sqrt{3}}$

$\therefore$ C

6. C

7. B

8. $x = \frac{p}{qr}$

$\ln x = \ln\frac{p}{qr}$

$= \ln p - \ln qr$

$= \ln p - (\ln q + \ln r)$

$= \ln p - \ln q - \ln r$

$\therefore$ B

9. $x^2 - kx + 2k = 0$

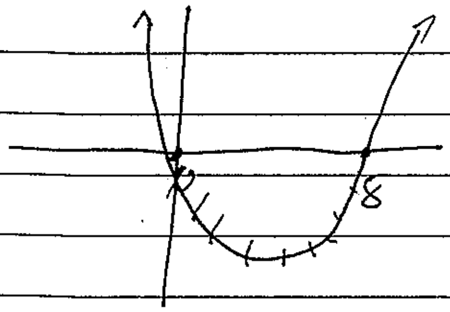
Positive definite :

$\Delta < 0$

$(-k)^2 - 4 \times 2k < 0$

$k^2 - 8k < 0$

$k(k-8) < 0$



$\therefore \Delta < 0$ for $0 < k < 8$

$\therefore$ A

10. D

SECTION II :

QUESTION 11

a) $14u = 28u^2$

$28u^2 - 14u = 0$

$14u(2u-1) = 0$

$u = 0$ or $2u - 1 = 0$

$\therefore u = 0$ and $u = \frac{1}{2}$

b) $\dfrac{a+b}{\frac{1}{a}+\frac{1}{b}} = \dfrac{a+b}{\frac{b+a}{ab}}$

$= \dfrac{ab\,\cancel{(a+b)}}{\cancel{a+b}}$

$= ab$

c) Monic quadratic $\therefore a = 1$

$\alpha + \beta = -\frac{b}{a}$ $\qquad$ $\alpha\beta = \frac{c}{a}$

$\therefore -\frac{b}{a} = -3$ $\qquad$ $\therefore \frac{c}{a} = 6$

$-\frac{b}{1} = -3$ $\qquad$ $\frac{c}{1} = 6$

$\therefore b = 3$ $\qquad$ $\therefore c = 6$

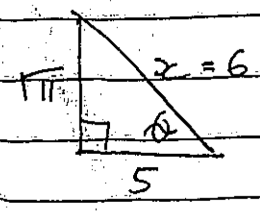
d) $|x-7| \le 3$

$\therefore -3 \le x-7 \le 3$

$-3+7 \le x \le 3+7$

$4 \le x \le 10$

e) $\tan\theta = \frac{\sqrt{11}}{5} = \frac{opp}{adj}$ $\qquad$ $\operatorname{cosec}\theta < 0$

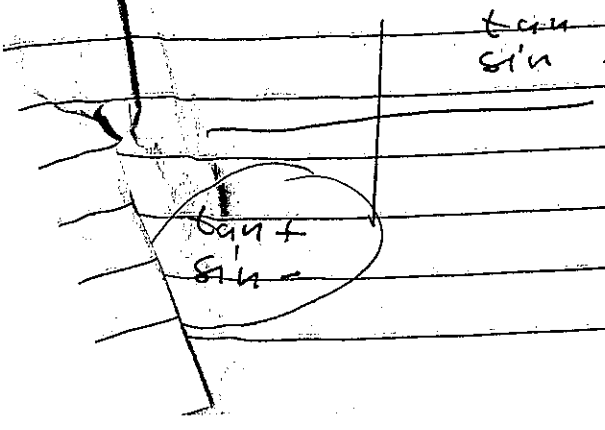


$x^2 = (\sqrt{11})^2 + 5^2$

$= 36$

$\therefore x = 6$

$\operatorname{cosec}\theta = \frac{1}{\sin\theta} < 0$



Since $\tan\theta > 0$

$\sin\theta < 0$

θ is in III quadrant

$\therefore \cos\theta < 0$

$\therefore \cos\theta = -\frac{5}{6}$

f) $\displaystyle\int_{\ln 2}^{\ln 3} (e^{3x} - 3)\,dx = \left[\frac{e^{3x}}{3} - 3x\right]_{\ln 2}^{\ln 3}$

$= \left(\frac{e^{3\ln 9}}{3} - 3\ln 9\right) - \left(\frac{e^{3\ln 3}}{3} - 3\ln 3\right)$

$= \left(\frac{e^{\ln 9^3}}{3} - \ln 9^3\right) - \left(\frac{e^{\ln 3^3}}{3} - \ln 3^3\right)$

$= \frac{3^6}{\cancel{3}} - \ln 3^6 - 9 + \ln 3^3$

$= 234 + \ln \frac{3^3}{3^6}$

$= 234 + \ln 3^{-3}$

$= 234 - 3\ln 3$

g) $(3-\sqrt{7}) + (3-\sqrt{7})^2 + (3-\sqrt{7})^3 + \dots$

$a = 3-\sqrt{7} \qquad r = 3-\sqrt{7}$

$S_\infty = \frac{a}{1-r}$ since $-1 < r < 1$

$= \frac{3-\sqrt{7}}{1-(3-\sqrt{7})}$

$= \frac{3-\sqrt{7}}{\sqrt{7}-2}$

QUESTION 12:

a) i) $y = \dfrac{x - e^x}{2x}$

$= \dfrac{x}{2x} - \dfrac{e^x}{2x}$

$= \dfrac{1}{2} - \dfrac{e^x}{2x}$

Let $u = -e^x \quad u' = -e^x$

$v = 2x \quad v' = 2$

$y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{-2xe^x + 2e^x}{4x^2}$

$= \dfrac{2e^x(1-x)}{4x^2}$

$= \dfrac{e^x(1-x)}{2x^2}$

ii) $y = \ln(x^2 - 4)$

$y' = \dfrac{1}{x^2-4} \times 2x$

$= \dfrac{2x}{x^2-4}$

b) i $\displaystyle\int_0^{\frac{\pi}{6}} \tan^2 x \, dx = \int_0^{\frac{\pi}{6}} (\sec^2 x - 1) dx$

$= \left[\tan x - x\right]_0^{\frac{\pi}{6}}$

$= \left(\tan\frac{\pi}{6} - \frac{\pi}{6}\right) - \left(\tan 0 - 0\right)$

$= \dfrac{1}{\sqrt{3}} - \dfrac{\pi}{6}$

$= \dfrac{\sqrt{3}}{3} - \dfrac{\pi}{6}$

$= \dfrac{2\sqrt{3} - \pi}{6}$

ii) $\displaystyle\int \frac{\cos 3x}{1 + \sin 3x} = \frac{1}{3}\int \frac{3\cos 3x}{1 + \sin 3x} dx$

$= \dfrac{1}{3}\ln(1 + \sin 3x) + C$

c) $P = \$15000$ $\qquad A = P(1+R)^n$

$A = \$25000$

$R = 9\%$ p.a.

$= 0.0075$ p. month

$n = ?$

$25000 = 15000(1+0.0075)^n$

$1.0075^n \approx 1.67$

$\therefore n \approx \dfrac{\ln 1.67}{\ln 1.0075}$

≈ 68.3651

≈ 68 months

(rounded to the near month)

It will take 2 years and 20 month approx.

d) From the graph of $f'(x)$

x		2		4	
$f'(x)$	$-$	0	$+$	0	$-$
slope	\	—	/	—	\

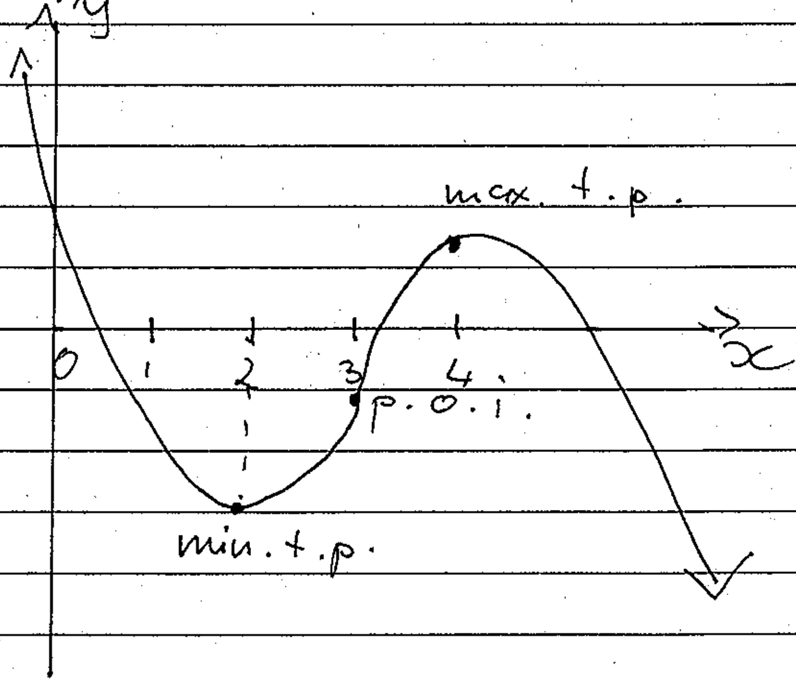
$\therefore$ at $x=2$ min t.p.

at $x=4$ max t.p.

From the graph of $f''(x)$

x		3	
$f''(x)$	$+$	0	$-$
conc	$\cup$	$\cdot$	$\cap$

$\therefore$ at $x=3$ p.o.i.



e) $y = \ln x$

$\therefore e^y = e^{\ln x}$

$e^y = x$

$$V = \pi \int_{-2}^{1} x^2 \, dy$$

$$= \pi \int_{-2}^{1} (e^y)^2 \, dy$$

$$= \pi \int_{-2}^{1} e^{2y} \, dy$$

$$= \pi \left[\frac{e^{2y}}{2} \right]_{-2}^{1}$$

$$= \pi \left(\frac{e^2}{2} - \frac{e^{-4}}{2} \right)$$

$$= \frac{\pi (e^2 - e^{-4})}{2} \; u^3$$

QUESTION 13.

a) $3^x + \frac{27}{3^x} = 28$ $\quad / \times e^x$

$3^{2x} + 27 = 28^x$

$3^{2x} - 28\,3^x + 27 = 0$

Let $3^x = u$

$u^2 - 28u + 27 = 0$

$(u-1)(u-27) = 0$

$\therefore u - 1 = 0$ or $u - 27 = 0$

$u = 1$ and $u = 27$

$\therefore 3^x = 1$ and $3^x = 27$

$x = 0$ $\quad 3^x = 3^3$

$\therefore x = 3$

b) $y = \frac{x^2}{4}$ $\qquad \frac{x}{2} = -2$

$y' = \frac{x}{2}$ - grad. function $\qquad \therefore x = -4$

For $x = -4$, $y = \frac{(-4)^2}{4}$

$= 4$

$P(-4, 4)$ $\qquad m = 2$

Point - gradient formula

$y - y_1 = m(x - x_1)$

$y - 4 = -2(x + 4)$

$y - 4 = -2x - 8$

$2x + y + 4 = 0$ - equation of the tangent

c) $l_1 : kx - 3y = 4$
$l_2 : (k+1)x - 4y = 7$

$l_1 : \quad 3y = kx - 4$
$\therefore y = \frac{k}{3}x - \frac{4}{3}$
$m_1 = \frac{k}{3}$

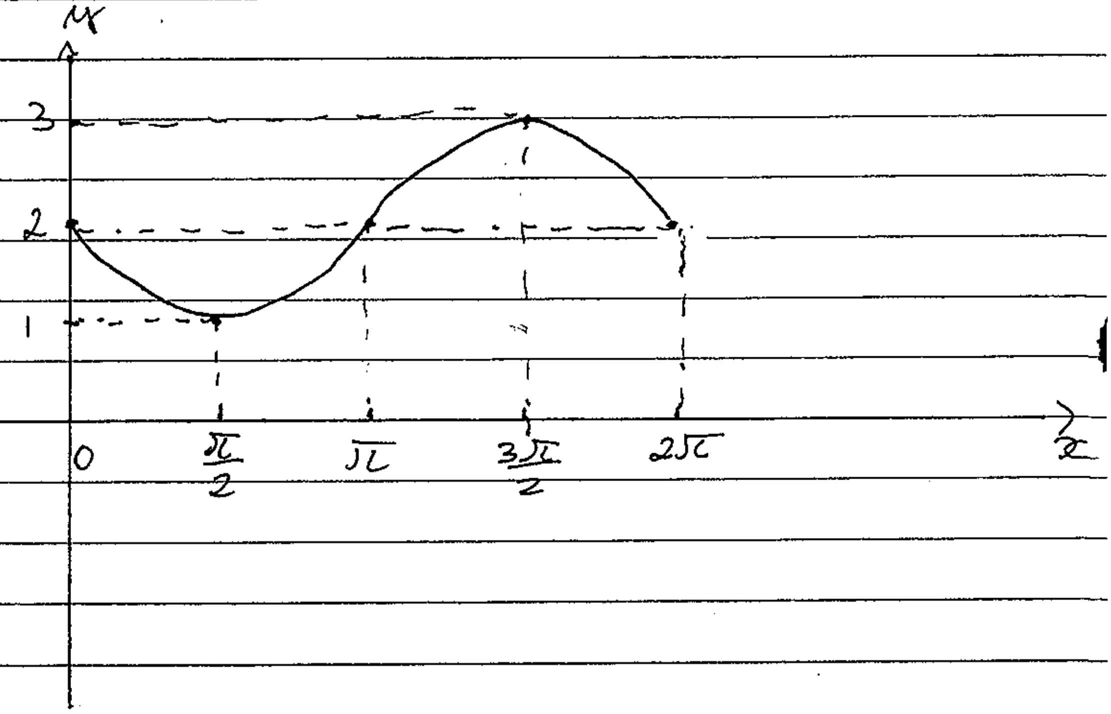
$l_2 : \quad 4y = (k+1)x - 7$
$\therefore y = \frac{k+1}{4}x - \frac{7}{4}$
$m_2 = \frac{k+1}{4}$

$l_1 \parallel l_2 \Rightarrow m_1 = m_2$

$$\frac{k}{3} = \frac{k+1}{4} \quad / \times 12$$
$$4k = 3(k+1)$$
$$4k = 3k + 3$$
$$\boxed{\therefore k = 3}$$

d) i) $f(x) = 2 - \sin x$
$f(x)$ is $\sin x$ first flipped about the x-axis and then shifted 2 units up
$\therefore$ range was $-1 \le y \le +1$
shifted up 2 units $1 \le y \le 3$

ii)



e) $\int_0^{\pi} x\cos x \, dx$

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$x\cos x$	0	$\frac{\pi\sqrt{2}}{8}$	0	$-\frac{3\pi\sqrt{2}}{8}$	$-\pi$

$$\int_0^{\pi} x\cos x\,dx = \int_0^{\frac{\pi}{2}} x\cos x\,dx + \int_{\frac{\pi}{2}}^{\pi} x\cos x\,dx$$

$$= \frac{\frac{\pi}{2}-0}{6}\left(0+4\times\frac{\pi\sqrt{2}}{8}+0\right)+\frac{\pi-\frac{\pi}{2}}{6}\left(0-4\times\frac{3\pi\sqrt{2}}{8}-\pi\right)$$

$$= \frac{\pi}{12}\left(\frac{\pi\sqrt{2}}{2}-\frac{3\pi\sqrt{2}}{2}-\pi\right)$$

$$= \frac{\pi}{12}\left(-\pi\sqrt{2}-\pi\right)$$

$$= -\frac{\pi^2}{12}\left(\sqrt{2}+1\right)$$

f) $T_3 = 21$
$T_{10} = 49$

Arithmetic sequence with first term a and common difference d.

$T_n = a + (n-1)d$

$\therefore T_3 = a + (3-1)d$
$T_3 = a + 2d$

$T_{10} = a + (10-1)d$
$= a + 9d$

Solve simultaneously:

$a + 2d = 21 \quad \ldots (1)$
$a + 9d = 49 \quad \ldots (2)$

(2) − (1)
$7d = 28$
$\therefore d = 4 \quad \ldots (3)$

Sub (3) → (1)
$a + 2\times 4 = 21$
$\therefore a = 13$

On the first day she was running for 13min and she was increasing the running time by 4min

QUESTION 14

a) $A(1,-3)$, $B(0,4)$, $C(3,8)$

i) Equation of the line throug B and C

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 4 = \frac{8-4}{3-0}(x - 0)$$

$$y - 4 = \frac{4}{3}x \quad /\times 3$$

$$3y - 12 = 4x$$

$$\therefore \; 4x - 3y + 12 = 0$$

ii) $$d(A, BC) = \frac{|4\times 1 - 3\times(-3) + 12|}{\sqrt{4^2 + (-3)^2}}$$

$$= \frac{|25|}{5}$$

$$= 5$$

$$d(B, C) = \sqrt{(8-4)^2 + (3-0)^2}$$

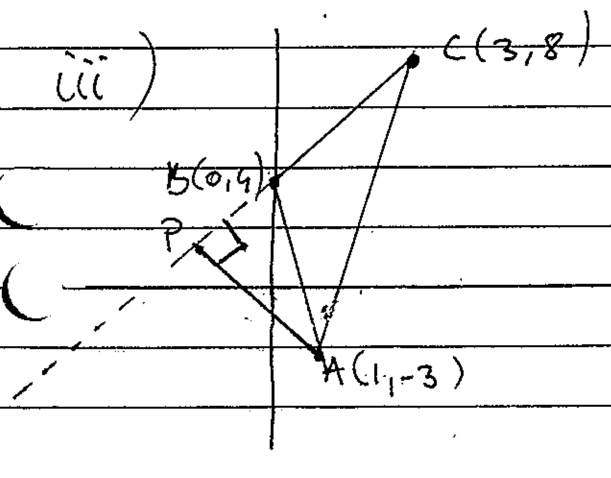
$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{25}$$

$$= 5$$

$$\therefore \; d(A, BC) = d(B, C) = 5$$

iii)



$$A = \frac{1}{2}bh$$

$$= \frac{1}{2}\times BC \times AP$$

$$= \frac{1}{2}\times 5 \times 5$$

$$= 12.5u^2$$

b) $y = (x+2)e^{-x}$

i) x - intercepts: sub $y=0$

$(x+2)e^{-x} = 0$ only if $x+2=0$

$\therefore x = -2$

y - intercepts: sub $x = 0$

$y = (0+2)e^{0}$

$\therefore y = 2$

ii) $y' = vu' + uv'$, $y = (x+2)e^{-x}$

$= e^{-x} - e^{-x}(x+2)$ $\quad u = x+2 \quad u' = 1$

$= e^{-x}(1-x-2)$ $\quad v = e^{-x} \quad v' = -e^{-x}$

$= -e^{-x}(x+1)$

$y' = 0$ only if $x+1 = 0$

$\therefore$ at $x = -1$ there is a stat. pt.

For $x = -1$, $y = e$

x	-2	-1	0
y'	e^2	0	-1
slope	/	—	\

$\therefore$ There is a max. turning point at $(-1, e)$

iii) $y'' = xe^{-x}$ $\quad y'' = 0$ only if $x = 0$

For $x = 0$, $y = 2$

x	-1	0	1
y''	$-e$	0	$1/e$
conc.	$\cap$	$\cdot$	$\cup$

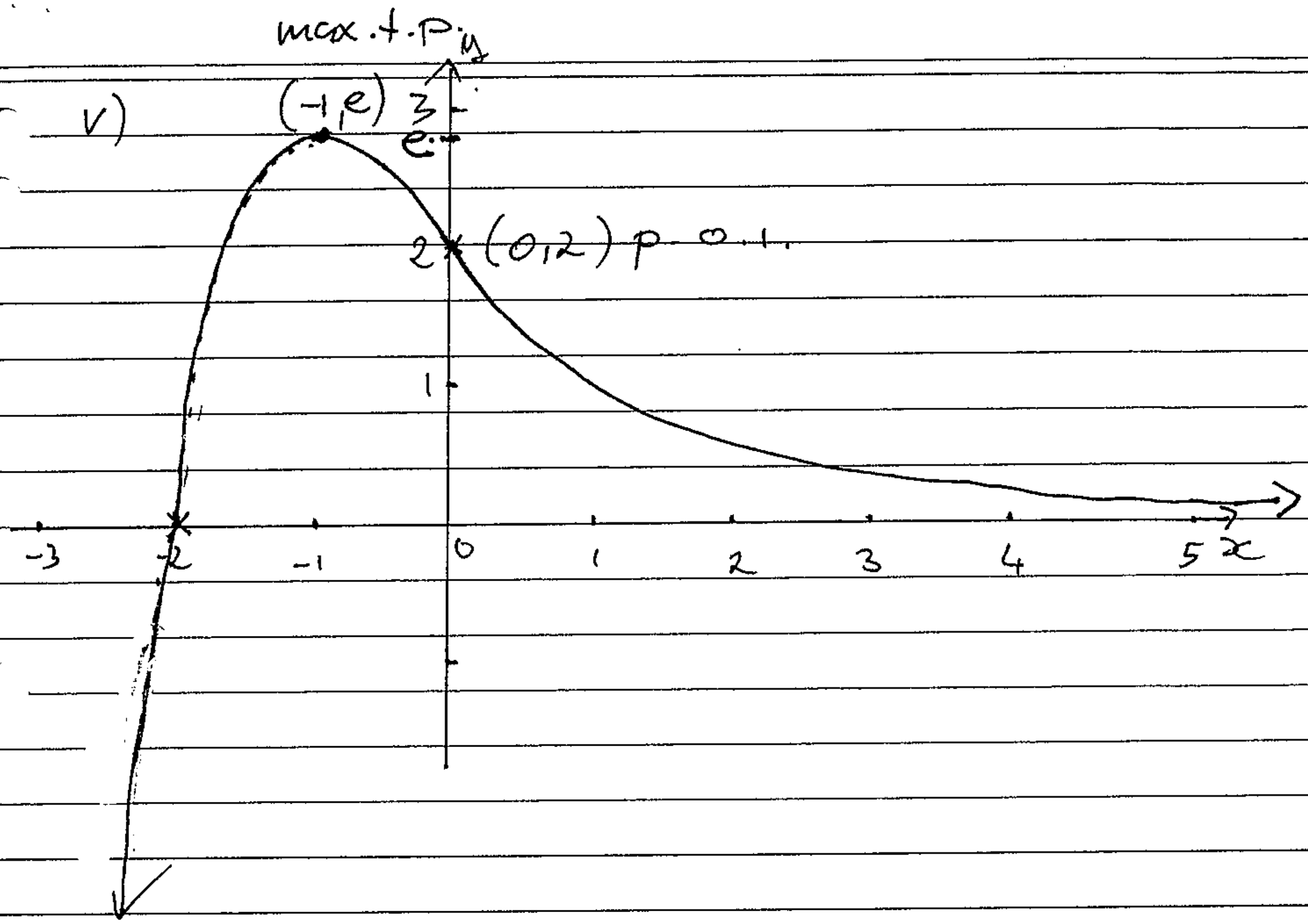
$\therefore$ There is a point of inflexion at $(0, 2)$.

iv) As $x \to \infty$, $y \to 0$ since the denominator of $\frac{x+2}{e^x}$ is growing much faster than its numerator

As $x \to -\infty$, $y \to -\infty$

Since $x+2$ will be a negative infin. big number and e^{-x} will be positive so their product is negative.

v)



QUESTION 15

a) $y = -x^3 + 2x^2 + 3x \ldots (1)$ $\qquad y = -x + 8 \ldots (2)$

i) $(1) = (2)$

$\therefore -x^3 + 2x^2 + 3x = -x + 8$

$-x^3 + 2x^2 + 4x - 8 = 0$

$-x^2(x-2) + 4(x-2) = 0$

$(x-2)(4-x^2) = 0$

$-(x-2)(x^2-4) = 0$

$-(x-2)(x-2)(x+2) = 0$

$\therefore -(x-2)^2(x+2) = 0$

$\therefore x - 2 = 0$ or $x + 2 = 0$

$x = 2$ and $x = -2$

ii) Area $= \int_{-2}^{2} (\text{line} - \text{cubic})\,dx$

$= \int_{-2}^{2} \left[-x + 8 - (-x^3 + 2x^2 + 3x)\right] dx$

$= \int_{-2}^{2} (-x + 8 + x^3 - 2x^2 - 3x)\,dx$

$= \int_{-2}^{2} (x^3 - 2x^2 - 4x + 8)\,dx$

$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - 2x^2 + 8x\right]_{-2}^{2}$

$= \left(\frac{2^4}{4} - \frac{2\times 2^3}{3} - 2\times 2^2 + 8\times 2\right) - \left(\frac{(-2)^4}{4} - \frac{2\times(-2)^3}{3} - 2\times(-2)^2 + 8\times(-2)\right)$

$= \left(\cancel{4} - \frac{16}{3} - \cancel{8} + 16 - \cancel{4} - \frac{16}{3} + \cancel{8} + 16\right)$

$= 32 - \frac{32}{3}$

$= 21.33u^2$

b) $F = F_0 e^{kt}$

i) $\therefore \frac{dF}{dt} = F_0 \times k e^{kt}$

$= k \underbrace{F_0 e^{kt}}_{F}$

$= kF$

$\therefore F = F_0 e^{kt}$ satisfies the given diff. equation

ii) When $t = 0$, $F = F_0$

$t = 3$, $F = 3F_0$

$\therefore$ For $t=2$, $3\cancel{F_0} = \cancel{F_0} e^{k \times 2}$

$\therefore e^{2k} = 3$ / ln

$\ln e^{2k} = \ln 3$

$2k = \ln 3$

$\therefore k = \frac{\ln 3}{2}$

$= 0.549306$ (6 d.p.)

iii) When $t = 6$, $F = 21000$

$\therefore 21000 = F_0 e^{0.549306 \times 6}$

$\therefore F_0 = 777.78$ (2d.p)

≈ 778 fish (777 accepted as well)

c)i) $y = x^2 \ln x$ $\qquad u = x^2 \quad u' = 2x$

$v = \ln x \quad v' = \frac{1}{x}$

$y' = vu' + uv'$

$= 2x \ln x + \frac{1}{x} \times x^2$

$= 2x \ln x + x$ $\qquad \therefore \frac{d}{dx}(x^2 \ln x) = 2x \ln x + x$

ii) $\therefore \int (2x \ln x + x)\,dx = x^2 \ln x + C_1$

$\int 2x \ln x\,dx + \int x\,dx = x^2 \ln x + C_1$

$\therefore \int 2x \ln x\,dx = x^2 \ln x - \int x\,dx + C_1$

$\int 2x \ln x\,dx = x^2 \ln x - \frac{x^2}{2} + C_2$

$\therefore \int x \ln x\,dx = \frac{x^2 \ln x}{2} - \frac{x^2}{4} + C_2$

d) $R = \frac{s^2}{100} + 40s + 5476$

i) $C = R \times t$, t = time , $t = \frac{d}{s}$

$C = \left(\frac{s^2}{100} + 40s + 5476\right) \times t$ $\quad = \frac{300}{s}$

$= \left(\frac{s^2}{100} + 40s + 5476\right) \times \frac{300}{s}$

$= 300\left(\frac{s}{100} + 40 + \frac{5476}{s}\right)$

ii) $C' = \frac{1}{100} - \frac{5476}{s^2}$

$C' = 0$, $\frac{1}{100} - \frac{5476}{s^2} = 0 \quad / \times 100s^2$

$s^2 - 547600 = 0$

$\therefore s^2 = 547600$

$s = 740$ km/h

Show that it is min. t. p.

$C'' = \frac{10952}{s^3} > 0$ for all s

$\therefore$ $s = 740$ km/h is the speed that will minimise the cost of the flight.

iii) For $s = 740$ km/h

$C = 300\left(\frac{740}{100} + 40 + \frac{5476}{740}\right)$

$= 16440$ cents

$= \$164.40$

QUESTION 16

a) $R = 3\%$ p.a $\qquad M = \$200$

$= 0.0025$ p. month

i) 1st install. will accumulate to $\$200 \times (1+0.0025)^n$

2nd install. will accumulate to $\$200 \times (1+0.0025)^{n-1}$

$\vdots$

nth install. will accumulate to $\$200 \times (1+0.025)$

After n months, $A_n = 200 \times 1.0025 + \dots + 200 \times 1.0025^n$

GP with $a = 200 \times 1.0025 \qquad r = 1.0025$

$$\therefore A_n = \frac{200 \times 1.0025(1.0025^n - 1)}{1.0025 - 1}$$

$$= 80200(1.0025^n - 1)$$

ii) $n = 9 \times 12$

$= 108$ months

$$A_{108} = 80200\,(1.0025^{108} - 1)$$

$$= \$24\,823.76$$

iii) $A_n = 15\,000 \quad , \; n = ?$

$$15000 = 80200\,(1.0025^n - 1)$$

$$1.0025^n - 1 = 0.187032$$

$$1.0025^n = 1.187032$$

$$\therefore n = \frac{\ln(1.187032)}{\ln 1.0025}$$

$$n \approx 68.67$$

$\therefore n \approx 69$ (rounded to the nearest month)

iv) $n = 4 \times 12$
$= 48$ months

$A_{48} = 20000$ $M = ?$

$$20000 = \frac{M \times 1.0025\,(1.0025^{48} - 1)}{1.0025 - 1}$$

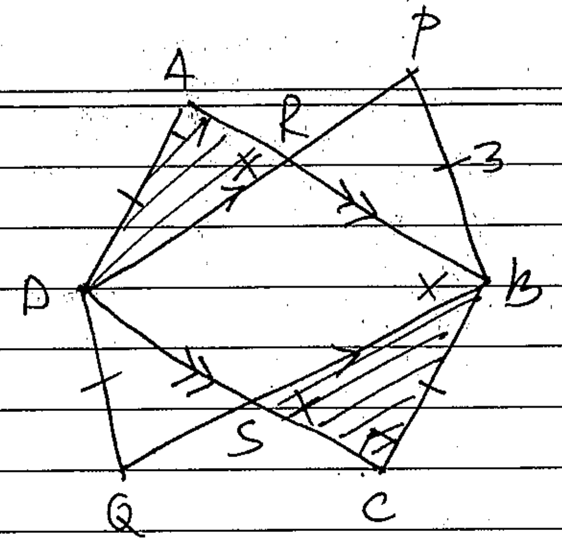
$$M \times 1.0025 = \frac{20000 \times 0.0025}{1.0025^{48} - 1}$$

$M \times 1.0025 \approx 392.69$ (2 d.p.)

$\therefore M \approx 391.71$ (2 d.p.)

New monthly installment would have to be $391.71.

b)



i) $\angle ARD = \angle ABQ$ (correspond. angles on // line ($DP \parallel QB$) are equal)

$\angle ABQ = \angle BSC$ (alternate angles on // lines ($AB \parallel DC$) are equal)

$\therefore \angle ARD = \angle BSC$

ii) $\angle DAR = \angle BCS$ (given, angles in a rectangle $= 90°$)

$AD = BC$ (given)

$\angle ARD = \angle BSC$ (proved above)

$\therefore \Delta ADR \equiv \Delta CBS$ (AAS)

iii) In ΔADR and ΔPRB

$\angle ARD = \angle PRB$ (vertically opp. $\angle$s are equal)

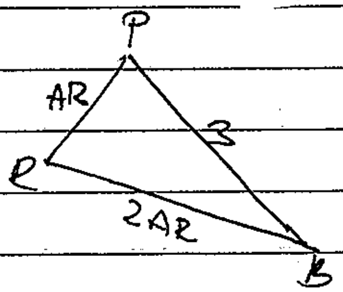
$\angle DAR = \angle BPR$ (given, angles of a rectangle)

$AD = PB$

$\therefore \Delta ADR \equiv \Delta PBR$ (AAS)

$\therefore AR = PR$ matching sides of congo. Δs

$\frac{AR}{RB} = \frac{1}{2}$ $\therefore RB = 2AR$



$(2AR)^2 = AR^2 - 3^2$

$4AR^2 = AR^2 - 3^2$

$3AR^2 = 9$

$AR^2 = 3$

$\therefore AR = \sqrt{3}$

iv) $A_{OCTAGON} = A_{RECTANGLE} + 2 \times A_{TRIANGLE}$

$$= 3 \times (\sqrt{3} + 2\sqrt{3}) + \cancel{2} \times \sqrt{3} \times 3 \times \frac{1}{\cancel{2}}$$

$$= 9\sqrt{3} + 3\sqrt{3}$$

$$= 12\sqrt{3} \cdot u^2$$