Question 1: (Start a new Booklet)

(i) Given that: $I = \dfrac{3E}{R+r}$, and E = 1.8, R = 1.5, r = 2.5, find I.

(ii) For the points A (2,-3), B(-3,1) find

(a) the gradient of the line AB

(b) the angle of inclination of the line AB, to the positive direction of the x axis. **(Answer to nearest degree)**

(iii) Simplify: $\dfrac{x^2+x-12}{x^2-9}$

(iv) Show that: $\dfrac{1}{3-\sqrt{2}}+\dfrac{1}{3+\sqrt{2}}$ is a rational number.

(v) Solve: $|3x-7| = 8$

(vi) Write $0.0\dot{4}\dot{1}$ in simplest fractional form.

Question 2: (Start a new booklet)

(i) The quadratic equation $3x^2 + 4x - 3 = 0$ has roots α and β.
Without finding α and β, evaluate the following:

(a) $\alpha+\beta$

(b) $\alpha\beta$

(c) $\alpha^2+\beta^2$

(ii) Write in simplest surd form,

(a) $\dfrac{\sin 60^\circ}{\cos 60^\circ}$

(b) $\cos 225^\circ$

(c) $\lim\limits_{x\to 3} \dfrac{x^3-27}{x-3}$

(iii) Prove that the line $4x + 3y + 18 = 0$ is a tangent to the circle, centre (-1,2) and radius 4 units.

Question 3: (Start a new booklet)

(i) Solve : $4 \tan\theta = -3$, for $0^\circ \le \theta \le 360^\circ$

(ii) For what values of k will the equation $x^2 - (k+4)x + (7+k) = 0$ have equal roots.

(iii) Find the indefinite integral of:

(a) $\dfrac{1}{x^3}$

(b) $(5x-2)^3$

(iv) Differentiate with respect to x,

(a) $y = \dfrac{(x+5)}{(1+3x)}$

(b) $y = \log_e(4-3x)$

Question 4: (Start a new booklet)

(i) Sketch: $y = |x-3|$, showing all main features.

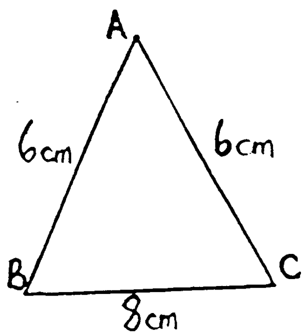
(ii) Solve: $\dfrac{9-Y}{3} > 2 - \dfrac{Y}{5}$

(iii) In the diagram **(not drawn to scale),**

AB = 6 cm

AC = 6 cm

BC = 8 cm



(a) Find $\angle CAB$ to the nearest minute.

(b) Hence, if D is on AC, such that DC = 2cm, find the area of ΔBCD .

(iv) A ship sails from a port on a bearing of $235°T$ for 150 nautical miles. It then turns and sails on a bearing of $120°T$ until it reaches its destination due south of its original position. Calculate the distance of the ship from the port correct to three significant figures.

Question 5: **(Start a new Booklet)**

(i) Find the size of $\angle EDC$, giving precise reasons, given $AB \parallel CD$.

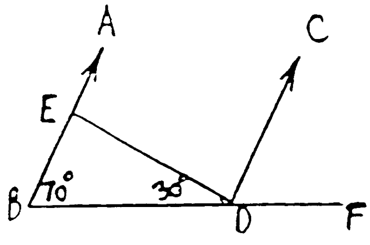


Diagram not to scale

(ii) Given, $ST \parallel QR$

ST = 9 cm

QR = 15 cm

TR = 3 cm

(a) Copy this diagram into your Examination Booklet.

(b) Prove: $\Delta PST \parallel \Delta PQR$.
give precise reasons

(c) Hence, find the length of PT.
give precise reasons

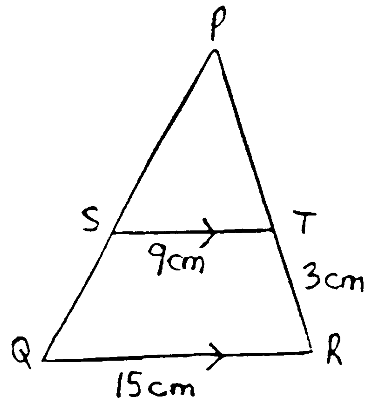


Diagram not to scale

(iii) ABCD is a parallelogram such that

AP = PQ = QC

Copy this diagram into your Examination Booklet.

Prove that DP = BQ.
give precise reasons

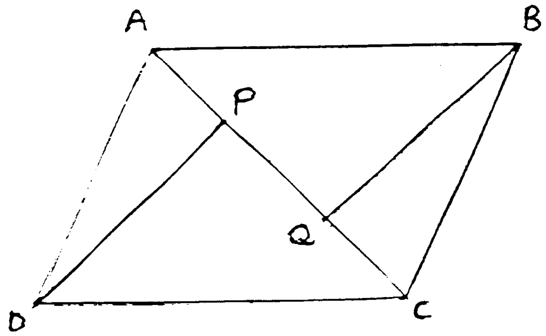


Diagram not to scale

Question 6: (Start a new Booklet)

(i) Sketch: $y = e^{-x}$, stating its domain and range.

(ii) Find the area between $y = e^{-x}$, the x-axis and the lines x = -1 and x = 1. Give the answer correct to three significant figures

(iii) Solve, $x^4 - 3x^2 + 2 = 0$.

(iv) From a set of 40 discs numbered 1 to 40, one disc is selected at random. What is the probability that the number is a multiple of either 5 or 7?

(v) In a certain arithmetic progression, the fourth term is 7, and the fourteenth term is 32.

(a) Find the first term and the common difference.

(b) Calculate the sum of the first 50 terms.

Question 7: (Start a new Booklet)

(i) Evaluate: $\sum_{n=1}^{\infty} 4 \times (\frac{1}{3})^{n-1}$

(ii) For $y = 2x^3 - 24x$

(a) determine the stationary points

(b) determine the nature of these stationary points

(c) determine points of inflexion

(d) sketch showing all important features.

Question 8: **(Start a new booklet)**

(i) At a seaside holiday resort, a fisherman is selling fish. He claims that all the fish are fresh, but in actual fact, only 70% of the fish are fresh.

(a) A woman chooses **TWO** fish at random. What is the probability that both fish are fresh?

(b) A tourist chooses **THREE** fish at random. What is the probability that at least one of the fish is fresh?

(ii) Find the equation of the tangent at the point $\log_e 2$ on the curve y = $\log_e(x+1)$.

(iii) For the parabola, $x^2 + 32y = 0$, find:

(a) the focal length

(b) the co-ordinates of the focus

(iv) Derive the equation of the locus P(x,y) which is equidistant from A(3,1) and B(2,5).

Question 9: **(Start a new Booklet)**

(i) Sketch: $y \le 3 - x^2$, showing all main features.

(ii) Evaluate: $\int_1^4 (x^2 + 1)\, dx.$

(iii) Use Simpson's Rule with 2 equal subintervals to find an approximation for the area between the curve $y = x^2 + 1$, the x-axis and x = 1 and x = 4.

(iv) ABCDE is a pentagon of fixed perimeter P cm. Its shape is such that ABE is an equilateral triangle and BCDE is a rectangle. If the length of AB is x cm:

(a) Show that the length BC is $\frac{P-3x}{2} cm$

(b) Show that the area of the pentagon is given by:

$$A = \frac{1}{4}(2Px - (6 - \sqrt{3})x^2)$$

(c) Find the value of $\frac{P}{x}$ for which the area of the pentagon is a maximum.

Question 10: (Start a new Booklet)

(i) Sketch the area between the curve $y = \sqrt{x}$, the y-axis and $y = 1$ and $y = 2$.

(ii) Find the volume of the solid obtained by rotating the area in (i) about the y-axis.

(iii) Find an expression for the volume of the solid obtained by rotating the area in (i) about the x-axis. **DO NOT EVALUATE.**

(iv) (a) Differentiate: e^{x^2}.

(b) Hence, or otherwise, find : $\int x\, e^{x^2}\, dx$.

(v) A loan of $6000 is to be repaid by equal annual instalments.
Compound Interest, calculated yearly is 9% p.a. If the annual instalment is $P, then:

$A_1 = \$(6000 \times 1.09 - P)$ is the amount owing at the end of **ONE** year.

$A_2 = \$\left(6000 \times (1.09)^2 - P(1 + 1.09)\right)$ is the amount owing at the end of **TWO** years.

(a) Write an expression for A_n, the amount owing at the end of n years.

(b) If the $6000 loan (including interest charges) is exactly repaid at the end of n years, write an expression for the annual instalment, $P.

(c) Calculate the annual instalment, $P, when n = 25.

END OF PAPER

BARKER 1990

VI UNIT EXAM SOLUTIONS

Q1

(i) $I = \frac{3(1.8)}{1.5+2.5}$

$= 1.35$

(ii) a) $m_{AB} = \frac{1+3}{-3-2} = -4/5$

b) $m = -4/5$

$\theta = 141°$

(iii) $\frac{x^2+x-12}{x^2-9} = \frac{(x-3)(x+4)}{(x-3)(x+3)}$

$= \frac{x+4}{x+3}$

(iv) $\frac{1}{3-\sqrt{2}} + \frac{1}{3+\sqrt{2}}$

$= \frac{3+\sqrt{2}}{9-2} + \frac{3-\sqrt{2}}{9-2} = \frac{6}{7}$

(v) $|3x-7| = 8$

$3x-7=8$ $\quad 3x-7=-8$

$3x=15$ $\quad 3x=-1$

$x=5$ $\quad x=-1/3$

(vi) $x = 0.0\dot{1}\dot{1}$

$100x = 4.1\dot{1}\dot{1}$

$99x = 4.1$

$x = \frac{41}{990}$

Q2

(i) $3x^2+4x-3=0$

a) $\alpha+\beta = -4/3$

b) $\alpha\beta = -3/3 = -1$

c) $\alpha^2+\beta^2 = (-4/3)^2 - 2(-1)$

$= 16/9 + 2$

$= 3\frac{7}{9}$ $(34/9)$

(ii) a) $\frac{\sin 60}{\cos 60} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$

b) $\cos 225° = -1/\sqrt{2}$

c) $\lim_{x\to3} \frac{x^3-27}{x-3} = \lim_{x\to3} \frac{(x-3)(x^2+3x+9)}{x-3} = 27$

(ii) $d = \frac{|4(-1)+3(2)+15|}{\sqrt{4^2+3^2}}$

$= \frac{|-4+6+15|}{5} = \frac{20}{5} = 4$

∴ d of line from centre = 4

Q3

(i) $4\tan\theta = -3$

$\tan\theta = -3/4$

$\theta = 143°8', 323°8'$

(ii) $x^2-(k+4)x+(7+k)=0$

for equal roots $\Delta=0$

$(k+4)^2 - 4(7+k) = 0$

$k^2+8k+16-28-4k=0$

$k^2+4k-12=0$

$(k+6)(k-2)=0$

$k=2$ or -6

(iii) a) $\int \frac{1}{x^3}\,dx = \frac{-1}{2x^2} + C$

b) $\int (5x-2)^3\,dx = \frac{(5x-2)^4}{20} + C$

(iv) a) $y' = \frac{(1+3x).1-(x+5).3}{(1+3x)^2}$

$= \frac{1+3x-3x-15}{(1+3x)^2}$

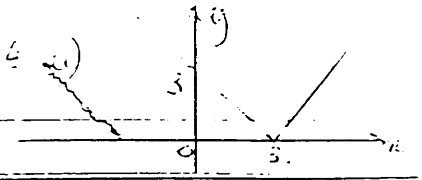
$= -14/(1+3x)^2$

b) $y' = \frac{1}{4-3x} \cdot -3$

$= \frac{-3}{4-3x}$

Q4

(i)



(ii) $\frac{y-7}{3} > 2 - \frac{y}{5}$

$5y-35 > 30-3y$

$15 > 2y$

$y < 7\frac{1}{2}$

(iii) a) $\cos C = \frac{6^2+6^2-8^2}{2\cdot6\cdot6}$

$C = 83°37'$

b) ∴ $\angle BCA = 45.2°$ (cosine)

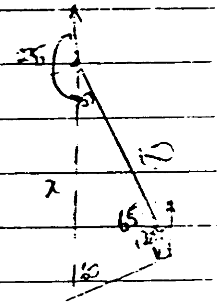
So $A_{\triangle ABC} = \frac{1}{2}\cdot 2\cdot 8 \sin 45.2°$

$= 5.46\ cm^2$

(iv) $\frac{x}{\sin 65} = \frac{150}{\sin 60}$

$x = \frac{150\sin 65°}{\sin 60}$

$= 156.977$



Q5

(i) $\angle BOC = 110°$ (co-int ∠s ...)

∴ $\angle BDC = 110 - 30°$ (adj ∠s)

$= 80°$

(ii) $\angle P$ common

$\angle PST = \angle PQR$ (corresp ∠s)

$\angle PTS = \angle PRQ$ (" ")

∴ $\triangle PST \parallel\parallel \triangle PQR$ (AAA)

∴ $\frac{PT}{9} = \frac{PT+3}{15}$

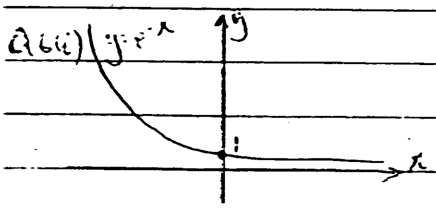
$15PT = 9PT + 27$

$6PT = 27$

$PT = 4\frac{1}{2}$

(iii) ∠OAP = ∠ACB (alt ∠s, ...)
AD = BC (opp sides of ...)
AP = BC (given).
∴ △APD ≡ △CBA (SAS)
∴ DP = BA (corresp. sides of cong. △s).

Q6(ii) $y = e^{-x}$



D: $x \in R$ R: $y > 0$

(iii) $A = \int_{-1}^{1} e^{-x}\,dx$
$= [-e^{-x}]_{-1}^{1}$
$= (-e^{-1} + e^{1})$
$= 2.35$

(iv) $x^4 - 3x^2 + 2 = 0$
$(x^2-2)(x^2-1) = 0$
$x = \pm\sqrt{2}, \pm 1$

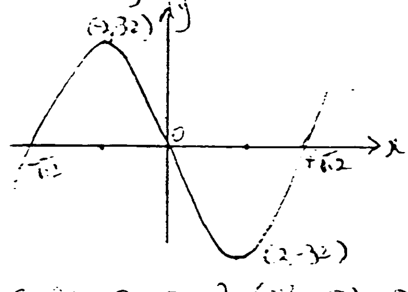
(ii) P(... 5, 7) $= \frac{12}{40} = 3/10$

(v) $T_4 = a + 3d = 7$
$T_{14} = a + 13d = 32$
∴ $10d = 25$
$d = 2\frac{1}{2}$
∴ $a = 7 - 3(2\frac{1}{2})$
$= -\frac{1}{2}$

a) $a = -\frac{1}{2}$ $d = 2\frac{1}{2}$
b) $S_{50} = \frac{50}{2}[-1 + 49(2\frac{1}{2})]$
$= 3037\frac{1}{2}$.

Q7
(i) $\sum_{n=1}^{\infty} 4 \times (\frac{1}{3})^{n-1}$
$a = 4$ $r = \frac{1}{3}$
$S_\infty = \frac{4}{1-\frac{1}{3}} = 4 \times \frac{3}{2} = 6$

(ii) $y = 2x^3 - 24x$.
a) $y' = 6x^2 - 24$
∴ $6x^2 - 24 = 0$ for stat pt.
$x^2 - 4 = 0$
$x = \pm 2$.
b) $y'' = 12x$ => $x=2$, $y'' = 24$ => min
$x = -2$, $y'' = -24$ => max
$x = 2$ => $y = -32$ ∴ min $(2, -32)$
$x = -2$ => $y = 32$ ∴ max $(-2, 32)$
c) $y'' = 0$ => $12x = 0$
∴ $x = 0$
check -1: $y'' = -12$ } Δ sign
1: $y'' = 12$ } inflex.
$x = 0$ => $y = 0$ ∴ inflex $(0,0)$.



$2x^3 - 24x = 0$ => $2x(x^2 - 12) = 0$
$x = 0, \pm\sqrt{12}$

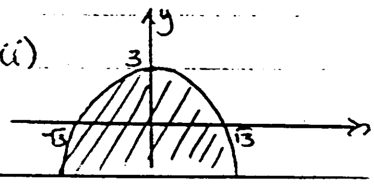
Q8. (i) a) P(FRESH) $= 0.7 \times 0.7$
$= 0.49$
b) P(at least 1) $= 1 - P(\text{none})$
$= 1 - (0.3)^3$
$= 0.973$.

(i) $y = \ln(x+1)$
$y' = \frac{1}{x+1}$
at $(1, \ln 2)$ $m = \frac{1}{1+1} = \frac{1}{2}$
∴ $y - \ln 2 = \frac{1}{2}(x-1)$
$x - 2y + 2\ln 2 - 1 = 0$

(iii) $x^2 + 32y = 0$ => $x^2 = -32y$.
∴ $4a = -32$
$a = -8$.
a) Focal length $= -8$.
b) focus $(0, -8)$ (as vertex at $(0,0)$).

(iv) $D_{PA} = \sqrt{(x-3)^2 + (y-1)^2}$
$D_{PB} = \sqrt{(x-2)^2 + (y-5)^2}$
$D_{PA} = D_{PB}$ => $D_{PA}^2 = D_{PB}^2$
ie $x^2 - 6x + 9 + y^2 - 2y + 1 = x^2 - 4x + 4 + y^2 - 10y + 25$
$2x - 8y + 19 = 0$.

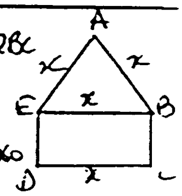
Q9 (i)



(ii) $\int_1^4 (x^2+1)\,dx = [\frac{x^3}{3} + x]_1^4$
$= \frac{64}{3} + 4 - \frac{1}{3} - 1$
$= 24$

(iii) $A \doteq \frac{2\frac{1}{2}}{3}[2 + 17 + 4(2\frac{1}{2}^2 + 1)]$
$= 40$

(iv) a) $P = x + x + x + 2BC$
∴ $BC = \frac{P - 3x}{2}$



b) $A = BC.x + \frac{1}{2}x.x.\sin 60$
$= \frac{P-3x}{2}.x + \frac{x^2}{2}.\frac{\sqrt{3}}{2}$
$= \frac{2Px - 6x^2 + \sqrt{3}x^2}{4} = \frac{1}{4}(2Px - (6-\sqrt{3}$

$\frac{dA}{dx} = \frac{1}{4}\left(2P - 2(6-\sqrt{3})x\right)$

so $2P - 2(6-\sqrt{3})x = 0$

$P - (6-\sqrt{3})x = 0$

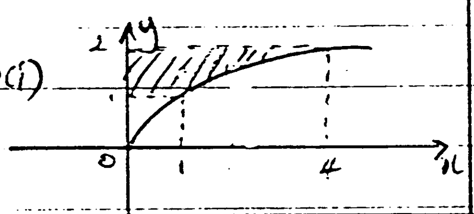
$P/x = 6-\sqrt{3}$.

$\frac{d^2A}{dx^2} = \frac{-2(6-\sqrt{3})}{4} < 0$

$\therefore$ concave up

so $P/x = 6-\sqrt{3}$ is a max value for area to be max

Q10(i)



(ii) $V = \pi\int_1^2 y^4\,dy$

$= \pi\left[\frac{y^5}{5}\right]_1^2$

$= \pi\left(\frac{32}{5} - \frac{1}{5}\right)$

$= \frac{31\pi}{5}\ u^3$.

(iii) $V = 4\pi - \pi + 4\pi.3 - \pi\int_1^4 x\,dx$

$= 3\pi + 12\pi - \pi\int_1^4 x\,dx$

$= 15\pi - \pi\int_1^4 x\,dx$.

(iv) a) $\frac{d}{dx}e^{x^2} = 2xe^{x^2}$

(b) $\int xe^{x^2}dx = \frac{1}{2}\int 2xe^{x^2}dx$

$= \frac{1}{2}e^{x^2} + c$

(v) a) $A_n = \$(6000(1.09)^n - P(1 + 1.09 + \dots + (1.09)^{n-1}))$

b) $P = \frac{6000(1.09)^n}{1 + 1.09 + \dots + (1.09)^{n-1}}$

c) $P = \frac{6000(1.09)^{25}}{\frac{(1.09)^{25}-1}{1.09-1}}$

$= \$610.84$