Barker College

Mathematics

Staff Involved:

- AES*
- TE*
- BJR
- JGD
- RMH
- GDH
- AJD
- MRB
- DOK
- BHC
- LJP
- CFR

200 copies

AM WEDNESDAY 8 AUGUST

Total marks (120)

- Attempt Questions 1 – 10

- All questions are of equal value

General Instructions

- Reading time – 5 minutes

- Working time – 3 hours

- Write using blue or black pen

- Make sure your Barker Student Number is on ALL pages

- Board-approved calculators may be used

- A table of standard integrals is provided on page 13

- ALL necessary working should be shown in every question

Total marks (120)
Attempt Questions 1 – 10
ALL questions are of equal value

Answer each question on a SEPARATE sheet of paper

		Marks

Question 1. [12 marks]   **[START A NEW PAGE]**

(a)   Evaluate $11 + |-7 + 3|$ **1**

(b)   Write down the exact value of $\sin 240°$ **1**

(c)   Write down the equation of a line perpendicular to $3x - 2y + 1 = 0$ **2**

(d)   Express $0.2\dot{5}$ in the form $\dfrac{p}{q}$ where p and q are whole numbers.   **2**

(e)   Solve for x if $(2x - 1)^2 = 4$ **2**

(f)   Simplify $\dfrac{6 \pm \sqrt{24}}{8}$. Leave your answer as a surd.   **2**

(g)   If $a^{\frac{1}{4}} = 3$ find $\sqrt[4]{a^{-1}}$ **2**

Question 2 [12 marks]   [START A NEW PAGE]

(a) Sketch the function $y = f(x)$

where $f(x) = \begin{cases} 3 & \text{for } x \leq 3 \\ x & \text{for } x > 3 \end{cases}$ 1

(b) Let $g(x) = \dfrac{1}{\sqrt{x+2}}$
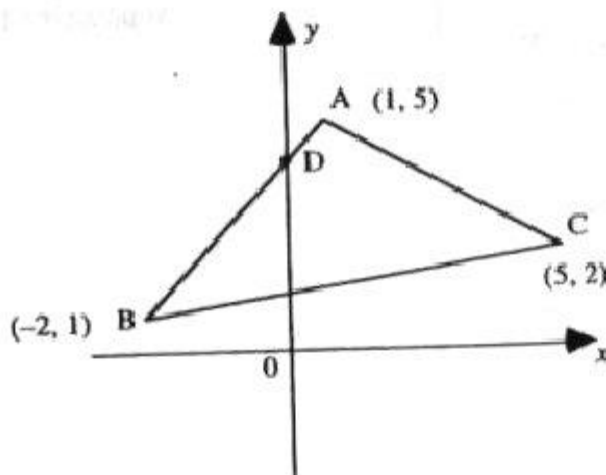
(i) Find $g(7)$, giving your answer with a rational denominator.   1

(ii) What is the domain of $g(x)$?   1

(c)

NOT DRAWN TO SCALE

The triangle ABC has vertices A $(1, 5)$, B $(-2, 1)$ and C $(5, 2)$ as shown.
D lies on AB and has coordinates $(0, a)$.

(i) Show that the gradient of AB is $\dfrac{4}{3}$.   1

(ii) Show that $\angle BAC$ is a right angle.   2

(iii) Show that the equation of the line AB is given by $4x - 3y + 11 = 0$.   2

(iv) Determine the coordinates of D.   1

(v) Given that the length of AB is 5 units, show that $\triangle ABC$ is isosceles.   2
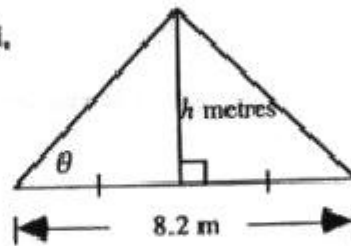
(vi) Find the area of $\triangle ABC$.   1

Question 3 [12 marks]   [START A NEW PAGE]

(a)   Write down B, correct to the nearest whole number, if

   $\cos B° = -0.7$ and $0 \leq B \leq 180$

1

(b)   A house gable is to be constructed,
        as shown in the diagram with
        pitch of angleθ.
        The pitch, θ, of this gable is to
        be $25°$.

NOT TO SCALE

h metres

θ

8.2 m
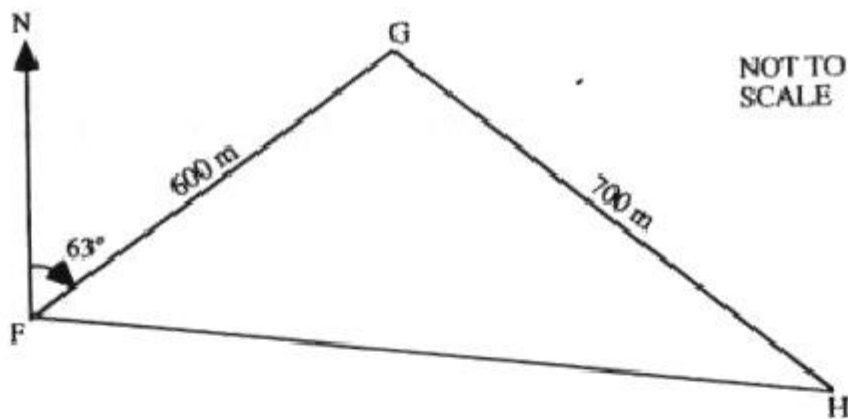
Calculate the height h metres, to 1 decimal place, of the house gable if the
width of the gable is to be 8.2 m and the gable is to be built symmetrically.

2

(c)   Mischa is setting up part of an orienteering course.
        She follows the course shown in the diagram below.

N

G

NOT TO SCALE

600 m

700 m

63°

F

H

(i)   If $\angle FGH$ is $108°$, show that the distance FH to the nearest metre
        is 1053 metres.

2

(ii)   Hence, or otherwise, calculate the size of $\angle GFH$ to the nearest degree.

2

(iii)   If the bearing of G from F is $063°$, calculate the bearing of H from F
        to the nearest degree.

1

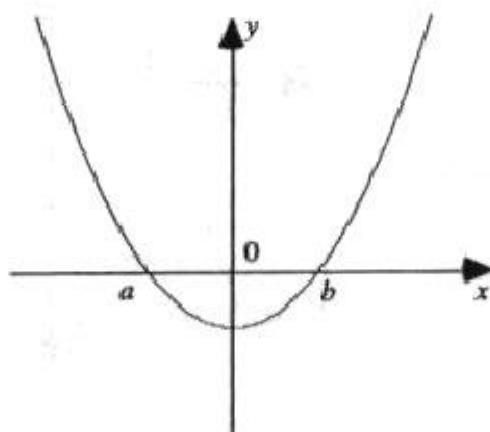(d)   Simplify $\dfrac{\sec^2 \theta \cot \theta}{\csc^3 \theta}$

2

(e)   If $x = 3 \cot \alpha$ and $y = 2 \csc \alpha$, show that $1 + \dfrac{x^2}{9} = \dfrac{y^2}{4}$

2

Question 4 **[12 marks]**   [START A NEW PAGE]

(a)   The parabola with equation $y = (x + 1)(2x - 7)$ is sketched below.

NOT TO
SCALE

(i)   Write down the values of a and b.   2

(ii)   Solve the inequation $(x + 1)(2x - 7) > 0$ 1

(iii)   Find the minimum value of y.   2

(b)   A parabola has equation $(x + 1)^2 = 12y + 12$

(i)   What is its focal length?   1

(ii)   What are the coordinates of its vertex?   1

(iii)   Draw a neat sketch of this parabola showing the coordinates
        of its vertex and focus.   1

(c)   The equation $x^2 - 6x + k = 0$ has roots α and β.

    If $\alpha = 2\beta$,

(i)   Show that $k = 8$ 2

(ii)   Find the value of $(\alpha + 1)(\beta + 1)$ 2

Question 5 [12 marks]   [START A NEW PAGE]

(a)

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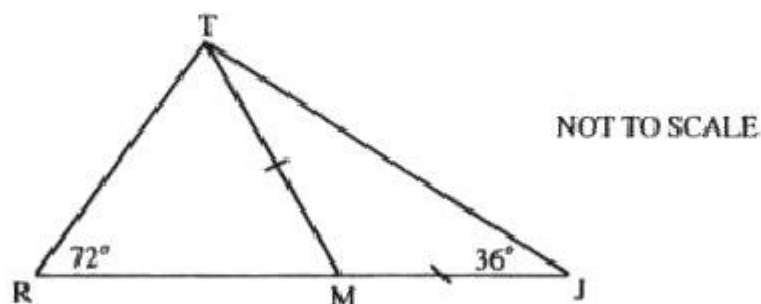
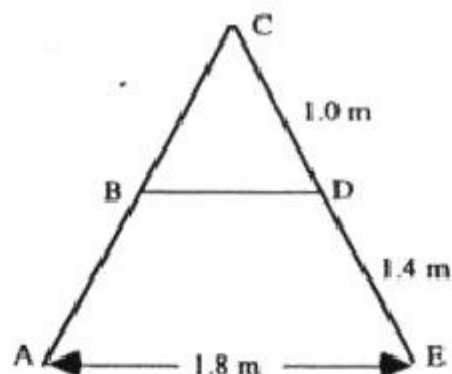Given that TM = MJ, show that triangle RTM is isosceles giving reasons in your answer.   3

(b)   The supports of a swing are constructed as in the following diagram.

(i)   If AE∥BD, prove that △ACE is similar to △BCD   2

(ii)   If AE = 1.8 m, CD = 1.0 m and DE = 1.4 m, find the length of BD. Justify your answer.   2
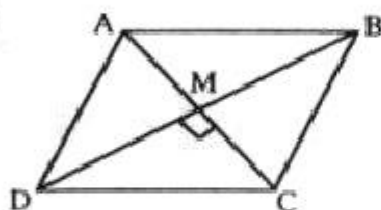

NOT TO SCALE

(c)

NOT TO SCALE

ABCD is a parallelogram whose diagonals AC and BD intersect at M. ∠DMC is a right angle.

(i)   Prove that △AMD is congruent to △CMD.   3
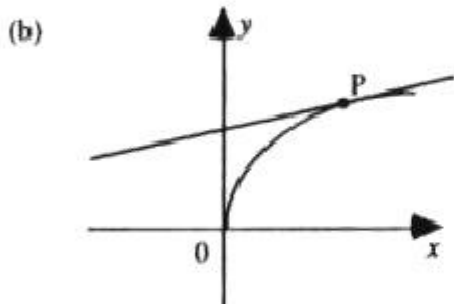
(ii)   Hence show that ABCD is a rhombus.   2

Question 6 [12 marks] [START A NEW PAGE]

(a) Differentiate $\left(4t^2 + 1\right)^3$ with respect to t. 2

(b)

A sketch of $y = 2\sqrt{x}$ is shown with a tangent drawn at P.

(i) Find $\dfrac{dy}{dx}$ 1

(ii) Show that if the gradient at P is 3, then the coordinates of P are
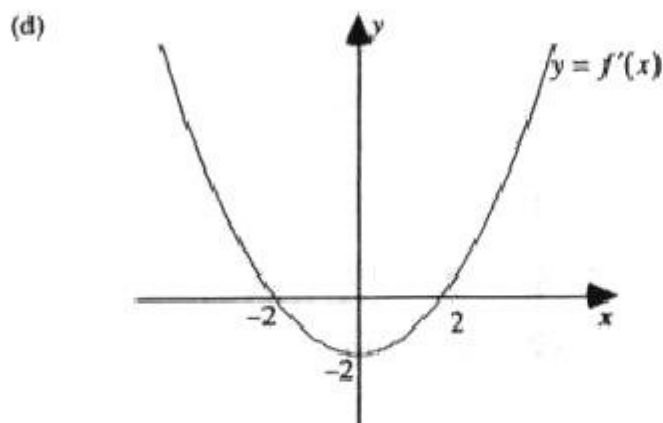
$$\left(\frac{1}{9}, \frac{2}{3}\right)$$ 2

(iii) Find the equation of the tangent at P. 2

(c) If $y = 2x^3 - 7x^2 - 3x + 1$

(i) Find $\dfrac{d^2y}{dx^2}$ 2

(ii) For what values of x is the curve of $y = 2x^3 - 7x^2 - 3x + 1$ concave up? 1

(d)

The gradient function $y = f'(x)$ has been drawn at left.

Using the information from this diagram, draw a sketch of a possible function for $y = f(x)$, given that $f(0) = -1$ 2

Question 7 **[12 marks]**   **[START A NEW PAGE]**

(a)   Evaluate $\sum_{k=1}^{10} k$ 1

(b)   A geometric series has a limiting sum of 12 and the first term is 8.
Find:

   (i)   the common ratio   2

   (ii)   the 5th term.   1

(c)   Insert two numbers between 64 and 27 so that the four numbers form a geometric
sequence.   2

(d)   A certain type of machine produces 200 articles per week. A production line starts
up with 8 such machines. At the start of each successive week one more machine
is brought into operation.

   (i)   How many articles can be manufactured in the first week?   1

   (ii)   How many articles can be manufactured in the first 2 weeks altogether?   1

   (iii)   Show that an expression for the number of articles that can be
manufactured in the first n weeks is $1500n + 100n^2$.   2

   (iv)   How many weeks will it take to manufacture sufficient articles to fill an
order for 45 000?   2

Question 8 [12 marks]   [START A NEW PAGE]

(a) A die is biased so that the face showing 1 is likely to come up with a probability of 0.5. However, the other five faces, numbered 2, 3, 4, 5, 6, are all equally likely to appear.

   (i)   Find the probability of a 6 appearing.   1

   (ii)   Find the probability of any odd number appearing.   1

(b) Eggs at a poultry farm are randomly sampled. It is found that 2% of these sampled eggs are underweight. Find the probability, correct to 3 decimal places, that in a sample of 3 eggs:

   (i)   exactly one egg is underweight   1

   (ii)   no eggs are underweight   1

   (iii) at least one egg is underweight.   1

(c)
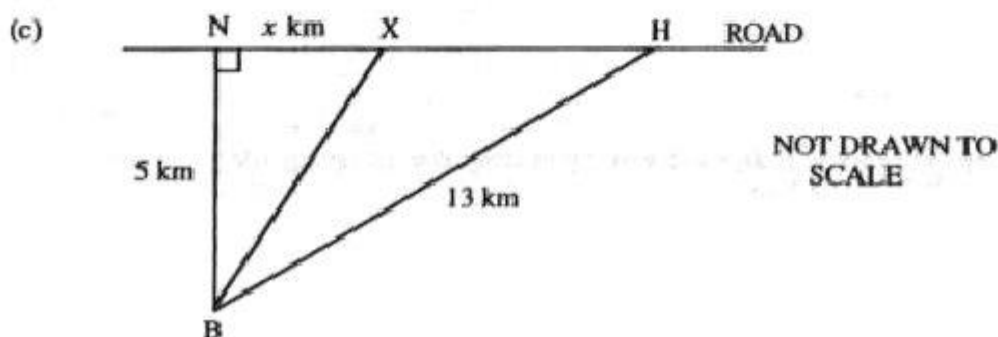
A bushwalker is in a forest at B, 5 kilometres from N on a road with a house at H. The bushwalker intends to travel directly through the forest to X which is on the road between N and H and then walk along the road to the house at H which is 13 km from B.

   (i)   If X is x km from N along the road, show that

   $$BX = \sqrt{25 + x^2} \text{ km and that}$$

   $$XH = (12 - x) \text{ km.}$$ 2

   (ii)   The bushwalker can travel at 3 km per hour through the forest and 5 km per hour on the road. The total time t hours taken to travel from B to X and then to H is given by:

   $$t = \frac{\sqrt{25 + x^2}}{3} + \frac{12 - x}{5}$$

   Find $\dfrac{dt}{dx}$.   2

   (iii) Show that the least amount of time taken to reach the house will occur when $x = 3\frac{1}{4}$.   3

Question 9 [12 marks]   [START A NEW PAGE]

(a)   Simplify $\log_4 64$ 1

(b)   Show that $\dfrac{d}{dx}\left(x^2 \log_e x\right) = x\left(2\log_e x + 1\right)$ 2

(c)   (i)   Factorise $u^2 - 6u - 16$ 1

       (ii)   Hence, or otherwise, solve for x
              $\left[\log_2 x\right]^2 - 6\left[\log_2 x\right] - 16 = 0$ 3

(d)   The growth of a bacterial culture is given by $N = 100e^{0.5t}$ where t is time in
       hours and N is the number of bacteria present.

       (i)   Find the number of whole bacteria present at time $t = 0$ and at $t = 2$.   2

       (ii)   Write down the average rate of growth per hour during the first 2 hours.   1

       (iii) Find $\dfrac{dN}{dt}$ and hence calculate the rate of bacterial growth at the end of
              the second hour.   2

Question 10 **[12 marks]** **[START A NEW PAGE]**

(a) A sketch of $y = (x + 1)^2$ is shown below
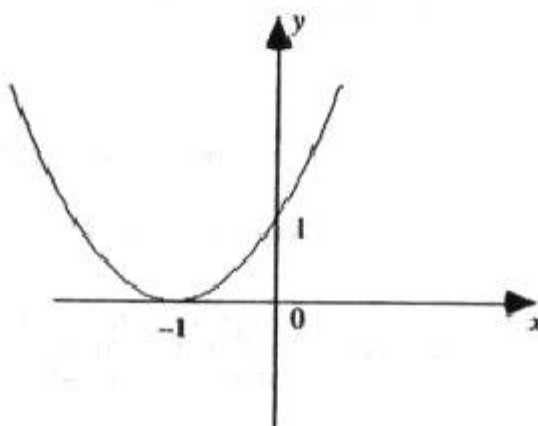
(i) Copy the sketch and shade in the area given by $\int_{-2}^{-1}(x + 1)^2 dx$ 1

(ii) State which **TWO** of the following A, B C and D are equal to

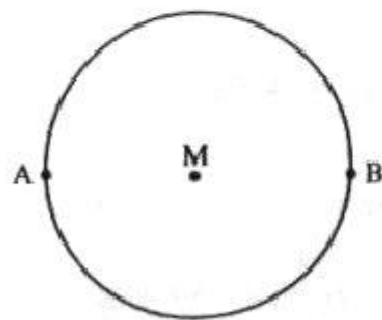$$\int_{-2}^{-1}(x + 1)^2 dx$$

(A) $-\int_{2}^{1}(x + 1)^2 dx$ (B) $-\int_{-1}^{-2}(x + 1)^2 dx$

(C) $\int_{1}^{2}(x + 1)^2 dx$ (D) $\int_{-1}^{0}(x + 1)^2 dx$ 2
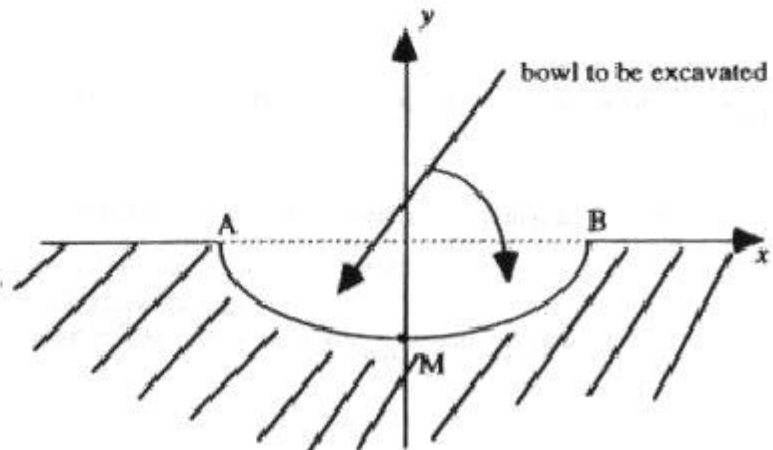
Question 10 continued over page

(b)   An engineer is required to calculate the volume of material to be excavated in the construction of a velodrome bowl whose cross sections are shown below.

PLAN (bird's eye view)

bowl to be excavated

SIDE ELEVATION OF SECTION A – B

The curve AMB is a parabola with equation given by $100y = x^2 - 625$, with units in metres and axes placed through A, B and M as shown.

(i)   Use this equation to show that the depth of the velodrome at M is 6.25 metres.   1

(ii)   Calculate the distance AB.   2

(iii)   Calculate the volume to be excavated correct to the nearest cubic metre.   6

END   OF   PAPER