Baulkham Hills High School 2004 Mathematics

Question 1

(a). Calculate $\dfrac{3.561+(0.4)^3}{88.2\times(1.35\times10^{-1})}$ correct to 2 significant figures

(b). Factorise fully $2x^3-16$.

(c). Given $g(x)=|x-3|$, for what value(s) of x is $g(x)=45$.

(d). Express $0.7\dot{4}$ as a fraction in its simplest form. Show all working.

(e). Solve $5-2x<12$.

(f). Find a and b given $\sqrt{24}+7\sqrt{54}=a\sqrt{b}$ when expressed in simplest form.

Question 2

(a). Find the equation of the tangent to $y=2e^{5x}$ at the point where $x=0$.

(b). Differentiate the following with respect to x.

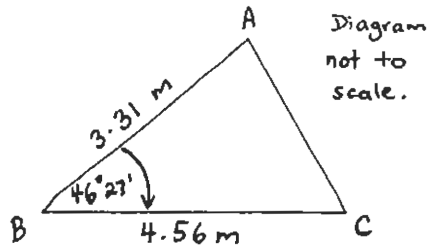
(i). $(5-8x^2)^4$

(ii). $x^2\ln(5x)$

(iii). $\dfrac{\cos 2x}{x}$

(c). (i). Find the length of AC, correct to 1 decimal place.

(ii). Calculate the area of ΔABC, correct to 1 decimal place.



Question 3

(a). Given the points $P(5,2)$, $Q(-3,8)$ and $R(11,10)$ -

(i). Find the exact length of interval QR.

(ii). Find the coordinates of S, the midpoint of QR.

(iii). Find the equation of the circle with QR as its diameter.

(iv). Find the gradient of line QR.

(v). Show that SP is perpendicular to QR.

(vi). Hence, or otherwise, find the area of ΔPQR.

(b). Solve $2\cos x=-1$ for $0\le x\le 2\pi$.

(c). Evaluate $\sum_{k=2}^{\infty}4^{-k}$.

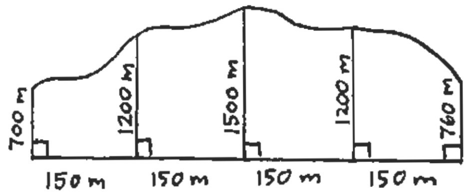
Question 4

(a). The first two terms of an arithmetic sequence are 10 and 7 respectively.

(i). Find the eighth term.

(ii). Find the sum of the first 30 terms.

(b). Prime land along a foreshore is to be reclaimed and developed as part of a housing estate. A plan of the land to be reclaimed is shown below.



Use Simpson's Rule with five function values to find an approximation of this area.

Question 4 (continued)

(c). A parabola has the equation $8y = x^2 + 4x - 28$.

(i). Find the coordinates of the vertex.

(ii). Find the coordinates of the focus.

(iii). Find the equation of the normal to the parabola at the point $(-2, -4)$. Write the equation of the line in general form.

Question 5

(a). Consider the curve $y = x^3 + 3x^2 - 9x$.

(i). Find the stationary points and determine their nature.

(ii). Find the point of inflexion.

(iii). Sketch the curve for $-4 \le x \le 2$.

(b). (i). Find $\int \left(\sec^2 x + \frac{x}{x^2 + 4} \right) dx$.

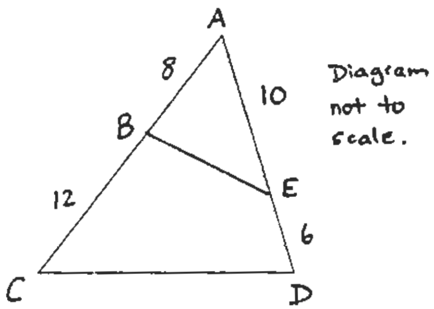
(ii). Evaluate $\int_2^3 \frac{x^2 + 5}{x^2}\, dx$.

Question 6

(a).

(i). Prove that ΔABE and ΔACD are similar.

(ii). If CD = 18 cm, find the length of BE, giving reasons.



(b). Paula deposits $50 into a superannuation fund at the start of each month. The fund pays 15% p.a. interest which is compounded at the end of each month.

(i). Find the value of the fund after 10 years.

(ii). How many months will Paula have to contribute to the fund if she wishes the fund to be worth $25 000?

Question 7

(a). In an experiment it was found that the temperature P of a body after t minutes is given by $P = 110e^{-0.04t}$ where P is in °C.

(i) What will the temperature of the body be after 10 minutes?

(ii). After how long is the temperature of the body 75°C?

(b). (i). Find $\frac{d}{dx}(\cos(x^3))$.

(ii). Hence, find $\int x^2 \sin(x^3)\, dx$.

(c). For the function $f(x) = 4\sin 2x - 1$

(i). Sketch the graph of $y = f(x)$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

(ii). What is the range of this function.

(iii). How many solutions are there to the equation $4\sin 2x = 1$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

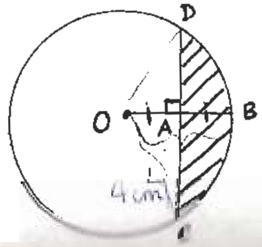
(iv). Determine the value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 4\sin 2x\, dx$.

Question 8

(a). The acceleration of a moving body is given by $a = \sqrt{2t + 1}$. If the body starts from rest, find its velocity after 4 seconds.

(b). (i). Find the exact area of the shaded region, given O is the centre, AC = AD and OA = AB = 2 cm.

(ii). Find the length of the arc CD.



.estion 8 (continued)

(c). The quadratic equation $P(x)$ is given by $P(x) = x^2 - 2(k-3)x + (k-1)$.

(i). Find the values of k for which $P(x) = 0$ has equal roots.

(ii). Find the range of values of k for which the quadratic function is positive definite.

Question 9

(a). (i). Given that $xy = 4$ and $y = (x-3)^2$ intersect at $A(1,4)$ and $B(4,1)$, sketch the curves and shade the region enclosed between them.

(ii). Find the area of this region.

(iii). Find the volume enclosed when this region is rotated about the x axis.

(b). (i). Use the change of base rule or otherwise to show that $\log_{a^x} b = \frac{\log_a b}{x}$.

(ii). Hence or otherwise, find the value of x, given
$\log_{\sqrt{a}}(x+2) - \log_{\sqrt{a}} 2 = \log_a x + \log_a 2$.

Question 10

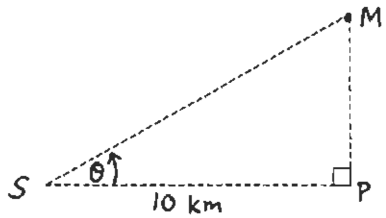
(a). A company makes 300 chairs per month. At $75 each, they can sell all the chairs. However, the price of each chair can be increased in increments of $3 but this will also result in a 4 chair reduction in sales for each $3 increment. The company also has fixed costs of $12 000 per month.

(i). If the number of $3 increments is x, show that the monthly profit $\$P$ is given by the formula $P = 10500 + 600x - 12x^2$.

(ii). Find the price that should be charged per chair to ensure the monthly profit is maximised. Also, find how many of the chairs would be sold in this circumstance.

Question 10 (continued)

(b). A missile, M, is fired vertically from a point P which is 10 km from a tracking station S at the same elevation as P, as shown below. For the first 20 seconds of flight its angle of elevation θ changes at a constant rate of 2° per second.



(i). Find θ as a function of t in the first 20 seconds.

(ii). Hence, express the distance x travelled by the missile, as a function of t.

(iii). Find the velocity of the missile when the angle of elevation is 30°, giving your answer in km/h.