(a.) Evaluate, correct to three significant figures

$$
\sqrt{\frac{(2.044)^{3}}{35.5-1.2^{2}}}
$$

(b.) Solve for $x: x^{3}=6 x^{2}$
(c.) Differentiate with respect to $x$

$$
y=e^{x^{2}-4 x}
$$

(d.) Solve the pair of simultaneous equations

$$
\begin{aligned}
& 2 x+3 y=-1 \\
& 3 x-y=15
\end{aligned}
$$

(e.) Find a primitive of $\sin \frac{x}{2}$
(f.) Solve $|7-3 x| \geq 2$ and graph your solution on the number line
(a.) If $\cos \theta=\frac{2}{5}$ and $\tan \theta<0$, find the exact value of $\sin \theta$
(b.) Differentiate
i.) $x^{2} \ln x$
2
ii.) $\frac{\sqrt{x}}{3 x-2}$
2
(c.) Find $\int \frac{x^{2}}{x^{3}+5} d x$

2
(d.) Evaluate $\int_{0}^{\ln 3} e^{2 x} d x$
(e.) Solve $e^{\log _{c} x^{3}}=27$


The diagram shows $\triangle \mathrm{ABC}$ with vertices $\mathrm{A}(-1,-2), \mathrm{B}(0,3)$ and $\mathrm{C}(7,4)$. Copy the diagram onto your answer sheet.
(a.) $E$ is the midpoint of AC. Find the coordinates of E
(b.) Find the gradient of AC
(c.) A line $l$ is drawn through B , perpendicular to AC .

Show that the equation of line $l$ is $4 x+3 y-9=0$
(d.) What is the angle that $l$ makes with the positive $x$ axis?
(e.) Find the perpendicular distance of B from AC
(f.) Find the area of $\triangle \mathrm{ABC}$
(g.) AC is the diameter of a circle
i.) Calculate the radius of the circle 1
ii.) Hence find the equation of the circle

QUESTION 4 [12 marks]
(a.) Given the parabola $y=2 x^{2}-8 x+1$, find:
i.) focal length
ii.) vertex
iii.) focus

```
iv.) directrix
(b.) The third term and the tenth term of an arithmetic series are 10 and 31 respectively. Find the:
i.) first term and the common difference
ii.) sum of the first ten terms of the series
(c.) Using Simpson's Rule with three function values, find an approximate value for the area represented by the definite integral
\[
\int_{2}^{3} \cos ^{2} x d x
\]
(d.) One hundred tickets are sold in a raffle. Two different tickets are to be drawn for first and second prizes. Bianca buys 15 tickets.
What is the probability that she:
i.) wins first prize
ii.) wins at least one prize

\section*{QUESTION 5 [12 marks]}
(a.) Find the equation of the tangent to the curve \(y=\ln \left(x^{2}+2\right)\) at the point where \(x=1\). Answer in general form
(b.) The gradient of the curve \(y=f(x)\) is given by \(f^{\prime}(x)=\frac{2 x^{2}+1}{x}\). Find the equation of the curve if it passes through the point \((1,5)\)
(c.)


Two athletes are jogging on separate roads which meet at right angles at town X . Athlete A is 8 km from X and is travelling at \(5 \mathrm{~km} / \mathrm{h}\) away from X . Athlete \(B\) is 10 km from X and is travelling at \(6 \mathrm{~km} / \mathrm{h}\) and travelling towards X .
i.) Show that the distance apart, \(P \mathrm{~km}\), after \(t\) hours is given by
\[
P=\sqrt{61 t^{2}-40 t+164}
\]
ii.) Hence find their minimum distance apart .
(a.) Consider the curve \(y=x(x-3)^{3}\)
i.) Find the coordinates of any stationary points and
4
determine their nature
ii.) Find the point(s) of inflexion

2
iii.) Sketch the curve, showing clearly all features
(b.) If \(\alpha, \beta\) are roots of the quadratic equation \(6 x^{2}-x+5=0\), find:
i.) \(\alpha+\beta\)
ii.) \(\alpha \times \beta\)
iii.) \(\alpha^{2}+\beta^{2}\)
(c.) Solve \(\log _{27} 32=x \log _{3} 2\) without the aid of a calculator.

Show all working

QUESTION 7 [12 marks]
(a.) i.) Draw a neat sketch of the function \(y=1-2 \sin x\) for \(0 \leq x \leq 2 \pi\)
ii.) Determine the exact values where the graph cuts the \(x\)-axis in the given domain
iii.) Calculate the area bounded by the curve \(y=1-2 \sin x\), the \(x\)-axis and the lines \(x=0\) and \(x=\frac{\pi}{2}\)
(b.) In the diagram PQ is an arc of a circle with centre \(O\) and radius 5 cm . The chord \(P Q\) has length 6 cm .

i.) Find the size of \(\angle \mathrm{POQ}\) in radians 2
ii.) Calculate the shaded area
(c.) Find the values of \(K\) for which the equation
\((2-K) x^{2}-4(K-2) x-5=0\), has two real distinct roots

QUESTION 8 [12 marks]
(a.) A particle, initially at rest at the origin, moves in a straight line with velocity \(V \mathrm{~m} / \mathrm{s}\) so that \(V=5 t(4-t)\) where \(t\) is the time in seconds. Find:
i.) the acceleration of the particle after 4 seconds
ii.) an expression for the displacement \(x\) metres of the particle in terms of \(t\)
iii.) the total distance travelled in the first 6 seconds

2
(b.)


Not to scale
\(A B C\) is right angled at \(B\) and \(D E\) is perpendicular to \(A C\).
i.) Prove that \(\triangle \mathrm{ABC}\) and \(\triangle \mathrm{CDE}\) are similar
ii.) Prove that \(\mathrm{BC} \times \mathrm{CE}=\mathrm{AC} \times \mathrm{CD}\) 2
iii.) Prove that \(\mathrm{DE}^{2}=\mathrm{AD} \times \mathrm{DC}-\mathrm{BE} \times \mathrm{EC}\)
(a.) A bottle of solvent is open and the solvent evaporates in such a way that the amount remaining, \(V \mathrm{ml}\), in the bottle is given by \(V=2000 e^{-0.005 t}\), where \(t\) is time in hours.
i.) How much solvent is in the bottle initially? \(\quad \mathbf{1}\)
ii.) How much solvent has evaporated out of the bottle after 30 hours?
iii.) How long is it before half the initial amount of solvent has evaporated from the bottle?
iv.) If the solvent continues to evaporate will the bottle ever

1 become empty? Explain.
(b.) Boxes in a storeroom are stacked in a pile such that there are 25 on the bottom row, 22 on the next, 19 on the next, and so on until 117 boxes are on the pile altogether.
i.) How many rows of boxes are there?
ii.) How many boxes are there on the top row?
(c.) By expressing 1.25 as the sum of an infinite geometric series, find the simple equivalent fraction for \(1.25^{\circ}\).
(d.) The graph \(y=e^{x}-1\) is rotated about the \(x\)-axis from \(x=1\) to \(x=3\).

3 Find the volume generated.

QUESTION 10 [12 marks]
Marks

(a.) The diagram shows a part of the circle \(x^{2}+y^{2}=25\). The point \(\mathrm{P}(x, y)\) is on the circle and O is the origin. Given \(\mathrm{M}(2,0)\) and \(\mathrm{N}(0,4)\) and \(\angle \mathrm{MOP}\) is \(\theta\) radians.
i.) Show that the area A , of the quadrilateral OMPN is given by
\[
A=5 \sin \theta+10 \cos \theta
\]
ii.) Find the value of \(\tan \theta\) for which A is maximum
iii.) Hence find (in surd form) the coordinates of P
for which A is maximum
(b.) Alice is retiring tomorrow and her Super Fund contains \(\$ 450,000\). The fund is earning \(15 \%\) p.a. compound interest, compounded monthly. Alice wishes to withdraw a regular amount of \(\$ 6000\) per month to cover her expenses.
i.) Show that after 1 month she will have an amount A in her account 1 where \(A_{1}=450000 \times 1.0125-6000\)
ii.) Find an expression for the amount remaining after \(n\) months
iii.) How many years will the money last?
iv.) If she wishes to withdraw \(\$ 6000\) per month from her account for 1 30 years, use your calculator to approximate the required interest rate.

\section*{End of Paper}



\[
\begin{aligned}
& \text { (iv) } \\
& \text { Try } r=16 \% \quad \therefore r=\frac{16}{100} \\
& \therefore 450000 \times(1.013)^{360}=6000\left(\left(1.013^{\circ}\right)^{360}-1\right) \\
& \therefore 450000 \times\left(1.013^{\circ}\right)^{360}=6000 \times\left(1.013^{\circ}-1\right) \\
& 450000 \times\left(1.013^{\circ}\right)^{360}=6000 \times(8753 \ldots) \\
& \frac{450000 \times\left(1.013^{\circ}\right)}{8753 \cdots} \doteq 6052 \div 6000 \text { (1) }
\end{aligned}
\]```

