**QUESTION 1.** Start on a new page

(a) Find the value of $e^{2\pi}$ correct to 2 significant figures. 2

(b) Factorise $3x^2 + x - 2$. 2

(c) Find the value(s) of $x$ for which $|2x-1| = 5$. 2

(d) Simplify $\frac{1}{2} - \frac{1}{x+1}$. 2

(e) If $\frac{20\sqrt{18}}{16\sqrt{24}} = a\sqrt{3}$ find the value of $a$. 2

(f) If $\alpha$ and $\beta$ are the roots of the equation $x^2 + 6x - 3 = 0$

evaluate $\alpha^2\beta + \beta^2\alpha$. 2

**QUESTION 2.** Start on a new page.

(a) Differentiate :

(i) $(e^x + 2)^8$ 2

(ii) $\frac{x-1}{\cos x}$ 2

(b) Find:

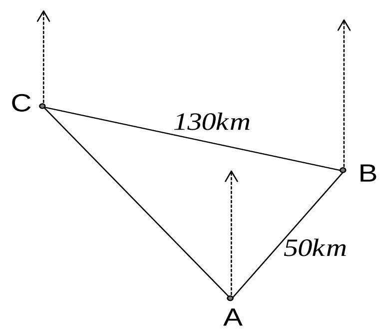
(i) $\int_0^{\frac{\pi}{3}} \sin 4x \, dx$ 3

(ii) $\int \frac{1-x^2}{x} \, dx$ 3

(c) Solve $2\log_3 5 - \log_3 x = 2$ 2

**QUESTION 3.** Start on a new page.

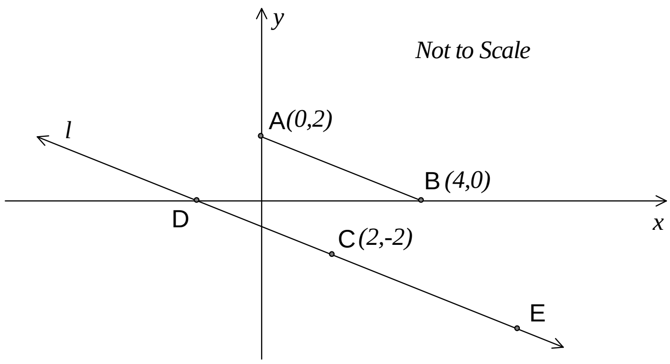
(a) A ship sails 50 km from port A to port B on a bearing of $063^\circ$. It then sails 130 km from port B to port C on a bearing of $296^\circ$.




(i) Show (with working ) $\angle ABC = 53^\circ$ 1

(ii) Find AC to the nearest kilometre. 2

(b)




(i) Using the diagram above find the gradient of AB. 1

(ii) Show that the distance AB is $2\sqrt{5}$ units. 1

(iii) Show that the equation of the line $l$, through C parallel to AB is $x+2y+2=0$. 2

(iv) Find the co-ordinates of the point D where $l$ cuts the $x$ axis. 1

(v) Show that the perpendicular distance from A to the line $l$ is $\frac{6}{\sqrt{5}}$ units. 1

(vi) If the area of the trapezium is 18 $units^2$ find the co-ordinates of the point E. 3

**QUESTION 4.** Start on a new page.

(a) For what value(s) of $k$ does the equation $x^2 - (k+2)x + 2k + 1 = 0$ have real roots. 3

(b) Let $f(x) = x(x-4)^3$.

(i) Show that $f'(x) = 4(x-4)^2(x-1)$. 2

(ii) Find the co-ordinates of the stationary points and determine their nature. 3

(iii) If $f''(x) = 12(x-2)(x-4)$ find the points of inflexion on the curve. 2

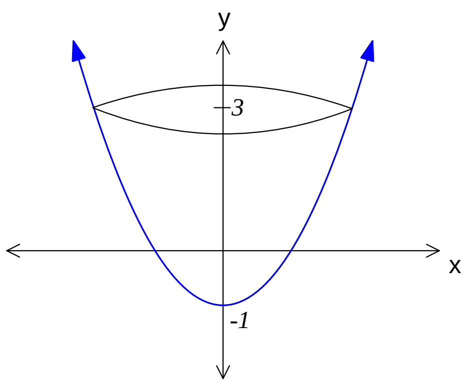
(iv) Sketch the curve $y = f(x)$ showing intercepts, stationary points and points of inflexion. 2

**QUESTION 5.** Start on a new page.

(a) (i) Find the probability that in a family of 4 children 2 are boys and 2 are girls. (You may wish to draw a tree diagram). 2

(ii) Two women each have had 4 children. What is the probability that neither woman has 2 boys and 2 girls? 2

(b) The shape of a glass is formed by rotating the curve $y = 2x^2 - 1$ about the $y$ axis from $y = -1$ to $y = 3$.




Find the volume of the glass. 3

(c) In 1990 a farmer harvested 900 tonnes of wheat. This amount increased by 4% each year.

(i) How much was harvested in the year 2000? 2

(ii) How much was harvested from 1990 to 2000? 3

**QUESTION 6.** Start on a new page.

(a) (i) Show that $\sin\theta\cot\theta = \cos\theta$ 1

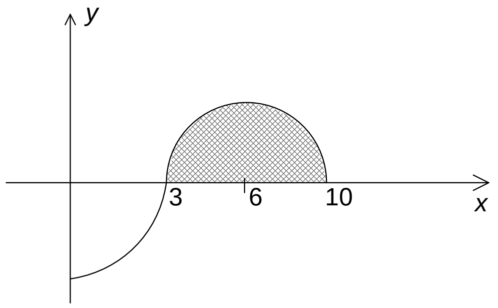
(ii) Hence solve $27\cos\theta\sin\theta\cot\theta = \sec\theta$ 3

(b) (i) If $y = \sqrt{\sin x}$ complete the table. 1

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $y$ | | | | | 0.95 |

(ii) Hence evaluate $\int_0^2 \sqrt{\sin x}\ dx$ using Simpson's rule with 5 function values. 3

(c) The graph shows the velocity- time graph of an object as it moves along the $x$ axis in 10 seconds.




(i) When does the object come to rest? 1

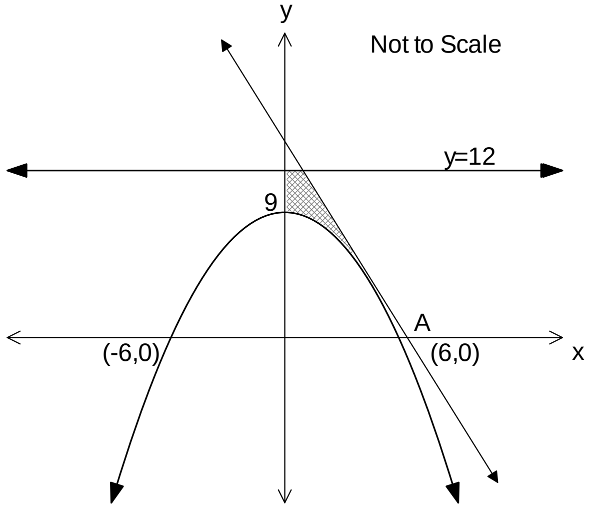
(ii) When is the acceleration of the object the greatest? 1

(iii) What does the shaded region represent? 1

(iv) At approximately what time will the object return to it's starting point? 1

**QUESTION 7.** Start on a new page.

(a) The graph of $y = 9 - \frac{x^2}{4}$ is drawn below. The tangent to the curve at A(6,0) is also drawn

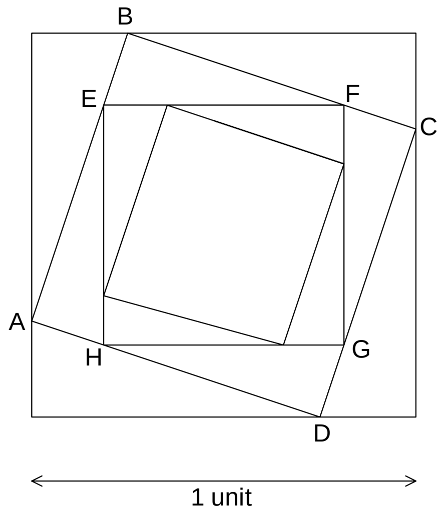



(i) Show that the equation of the tangent to the curve at A is given by

$y = -3x + 18$. 2

(ii) What are the co-ordinates of the focus of the parabola $y = 9 - \frac{x^2}{4}$? 2

(iii) Find the area of the shaded region. 4

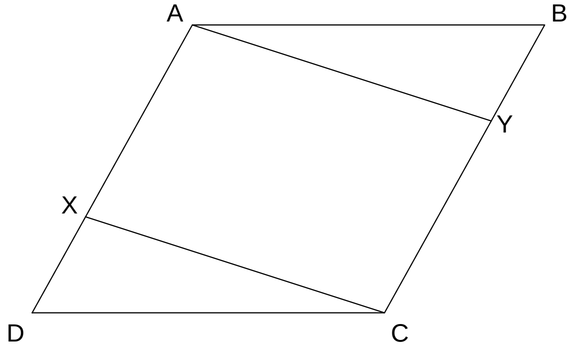



(b) In the diagram above the largest square has sides 1 unit. This pattern of squares has consecutive squares whose dimensions are $\frac{3}{4}$ that of the previous square. For example square ABCD has side lengths of $\frac{3}{4}$ of a unit.

(i) What is the area of square EFGH? 1

(ii) What is the sum of the area of all the squares if this pattern continued indefinitely? 3

**QUESTION 8.** Start on a new page.




(a) ABCD is a rhombus. X and Y lie on AD and BC such $XC \perp AD$ and $AY \perp BC$.

(i) Prove $\Delta ABC \equiv \Delta DXC$. 2

(ii) If the side length of the rhombus is 10 cm and $\angle DAB = 120^\circ$, find the length of the diagonal AC. 2

(iii) Find the area of the rectangle AYCX. 2

(b) The rate (R) at which green house gases are released into the atmosphere from a town in tonnes/hour is given by: $R = 20 + \dfrac{100}{(1+t)^2}$, where $t$ is in hours.

(i) At what rate are the green house gases released initially? 1

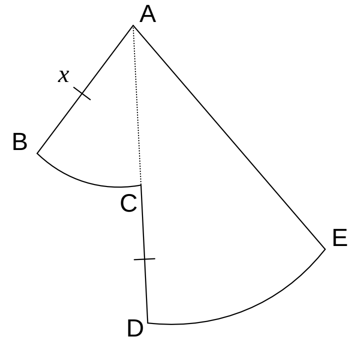
(ii) What is the rate at which green house gases are released as time increases indefinitely? 1

(iii) Without using calculus draw a graph of R as a function of time. 1

(iv) How much gas was released into the atmosphere in the first 2 hours? 3

**QUESTION 9.** Start on a new page.

(a) Two sectors make up a company logo.




Both sectors have centre A , AB=CD, AB= $x$ and AC bisects $\angle$BAE.

Let $\angle$BAC= $\theta$.

(i) If the area of the logo is $8\pi$ $units^2$ show that: $\theta = \dfrac{16\pi}{5x^2}$ 2

(ii) Show that the perimeter (P) of the logo is given by

$$P = 4x + \frac{48\pi}{5x}$$ 2

(iii) Find the value of $x$ which makes the perimeter of the logo a minimum. 3

(b) (i) Sketch the curve $y = 1 - 3\cos 2x$ *for* $-\pi \le x \le \pi$ . 3

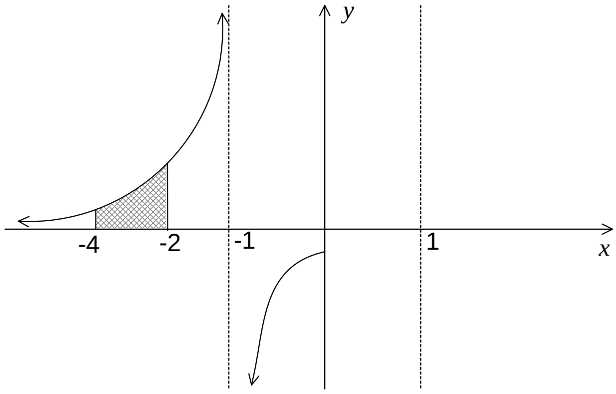
(ii) If $3\cos 2x = 1 - k$ has 3 solutions in the given domain, find the value of $k$ . 2

**QUESTION 10.** Start on a new page.

(a) (i) Show that the derivative of $\log_e \dfrac{x-1}{x+1}$ is $\dfrac{2}{(x-1)(x+1)}$ 2

(ii) Show that $f(x) = \dfrac{1}{(x-1)(x+1)}$ is an even function. 2

(iii)




A portion of the graph of $f(x) = \dfrac{1}{(x-1)(x+1)}$ is drawn above.

Find the shaded area. 3

(b) Evaluate $\displaystyle\sum_{k=4}^{20} 2(2^{k-4} + 2k - 3)$ 5

3 b) (i) $m_{AB} = -\frac{1}{2}$ (1)

(ii) distance $AB = \sqrt{4^2+2^2}$
$= \sqrt{20}$ (1)
$= 2\sqrt{5}$

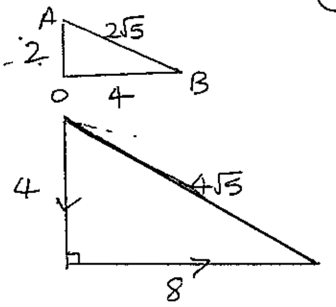
(iii) $y+2 = -\frac{1}{2}(x-2)$ (1) (1)
$2y+4 = -x+2$ no ise.
$x+2y+2=0$

(iv) at D $y=0$
$\therefore x+0+2=0$
$\therefore x=-2$
$D(-2,0)$ (1)

(v) $d = \frac{|0(1)+2(2)+2|}{\sqrt{1^2+2^2}}$ (1)
$= \frac{6}{\sqrt{5}}$

(vi) $\frac{1}{2}\times\frac{6}{\sqrt{5}}(a+2\sqrt{5}) = 18$
$a = 4\sqrt{5}$ (1)
$a = DE$.
$\Delta ABO \;|||\; \Delta DFE$.

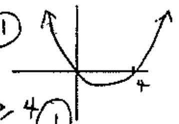
$\therefore AB : DE = 1:2$ (1)




$\therefore E(-2+8,\ 0-4)$
$E(6,-4)$ (1)

## Question 4

a) Real roots when $\Delta \ge 0$
$[-(k+2)]^2 - 4(2k+1) \ge 0$ (1)
$k^2+4k+4-8k-4 \ge 0$
$k^2-4k \ge 0$
$k(k-4) \ge 0$ (1)




$\therefore k \le 0,\ k \ge 4$ (1)

b) (i) $f(x) = x(x-4)^3$
$f'(x) = 1(x-4)^3 + 3(x-4)^2.x$ (1)
$= (x-4)^2[(x-4)+3x]$
$= 4(x-4)^2(x-1)$ (1)

(ii) st. pts when $y'=0$
$4(x-4)^2(x-1)=0$ when
$x=1$, $y=-27$; $x=4$, $y=0$
$\therefore (1,-27)$ $(4,0)$ (1)

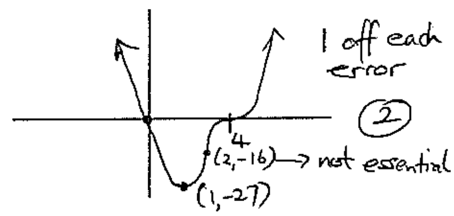
Test:

| | | | | | |
|---|---|---|---|---|---|
| | 0 | 1 | 2 | 4 | 5 |
| $y'$ | $4\times16\times-1$ $-16$ | 0 | $4(4)(1)$ 16 | 0 | $4\cdot1\cdot4$ ↗ |
| | ↘ | ⟷ | ↗ | ⟷ | |

$\therefore$ Min (1)
horizontal $\therefore$ inflex (1)

(iii) Pt of inflexion at $(4,0)$ (partic)
$y''=0$ $12(x-2)(x-4)$
$x=2$ $y=-16$ $(2,-16)$ (1)
Test concavity change.
when $x=1$ $y''=+36$ (1)
when $x=3$ $y''=-12$
Concavity change $\therefore$ pts o.i.
$(2,-16)$ & $(4,0)$

(iv)




1 off each error (2)

1. a) $e^{2\pi} = 535.49\ldots$ (1)
$= 540$
If error must show full working to get (1)

b) $(3x-2)(x+1)$ (2)
(1) mark for $(3x \mp 2)(x - 1)$ $\pm 1$ $\pm 2$

c) $|2x-1| = 5$
$2x-1=5$, $x=3$ (1)
$-2x+1=5$, $x=-2$ (1)

d) $\$979 = 110\%$ of orig.
$1\% = \frac{979}{110}$ (1)
Orig. Price $= \frac{979}{110}\times 100$
$= \$890$ (1)

e) $\frac{20\sqrt{18}}{16\sqrt{24}} = \frac{5}{4}\frac{\sqrt{3}}{\sqrt{4}}$
$= \frac{5}{8}\sqrt{3}$
$a = \frac{5}{8}$ (2)
1 off for not simplifying.

Solutions:- 2U 09 Trial

f) $\alpha+\beta = -6$ $\alpha\beta = -3$ (1)
$\alpha^2\beta+\beta^2\alpha = \alpha\beta(\alpha+\beta)$
$= -3\times-6$
$= 18$ (1)

2a) (i) $y = (e^x+2)^8$
$y' = 8e^x(e^x+2)^7$ (1) (1)

(ii) $y = \frac{x-1}{\cos x}$ 1 off each error
$y' = \frac{1.\cos x + \sin x(x-1)}{\cos^2 x}$ (2)
$= \frac{\cos x + x\sin x - \sin x}{\cos^2 x}$ (no ise if they cancel)

b (i) $\int_0^{\frac{\pi}{3}} \sin 4x \, dx = -\frac{1}{4}\left[\cos 4x\right]_0^{\frac{\pi}{3}}$ (1)
$= -\frac{1}{4}\left[\cos\frac{4\pi}{3} - \cos 0\right]$ (1)
$= -\frac{1}{4}\left[-\frac{1}{2} - 1\right]$
$= \frac{3}{8}$ (1)

(ii) $\int \frac{1-x^2}{x}\,dx = \int \frac{1}{x} - x\,dx$ (1)
$= \ln x - \frac{x^2}{2} + c$
(1) (1) (ignore c)

c) $2\log_3 5 - \log_3 x = 2$
$\log_3 25 - \log_3 x = 2$
$\log_3\left(\frac{25}{x}\right) = 2$ (1)
$3^2 = \frac{25}{x}$
$x = \frac{25}{9}$ (1)

3a) (i) $\angle NAB = 63°$
$\angle CBX = 64°$ ($\angle$'s at a pt)
$\angle ABC = 53°$ (Cointerior $\angle$'s on || lines)
(1) accept $180-(63+64)$ many methods.. not essential to have reasons.

(ii) $AC^2 = 50^2 + 130^2 - 2.50.130.\cos 53$ (1)
$= 107.59$
$= 108$ km. (1)

7 a)(ii) $y = 9 - \frac{x^2}{4}$

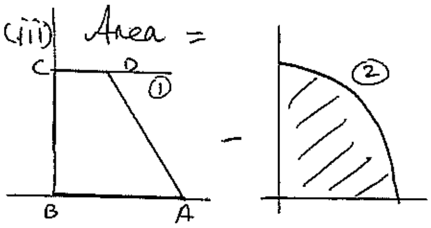
$\frac{x^2}{4} = -y + 9$

$x^2 = -4(y - 9)$ (1)

$\therefore 4a = 4 \quad a = 1$

Focus $(0, 8)$ (1)

(iii) Area =




at D $y = 12$ & $y = -3x + 18$

$\therefore 12 = -3x + 18$

$x = 2$

$\therefore$ Area of trap. $= \frac{1}{2} \cdot 12(2+6)$
$= 48$ units$^2$ (1)

Area② ⇒ make $x$ the subject

$x^2 = -4y + 36$

$x = \sqrt{36 - 4y}$
$= 2(9 - y)^{\frac{1}{2}}$ (1)

$\therefore$ Area $= 2\int_0^9 (9-y)^{\frac{1}{2}}$

$= 2\left[-\frac{2}{3}(9-y)^{\frac{3}{2}}\right]_0^9$ (1)

$= -\frac{4}{3}\left[\sqrt{(9-y)^3}\right]_0^9$

$= -\frac{4}{3}\left[0 - 27\right]$

$= \frac{108}{3} = 36$

$\therefore$ Shaded area $= 48 - 36$ (1)

(many ways of doing this) $= 12$ units$^2$

b) (i) $\left(\frac{3}{4} \times \frac{3}{4}\right)^2 = \frac{81}{256}$ units$^2$ (1)

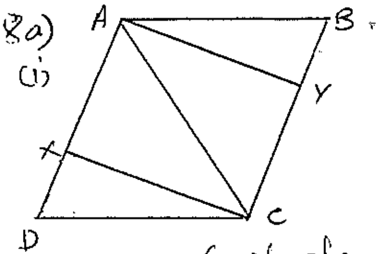
(ii) Area of squares =

$1 + \frac{9}{16} + \frac{81}{256} + \dots$ (1)

$S_\infty = \frac{1}{1 - \frac{9}{16}}$ (1)

$= \frac{16}{7}$ units$^2$. (1)

8a) (i)




AB = CD (sides of a rhombus)

$\angle AYB = 90°$ ($AY \perp BC$) (1)

$AXC = 90°$ ($AD \perp XC$)

$\angle B = \angle D$ (opp. $\angle$'s of a rhombus)

$\therefore \Delta ABY \equiv \Delta DXC$ (AAS) (1)

(ii) Construct AC

$\angle DAC = 60°$ (diagonal bisects $\angle$ at the corner)

(1) $\angle ADC = 60°$ (Co-interior $\angle$'s on $\parallel$ lines)

(1) $\Delta ADC$ is equilateral — not essential

$\therefore AC = 10$ cm.

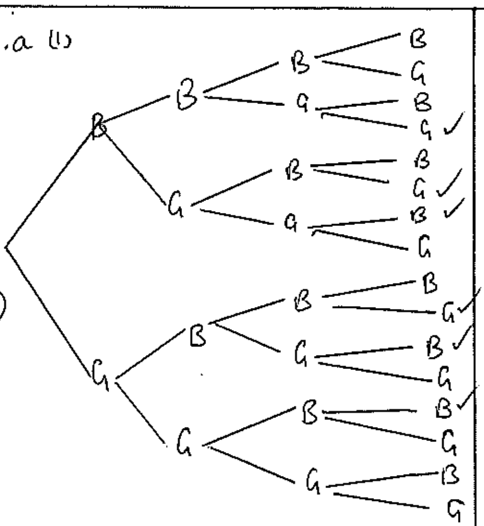
(iii) $AX = 5$cm (Perpendicular XC bisects base) (1)

$\sin 60° = \frac{XC}{10}$

$\therefore XC = 10 \cdot \frac{\sqrt{3}}{2}$ (1)
$= 5\sqrt{3}$

$\therefore$ Area $= 5 \times 5\sqrt{3} = 25\sqrt{3}$

5.a (i)



(1)

$P(2B2G) = \frac{6}{16} = \frac{3}{8}$ (1)

(ii) $\left(1 - \frac{3}{8}\right)\left(1 - \frac{3}{8}\right) = \frac{55}{64}$ (1) (1)

b) $y = 2x^2 - 1$

$x^2 = \frac{y}{2} + \frac{1}{2}$

$V = \pi \int_{-1}^{3} \frac{y}{2} + \frac{1}{2}$ (1)

$= \frac{\pi}{2}\left[\frac{y^2}{2} + y\right]_{-1}^{3}$ (1)

$= \frac{\pi}{2}\left[\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} - 1\right)\right]$

$= 4\pi$ units$^3$ (1)

c) (i) $900 \times 1.04^{10}$ (1)

$= 1332.22$ t. (1)

(ii) Total $= 900 + 900 \times 1.04 + \dots + 900 \times 1.04^{10}$ (1)

G.P. $a = 900$ $r = 1.04$ $n = 11$

$S_n = \frac{900(1.04^{11} - 1)}{1.04 - 1}$ (1)

$= 12137.72$ t. (1)

6(a)(i) LHS. $\sin\theta \cot\theta$

$= \sin\theta \cdot \frac{\cos\theta}{\sin\theta}$ (1)

$= \cos\theta$

(ii) $27\cos\theta \cot\theta \sin\theta = \sec\theta$

(1) $27\cos\theta \cdot \cos\theta = \frac{1}{\cos\theta}$

$27\cos^3\theta = 1$

$\cos^3\theta = \frac{1}{27}$

$\cos\theta = \frac{1}{3}$ (1)

$\theta = 70°32'$ (1)

or $\theta = 1.23$ radians

$y = \sqrt{\sin x}$

b)

| $x$ | 0 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|
| $y$ | 0 | 0.69 | 0.92 | 1.0 | 0.95 |

(1)

$A = \frac{0.5}{3}\left[0 + 0.95 + 4(0.69 + 1) + 2(0.92)\right]$ (1) (1)

$= 1.5916$

$= 1.6$ (1)

c) (i) $t = 3, 10$ (1)

(ii) $t = 3$ (1)

(iii) The distance travelled by the object between $t = 3$ & $t = 10$ (1)

(iv) $t = 6$. because total velocity to the left = " " to the right or = distance travelled left & right equal at $t = 6$. (1)

7 a) $y = 9 - \frac{x^2}{4}$

$y' = -\frac{2x}{4}$

at $x = 6$ $y' = -3$ (1)

$\therefore$ Tangent $y - 0 = -3(x - 6)$

$y = -3x + 18$ (1)

10 a) (i) $y = \ln\left(\frac{x-1}{x+1}\right)$

$y = \ln(x-1) - \ln(x+1)$ (1)

$\frac{dy}{dx} = \frac{1}{x-1} - \frac{1}{x+1}$

$= \frac{x+1-(x-1)}{(x-1)(x+1)}$ (1)

$= \frac{2}{(x-1)(x+1)}$

(ii) $f(x) = \frac{1}{(x-1)(x+1)}$

$f(-x) = \frac{1}{(-x-1)(-x+1)}$ (1)

$= \frac{1}{-(x+1)-(x-1)}$ (1)

$= \frac{1}{(x+1)(x-1)}$

$f(x) = f(-x)$ ∴ Even

$\int_{-4}^{-2} \frac{dx}{(x-1)(x+1)} = \int_{2}^{4} \frac{dx}{(x-1)(x+1)}$ (1)

since $y = f(x)$ is even

$\int_{2}^{4} \frac{dx}{(x-1)(x+1)} = \frac{1}{2}\left[\ln\left(\frac{x-1}{x+1}\right)\right]_2^4$ (1)

$= \frac{1}{2}\left[\ln\left(\frac{3}{5}\right) - \ln\left(\frac{1}{3}\right)\right]$

$= \frac{1}{2}\ln\left(\frac{3}{5}\times\frac{3}{1}\right)$ (1)

$= \frac{1}{2}\ln\left(\frac{9}{5}\right) = 0.2938...$

b) $\sum_{k=4}^{20} 2(2^{k-4} + 2k - 3)$

$= 2\left[\sum_{k=4}^{20} 2^{k-4} + \sum_{k=4}^{20} 2k-3\right]$ (1)

$= 2\left[1+2+\dots 2^{16}\right] + 2\left[5+7+\dots+37\right]$
(1) (1)

$2\left[\frac{1(2^{17}-1)}{2-1}\right] + 2\left[\frac{17}{2}(5+37)\right]$

$= 262142$ (1) $+ 714$ (1)

$= 262\,856$

for 10b)

(1) – For substitution

(1) – for $n = 17$.

b) (i) $R = 20 + \frac{100}{(1+2t)^2}$

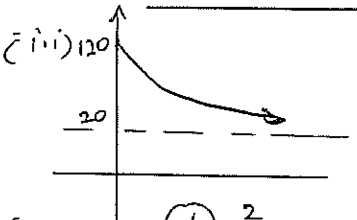
$t = 0 \quad R = 20 + \frac{100}{1}$

$= 120\,t/hr$ (1)

(ii) as $t \to \infty \quad \frac{100}{(1+2t)^2} \to 0$

∴ as $t \to \infty \quad R \to 20\,t/hr$ (1)

(iii)




(1)

(iv) Area = (1) $\int_0^2 20 + 100(1+2t)^{-2}$

$= \left[20t + \frac{100(1+2t)^{-1}}{-2}\right]_0^2$

$= \left[20t - \frac{50}{1+2t}\right]_0^2$ (1)

$= \left(40 - \frac{50}{5}\right) - (0 - 50)$

$= 80$ tonnes. (1)

9 (i) Area $= \frac{1}{2}x^2\theta + \frac{1}{2}(2x)^2\theta$ (1)

$= \frac{1}{2}x^2\theta + 2x^2\theta$

$A = \frac{5x^2\theta}{2}$

If $A = 8\pi$

$8\pi = \frac{5x^2\theta}{2}$ (1)

$\theta = \frac{16\pi}{5x^2}$

(ii) $P = x + x\theta + x + 2x\theta + 2x$ (1)

$= 4x + 3x\theta$

$P = 4x + 3x.\frac{16\pi}{5x^2}$ (1)

$= 4x + \frac{48\pi}{5x}$

(iii) $P' = 4 - \frac{48\pi}{5}x^{-2}$

$= 4 - \frac{48\pi}{5x^2}$ (1)

Min. when $P' = 0 \quad P'' > 0$

$4 - \frac{48\pi}{5x^2} = 0$

$20x^2 = 48\pi$

$x^2 = \frac{48\pi}{20} = \frac{12\pi}{5}$

∴ $x = \pm\sqrt{\frac{12\pi}{5}}$ but $x > 0$

∴ $x = \sqrt{\frac{12\pi}{5}}$ (1)

Test. $P'' = \frac{96\pi x}{5}$

when $x = \sqrt{\frac{12\pi}{5}} \quad P'' = \frac{96\pi}{5}.\sqrt{\frac{12\pi}{5}}$

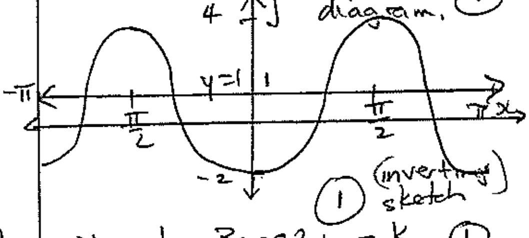
$> 0$ (1)

∴ Min. occurs

when $x = \sqrt{\frac{12\pi}{5}} =$

(b)(i) $y = 1 - 3\cos 2x$

Amp = 3 (1) Period = $\pi$ from diagram. (1)




(1) (inverting sketch)

(ii) $1 - 3\cos 2x = k$ (1)

ie $3\cos 2x = 1 - k$

will have 3 solutions if

$k = -2$ (1) answer only (½)

Because $y = -2$ & $y = 1 - 3\cos 2x$ have 3 pts of intersection