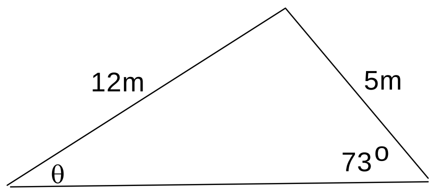
QUESTION 1

a) Evaluate $2e^{-0.72}$ correct to 2 significant figures **2**

b) Simplify fully $\sqrt{27} + \sqrt{48} - 2\sqrt{75}$ **2**

c) Solve $|2x + 3| = 3x$ **2**

d) Find the value of θ correct to the nearest degree



2

e) Solve $5 - 5x \leq 4$ **2**

f) Find the limiting sum of the geometric series $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \cdots$ **2**

QUESTION 2 (START A NEW PAGE NOW)

a) Differentiate with respect to x:

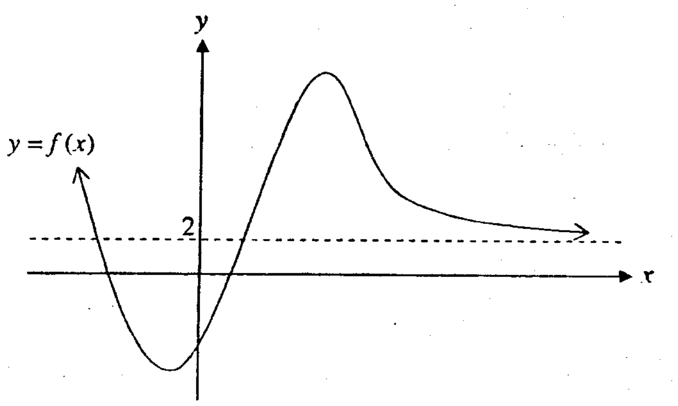
(i) $x^2 . \cos x$ **2**

(ii) $tan^2(4x + 1)$ **2**

b) Find $\int \frac{x}{3x^2+1}\, dx$ **2**

c) Evaluate $\int_0^{\frac{\pi}{9}} \sin 3x \, . dx$ **3**

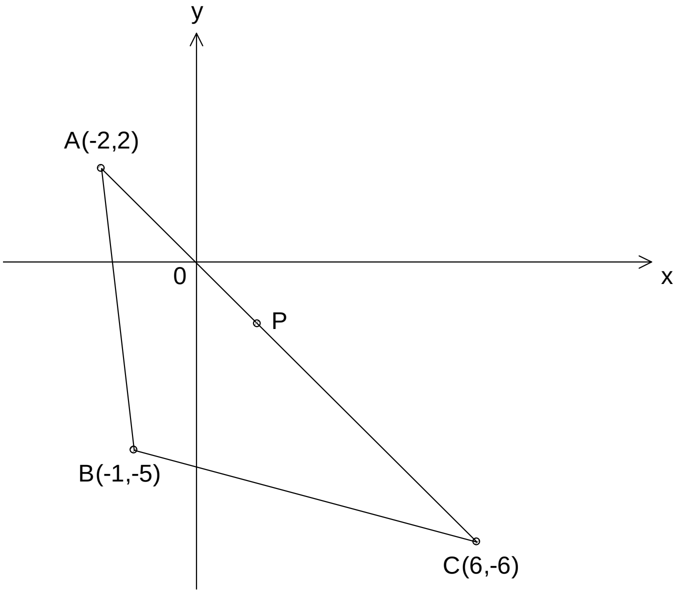
d) The diagram below shows a sketch of the curve $y = f(x)$. **3**



Copy the diagram onto your answer sheet and use it to draw a sketch of the gradient function $y = f'(x)$

QUESTION 3 (START A NEW PAGE NOW)

a) The diagram (not to scale) shows ΔABC with vertices A(-2,2) , B(-1,-5) and C(6,-6)



Copy the diagram neatly onto your answer sheet

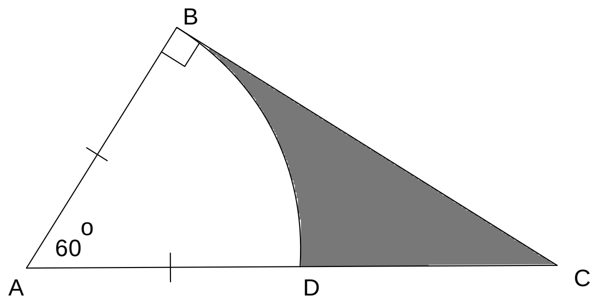
(i)	P is the midpoint of AC. Show that the coordinates of P are (2,-2).	**1**
(ii)	Find the gradient of BP.	**1**
(iii)	Show that BP is perpendicular to AC.	**2**
(iv)	Show that the equation of BP is $y = x - 4$.	**1**
(v)	Find the coordinates of D, if P is the midpoint of BD.	**2**
(vi)	Which shape best describes the geometric figure ABCD? Explain.	**2**

b) Find the value(s) of k such that the roots of

$x^2 + (3k+1)x + 4k + 5 = 0$ are real and equal. **3**

QUESTION 4 (START A NEW PAGE NOW)

a) In the diagram, $\angle B = 90^o$, $\angle A = 60^o$ and AB=AD=10m. BD is an arc of the circle with centre A



Calculate the shaded area in exact form. **3**

b) In an arithmetic progression, $T_2 = 7 \ \ and \ \ T_7 = 52$.

(i) Find the common difference and the first term. **2**

(ii) Find the **value** of the first term which is greater than 1000. **2**

c) α and β are the roots of $2x^2 - 9x - 3 = 0$. Find the value of:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **1**

(iv) $\alpha^3 + \beta^3$ **2**

QUESTION 5 (START A NEW PAGE NOW)

a) Find the coordinates of the point on the curve $y = 3x^2 - 2x - 1$ where the tangent is parallel to $4x - y - 1 = 0$. **3**

b) Solve $\sin x = \frac{1}{2}$ where $0 < x < 2\pi$ **2**

c) Evaluate **1**

$$\sum_{n=4}^{7} \frac{1}{n-2}$$

d) The equation $(x-1)^2 = -4y + 2$ represents a parabola.

(i) Find the coordinates of the vertex. **1**

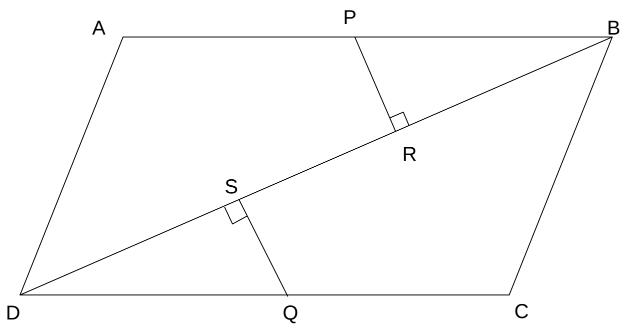
(ii) Find the focal length. **1**

(iii) Sketch the parabola, clearly showing the directrix, the focus and the x-intercepts **4**

QUESTION 6 (START A NEW PAGE NOW)

a) ABCD is a parallelogram and P, Q are the midpoints of AB, DC respectively.

The intervals PR and QS are perpendicular to the diagonal DB.



(i) Prove ΔBPR and ΔDQS are congruent. **3**

(ii) If AB=10cm, PR=3cm and BD=14cm find the length of SR. **1**

b) (i) Sketch the graph of $y = 2\cos 2x$ for $-\pi \leq x \leq \pi$. **2**

(ii) On the same diagram, sketch the line $x + y = 1$. **1**

(iii) Hence determine the **number** of solutions of the equation $2\cos 2x = 1 - x$. **1**

(iv) Let the negative solution to $2\cos 2x = 1 - x$ be $x = N$. Indicate N on the x-axis of your diagram. **1**

c) Calculate the exact area of the region bounded by the curve $y = e^{2x}$, the y-axis and the line $y = e^4$. **3**

QUESTION 7 (START A NEW PAGE NOW)

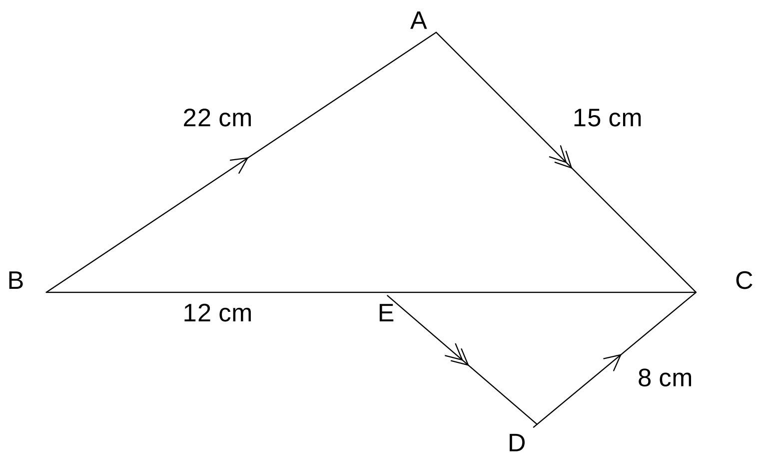
a) A circle has radius 2 cm. Find the size of the angle subtended at the centre of this circle by an arc of length 2.2 cm. Answer correct to the nearest minute . **2**

b) Solve $log_e x - log_e(x-4) = \ log_e 2$ **2**

c) Find all values of k for which $y = e^{kx}$ is a solution of $y'' - y' - 12y \geq 0$ **4**

d) In the diagram (not to scale), CD is parallel to AB and DE is parallel to CA.

AC=15 cm, AB=22 cm, CD=8 cm and BE=12 cm.



(i) Prove triangle ABC is similar to triangle DCE **2**

(ii) Hence find the length of BC **2**

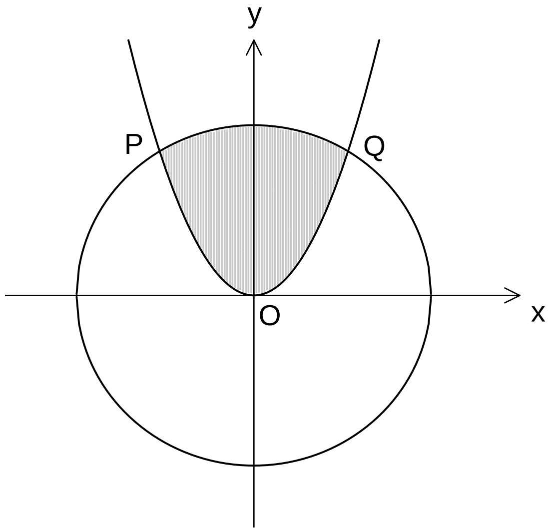
QUESTION 8 (START A NEW PAGE NOW)

a) (i) Show that $\int_0^2 \frac{1}{1+x}\, dx = \ln 3$ **1**

(i) Hence use Simpson's rule with three function values to find an approximation to $\ln 3$.

2

b) The region bounded by the parabola $y = x^2$ and the circle $x^2 + y^2 = 12$ is shown. The parabola and circle intersect at P and Q.



(i) P and Q have the same y-coordinate. Find its value. **1**

(ii) Find the exact value of the solid generated when the shaded region is rotated about the y-axis. **5**

c) Kim and Lee toss a biased coin alternately, with Kim going first. The probability that the coin shows 'heads' on any toss is $\frac{1}{3}$. The first person to throw a head wins the game. What is the probability that:

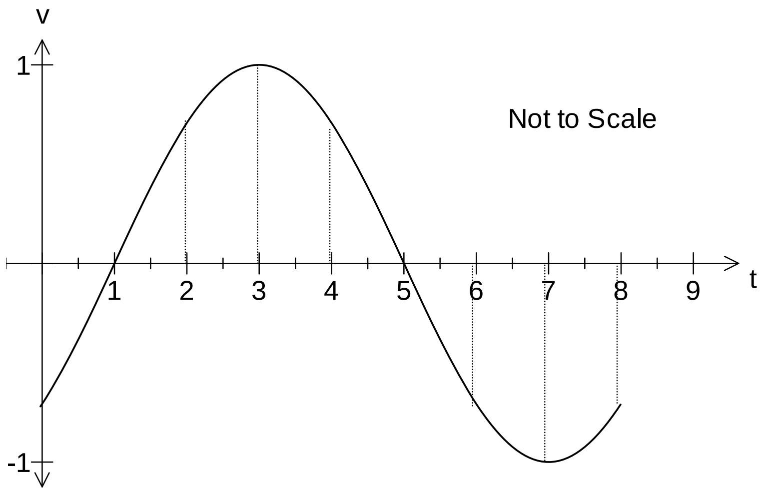
(i) Kim wins the game on her first throw? **1**

(ii) Lee wins on his first throw? **1**

(iii) after 4 tosses of the coin, there is no winner? **1**

QUESTION 9 **(**START A NEW PAGE NOW)

a) The diagram shows the velocity-time graph for a particle moving in a straight line.



State the times between t=0 and t=8 at which:

(i) The velocity is zero **1**

(ii) The acceleration is zero **1**

(iii) The speed is increasing **2**

b) The population of Town a is $P_A = 8000e^{0.02t}$ while the population of Town B is $P_B = 12000e^{-0.01t}$, where t is the time in years. How many years will it be until the two populations are equal? Answer correct to 1 decimal place. **2**

c) When Susan was born, her father deposited \$200 into a Trust account earning 12% p.a. interest compounding annually. The interest is paid on her birthday each year. Susan's father decided to deposit an additional \$200 into this account on her birthday each year, immediately after the interest is received.

(i) Find the value that the initial deposit would amount to on her 18th birthday. **1**

(ii) Let A_n=total amount in her account on her nth birthday

Show that $A_n = \frac{5000(1.12^{n+1}-1)}{3}$ **3**

(iii) Susan's father made his last deposit of \$200 on her 17th birthday. On her eighteenth birthday, he gave all the money in the account to Susan. How much money did she receive? **2**

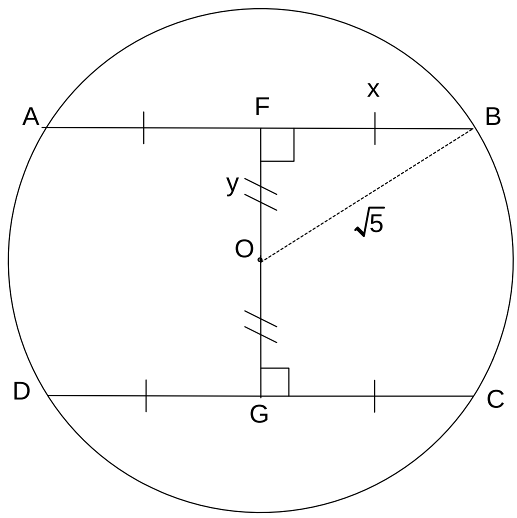
QUESTION 10 (START A NEW PAGE NOW)

a) The gradient function of a curve $y = f(x)$ is given by $f'(x) = x^2(3 - x)$.

(i) Show that the curve $y = f(x)$ has two stationary points and determine their nature. **4**

(ii) If $f(0) = 2$ and the maximum value of $f(x)$ is $8\frac{3}{4}$, draw a possible sketch of $y = f(x)$ **2**

b) A circular window of radius $\sqrt{5}$ metres requires three metal strips AB, DC and FG for reinforcement as shown. O is the centre of the window, and OF=OG=y metres and FB=FA=CG=GD=x metres.



(i) Given that L is the total length of the metal strips (ie. L=AB+CD+FG) show that

$$\mathrm{L} = 4x + 2\sqrt{5 - x^2}$$ **1**

(ii) The window will have maximum strength when the total length L is a maximum. Find the value of x for which the window has maximum strength. **5**

End of examination

Solutions : 2 unit Trial 2010

QUESTION 1

Total /120

a) 0.9735045 (1)

$= 0.97 \quad (2\,sf)$ (1)

b) $3\sqrt{3} + 4\sqrt{3} - 10\sqrt{3}$

$= -3\sqrt{3}$ (2) -1 any error

c) $2x + 3 = 3x$ or $2x + 3 = -3x$

$3 = x$ $\qquad 5x = -3$

$x = -\frac{3}{5}$ (1) both solns

Test
$LHS = |6+3| = 9$
$RHS = 3 \times 3 = 9$

Test
$LHS = \left|-\frac{6}{5} + 3\right| = \frac{9}{5}$
$RHS = -\frac{9}{5}$ not equal

$\therefore \; x = 3$ only. (1) final one soln only.

d) $\dfrac{5}{\sin\theta} = \dfrac{12}{\sin 73}$ (1)

$\cancel{12}\sin\theta = \dfrac{5\sin 73}{12} = 0.3985$

$\theta = 23^\circ$ (1)

e) $-5x \le -1$ (1)

$x \ge \frac{1}{5}$ (1)

f) $a = 2, \; r = -\frac{1}{3}$ (1)

$S_\infty = \dfrac{a}{1-r} = \dfrac{2}{1 - -\frac{1}{3}}$

$= \dfrac{2}{\frac{4}{3}}$

$= 1.5$ (1)

QUESTION 2.

a) i) $y = x^2 . \cos x$

$y' = uv' + vu'$

$\left[\begin{array}{ll} u = x^2 & u' = 2x \\ v = \cos x & v' = -\sin x \end{array}\right]$

$= x^2 . -\sin x + \cos x . 2x$ (1) (1)

$= -x^2 . \sin x + 2x . \cos x$

ii) $y = \tan^2(4x+1)$

$y' = 2\tan(4x+1) . \sec^2(4x+1) . 4$

$= 8\tan(4x+1) . \sec^2(4x+1)$ (2)

b) $\displaystyle\int \frac{x}{3x^2+1}\, dx$

$= \dfrac{1}{6}\displaystyle\int \frac{6x}{3x^2+1}\, dx \qquad \frac{f'(x)}{f(x)}$

$= \dfrac{1}{6}\ln(3x^2+1) + c.$ (1) for $\ln(3x^2+1)$ (1) for $\frac{1}{6}$

c) $\displaystyle\int_0^{\pi/9} \sin 3x .\, dx$

$= -\dfrac{1}{3}\Big[\cos 3x\Big]_0^{\pi/9}$ (2)

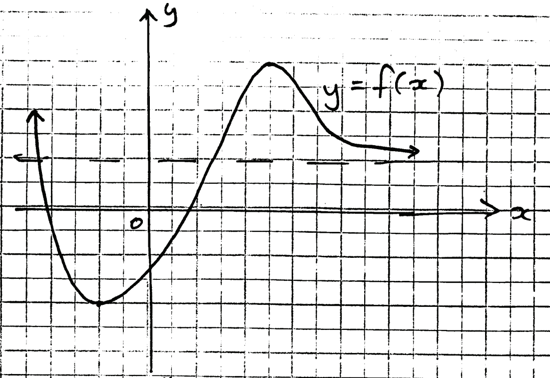
$= -\dfrac{1}{3}\left(\cos\dfrac{\pi}{3} - \cos 0\right)$

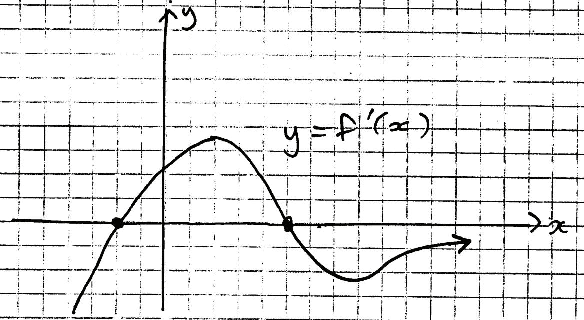
$= -\dfrac{1}{3}\left(\dfrac{1}{2} - 1\right)$

$= -\dfrac{1}{3} \times -\dfrac{1}{2}$

$= \dfrac{1}{6}$ (1)

d)





(1) x-ints
(1) max tshape
(1) asymptote

QUESTION 3

a) i) $P = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{-2+6}{2}, \frac{2-6}{2}\right)$ (1)

$= \left(\frac{4}{2}, \frac{-4}{2}\right) = (2, -2)$

ii) $m_{BP} = \frac{y_2 - y_1}{x_2 - x_1}$ $B(-1, -5)$, $P(2, -2)$

$= \cancel{\frac{-2+6}{2+1}} \quad \frac{-2+5}{2+1}$

$= \frac{3}{3} = 1$ (1)

iii) $m_{AC} = \frac{6-2}{6--2} = \frac{-8}{8} = -1$ (1)

$\therefore m_{BP} \times m_{AC} = 1 \times -1 = -1 \quad \therefore$ Perp. (1)

iv) BP has $m = 1$ and thr. $(-1, -5)$

$y + 5 = 1(x + 1)$ (1)

$y + 5 = x + 1$

$y = x - 4$

v) $\cancel{x+}\ \frac{x-1}{2} = 2 \qquad \frac{y-5}{2} = -2$

$x - 1 = 4 \qquad y - 5 = -4$

$x = 5 \qquad y = 1$

$\therefore D(5, 1)$ (1) for x, (1) for y

vi) Diagonals bisect and are perpendicular (1)

$\therefore$ Rhombus (1)

b) $\Delta = (3k+1)^2 - 4.1(4k+5)$

$= 9k^2 + 6k + 1 - 16k - 20$

$= 9k^2 - 10k - 19$ (1)

$= 0$ for real + equal roots. (1)

$k = \frac{10 \pm \sqrt{(-10)^2 - 4 \times 9 \times -19}}{2 \times 9}$

$= \frac{10 \pm \sqrt{784}}{18}$

$= \frac{38}{18}, -1 = \frac{19}{9}, -1$ (1)

QUESTION 4

a) Area shaded

$= \frac{1}{2} \cdot \cancel{AB}^{10} \cdot BC - \frac{1}{6} \cdot \pi \cdot 10^2$

$= 5 \cdot BC - \frac{100\pi}{6}$ ← (1)

Now $\tan 60^\circ = \frac{BC}{10}$, $BC = 10 \tan 60^\circ$

$= 10\sqrt{3}$ ← (1)

$\therefore$ Area $= 5 \times 10\sqrt{3} - \frac{100\pi}{6}$

(1)

$= 50\sqrt{3} - \frac{50\pi}{3}$ units^2

b) i) $T_2 = a + d = 7$

$T_7 = a + 6d = 52$

$5d = 45$

$d = 9$ (1)

$a + 9 = 7$

$a = -2$ (1)

ii) $T_n = a + (n-1)d$

$= -2 + (n-1) \cdot 9$

$= -2 + 9n - 9$

$= 9n - 11 > 1000$ (1)

when $9n > 1011$

$n > 112.\dot{3}$

$\therefore T_{113} = 9(113) - 11 = 1006$ (1)

c) $\cancel{2}x^2 - \frac{9}{2}x - \frac{3}{2} = 0$

(i) $\alpha + \beta = \frac{9}{2}$ (1)

(ii) $\alpha\beta = -\frac{3}{2}$ (1)

(iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{9}{2}\right)^2 - 2\left(-\frac{3}{2}\right)$

$= 23\frac{1}{4}$ or $\frac{93}{4}$ (1)

(iv) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$ (1)

$= \frac{9}{2} \times \left(23\frac{1}{4} - -\frac{3}{2}\right)$

$= 111.375$ or $\frac{891}{8}$ (1)

QUESTION 5

a) $m = \frac{dy}{dx} = 6x - 2$

Line has $m = 4$ (1)

$\therefore 6x - 2 = 4$

$6x = 6$

$x = 1$ (1)

$y = 3(1)^2 - 2(1) - 1 = 0$

$\therefore$ At point $(1, 0)$ (1)

b) $x = \frac{\pi}{6}, \frac{5\pi}{6}$. (2) [1 each]

−1 for degrees.

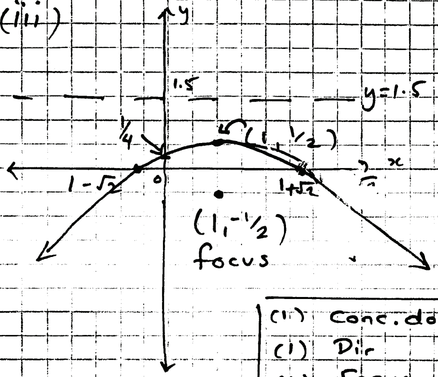
c) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = 1\frac{17}{60}$ or $\frac{77}{60}$ (1)

d) $(x-1)^2 = -4(y-\tfrac{1}{2})$

(i) Vertex $(1, \tfrac{1}{2})$ (1)

(ii) $a = 1$ (1)

(iii)



If $y = 0$

$(x-1)^2 = 2$

$x = 1 \pm \sqrt{2}$ (x-ints)

If $x = 0$

$\cancel{y =}\ 1 = -4y + 2$

$-1 = -4y$

$y = \tfrac{1}{4}$ (y-int)

y-int. Not needed.

(1) Conc. down, vertex
(1) Dir
(1) Focus
(1) x-int (one pos, one neg)

QUESTION 6.

a) i) In $\triangle BPR$, $\triangle DQS$:

$\underline{\angle PRB = \angle DSQ}$ (given, 90° each)

$AB \parallel DC$ (opp sides of parm. $\parallel$)

$\underline{\angle PBR = \angle SDQ}$ (alternate $\angle$s, $AB \parallel DC$) (1)

$BP = \tfrac{1}{2} \cdot AB$

$DQ = \tfrac{1}{2} \cdot DC$ (P, Q are midpts of sides)

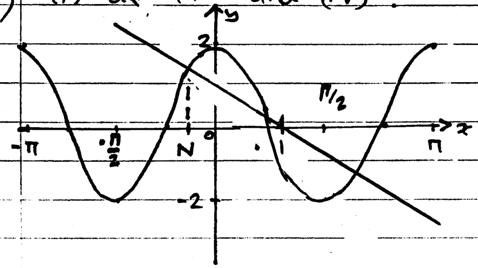
but $AB = DC$ (opp sides of parm equal)

$\therefore \underline{BP = DQ}$ (halving equal sides) (1)

$\therefore \triangle BPR \equiv \triangle DQS$ by AAS. (1)

(ii) $14 - 4 - 4 = 6$ cm (1)

b) (i) and (ii) and (iv):



(i) Pd = $\cancel{4}\pi$ (1)

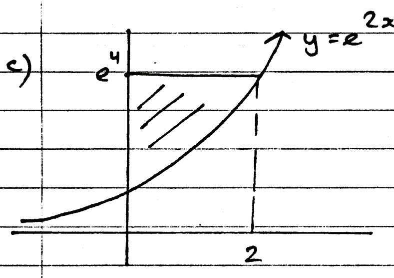
Amp = 2 (1)

(ii) Line (1)

(iii) 3 (1)

(iv) Show N (1)

c)



Area under curve

$= \int_0^2 e^{2x} \, dx$

$= \left[\tfrac{1}{2} e^{2x} \right]_0^2$ (1)

$= \tfrac{1}{2} e^4 - \tfrac{1}{2} e^0$

$= \tfrac{1}{2} e^4 - \tfrac{1}{2}$ (1)

Area shaded $= 2e^4 - \left(\tfrac{1}{2} e^4 - \tfrac{1}{2} \right)$ (1)

$= \tfrac{3}{2} e^4 + \tfrac{1}{2}$ units^2

QUESTION 7

a) $l = r\theta$ $\quad 2.2 = 2\theta$

$\theta = 1.1$ radians (1)

$= 1.1 \times \tfrac{180}{\pi}$ deg.

$= \underline{63°2'}$ (1)

Must be nearest

b) $\log_e \frac{x}{x-4} = \log_e 2$ (1)

$\frac{x}{x-4} = 2$

$x = 2x - 8$

$\underline{8 = x}$ (1)

c) $y = e^{kx}$

$y' = ke^{kx}$ (1)

$y'' = k^2 e^{kx}$ (1)

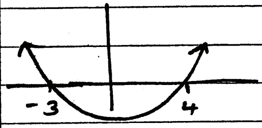
$k^2 e^{kx} - ke^{kx} - 12e^{kx} \geq 0$

$e^{kx}(k^2 - k - 12) \geq 0$

[$\div e^{kx}$ and factorise] $(k-4)(k+3) \geq 0$ (1)

Roots : $k = 4, -3$

Soln :



$k \geq 4, \quad k \leq -3$ (1)

d) (i) $\angle DCE = \angle ABC$ (alternate $\angle$s, $AB \parallel DC$) } (1)

$\angle CED = \angle ACB$ (alternate $\angle$s, $AC \parallel DE$)

$\therefore \triangle ABC \,|||\, \triangle DCE$ (matching $\angle$s equal) (1)

(ii) Let $EC = x$ $\quad \frac{x}{x+12} = \frac{8}{22} = \frac{4}{11}$ (1)

$11x = 4x + 48$

$7x = 48$

$x = \frac{48}{7}$

$\therefore BC = 12 + \frac{48}{7} = \frac{132}{7}$ (1)

QUESTION 8

a) (i) $\int_0^2 \frac{1}{1+x}\,dx = \left[\ln(1+x)\right]_0^2$

$= \ln 3 - \underbrace{\ln 1}_{0}$ (1)

$= \underline{\ln 3}$

(ii)

x	0	1	2
$y = \frac{1}{x+1}$	1	$\frac{1}{2}$	$\frac{1}{3}$

(1)

$\ln 3 \doteqdot \frac{h}{3}(y_0 + 4y_1 + y_2)$

$= \frac{1}{3}\left(1 + 4 \times \frac{1}{2} + \frac{1}{3}\right)$ (1)

$= \frac{10}{9}$.

b) (i) $y = x^2, \quad x^2 + y^2 = 12$:

$y + y^2 = 12$

$y^2 + y - 12 = 0$

$(y+4)(y-3) = 0$

$\cancel{y = -4}$ or $\underline{y = 3}$ (1)

(ii) $V_1 = \pi \int_0^3 y \,.\, dy$

$= \pi \left[\frac{y^2}{2}\right]_0^3$

$= \frac{9\pi}{2}$ (1)

Limits, $3, \sqrt{12}$: (1

$V_2 = \pi \int_3^{\sqrt{12}} 12 - y^2 \,.\, dy$

$= \pi \left[12y - \frac{y^3}{3}\right]_3^{\sqrt{12}}$ (1)

$= \pi \left(\left(12\sqrt{12} - \frac{12\sqrt{12}}{3}\right) - (36 - 9) \right)$

$= \pi \left(8\sqrt{12} - 27\right)$

$= \pi\,(16\sqrt{3} - 27)$ (1)

$\therefore$ Total vol $= V_1 + V_2$

$= \frac{9\pi}{2} + \pi\,(16\sqrt{3} - 27)$ (1)

$= \pi\left(\frac{9}{2} + 16\sqrt{3} - 27\right)$

$= \pi\,(16\sqrt{3} - \frac{45}{2})$

c) (i) $\frac{1}{3}$ (1)

(ii) $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$ (1)

(iii) $\left(\frac{2}{3}\right)^4 = \frac{16}{81}$ (1)

QUESTION 9.

a) (i) $t = 1, 5$ (1)

(ii) $t = 3, 7$ (1)

(iii) $\left.\begin{array}{l} v \text{ pos} \\ a \text{ pos} \end{array}\right\}$ $\therefore\ 1 < t < 3$ (1)

or

$\left.\begin{array}{l} v \text{ neg} \\ a \text{ neg} \end{array}\right\}$ $\therefore\ 5 < t < 7$ (1)

b) $8000e^{0.02t} = 12\,000\,e^{-0.01t}$

$e^{0.03t} = 1.5$ (1)

$0.03t = \ln 1.5$

$t = \frac{\ln 1.5}{0.03} = 13.5$ yrs (1)

c) (i) $A = 200(1.12)^{18} = \$1537.99$ (1)

(ii) $A_1 = 200(1.12) + 200$

$A_2 = A_1 \times 1.12 + 200$

$= 200(1.12)^2 + 200(1.12) + 200$ Developme

$A_3 = A_2 \times 1.12 + 200$ (1)

$= 200(1.12)^3 + 200(1.12)^2 + 200(1.12) + 200$

$= 200(1.12^3 + 1.12^2 + 1.12 + 1)$

Continuing the pattern:

$A_n = 200\,(1.12^n + 1.12^{n-1} + \ldots + 1.12 + 1)$ (1

$= 200 \times \frac{1\,(1.12^{n+1} - 1)}{1.12 - 1}$ geom serie $a = 1$ $r = 1.12$

$= \frac{200 \times (1.12^{n+1} - 1)}{0.12}$ (1) $(n+1)$ term $S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{5000(1.12^{n+1} - 1)}{3}$

(iii) Find amt $= A_{17} \times 1.12$ or $A_{18} - 200$

$= \$11\,150 \times 1.12$ or $\$12\,688 - 200$

$= \$12\,488$ (2)

-1 if out by 200

QUESTION 10

a)(i) Stat pts when $f'(x) = x^2(3-x) = 0$ (1)

i.e. When $\underline{x=0}$ or $\underline{x=3}$ (1)

Testing

x	-1	0	1
$f'(x)$	$+$	0	$+$

$(-1)^2.(3+1) = 1\times 4$ $\quad$ $1^2(3-1) = 1\times 2$

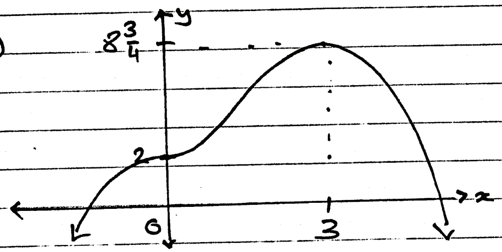
∴ Horizontal pt of inflex when $x=0$. (1)

x	2	3	4
$f'(x)$	$+$	0	$-$

$2^2(3-1) = 4\times 1$ $\quad$ $4^2(3-4) = 16\times -1$

∴ Max turn pt at $x=3$ (1)

(ii)



(2)

b)(i) $L = AB + CD + FG$

$= 2x + 2x + 2y$ $\quad$ but $\begin{cases} x^2+y^2=5 \\ y=\sqrt{5-x^2} \end{cases}$

$= 4x + 2.\sqrt{5-x^2}$ (1)

(ii) For max L, $L'=0$ and $L''<0$

$L' = 4 + \cancel{2}.\frac{1}{\cancel{2}}(5-x^2)^{-1/2}.-2x$ (1)

$= 4 - \frac{2x}{\sqrt{5-x^2}} = 0$ when ...

$\frac{2x}{\sqrt{5-x^2}} = 4$ ~~(1)~~

$\cancel{2}x = 2\cancel{4}.\sqrt{5-x^2}$

Square both sides:

$x^2 = 4(5-x^2)$

$x^2 = 20-4x^2$

$5x^2 = 20$

$x^2 = 4$

$x = \pm 2$ but $x>0$

∴ $\underline{x=2}$ (1)

Testing with L' (since L'' is messy) [Can't test with $x=$

x	1	2	2.1
L'	$+$	0	$-$

$4 - \frac{2}{\sqrt{4}}$ $\quad$ $4 - \frac{4.2}{\sqrt{5-4.41}}$ (2)

Deduct 1 if uncle

∴ L is a $\underline{max}$ when $x=2$.

If they use L'' (not recommended)

$$L' = 4 - \underbrace{2x(5-x^2)^{-1/2}}.$$

$u = 2x$ $\quad u' = 2$

$v = (5-x^2)^{-1/2}$ $\quad v' = -\frac{1}{2}(5-x^2)^{-3/2} \cdot -2x$

$= x(5-x^2)^{-3/2}$

(product rule)

$$L'' = 0 - \left(\begin{array}{l} 2x \cdot x(5-x^2)^{-3/2} + \\ (5-x^2)^{-1/2} \cdot 2 \end{array} \right)$$

$$= -\frac{2x}{(5-x^2)^{3/2}} - \frac{2}{\sqrt{5-x^2}} \quad (1)$$

$$= \frac{-4}{1^{3/2}} - \frac{2}{\sqrt{5-4}} \quad \text{when } x = 2$$

$$= -4 - 2 \quad (1)$$

$$= -6 < 0 \qquad \text{Max } L \text{ when } x = 2$$

OR

$$L'' = 4 - \left[\frac{2x}{(5-x^2)^{1/2}} \right]$$

$u = 2x$ $\quad u' = 2$

$v = (5-x^2)^{1/2}$ $\quad v' = \frac{1}{2}(5-x^2)^{-1/2} \cdot -2x$

$= \frac{-x}{\sqrt{5-x^2}}$

$$= -\left(\frac{\sqrt{5-x^2} \cdot 2 - 2x \cdot \frac{-x}{\sqrt{5-x^2}}}{5-x^2} \right) \quad (1)$$

$$= -\left(\frac{2\sqrt{5-x^2} + \frac{2x^2}{\sqrt{5-x^2}}}{5-x^2} \right)$$

$$= -\left(\frac{2\sqrt{1} + 8/\sqrt{1}}{\ } \right) \text{ when } x = 2 \quad (1)$$

$= -(2+8) < 0 \therefore$ Max L when