BAULKHAM HILLS HIGH SCHOOL

2011

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of the cover sheet.
- All necessary working should be shown in every question
- Marks may be deducted for careless or badly arranged work

Total marks – 120

- Attempt Questions 1 – 10
- All questions are of equal value
- Start each question on a new sheet of paper.
- Write your student number and the question number at the top of each sheet.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0,\ if\ n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \qquad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0,\ -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

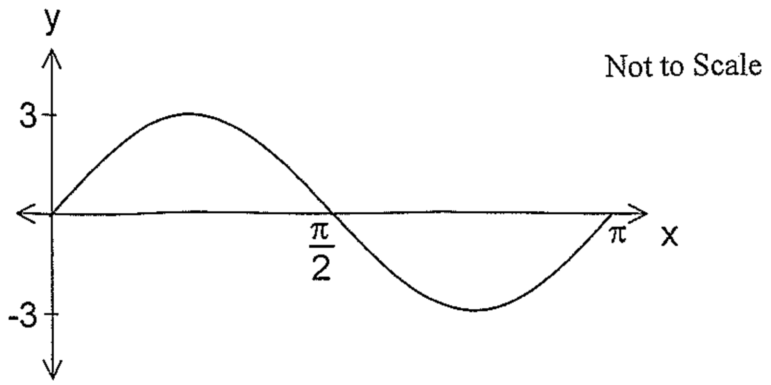
$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Question 1 **(12 marks)** **Marks**

a) Solve $|3x - 2| < 1$ **2**

b) The graph below can be expressed in the form $y = A\sin(nx)$ **2**



What are the values of A and n?

c) Find the values of a and b such that $\frac{2}{\sqrt{10}+3} = a + b\sqrt{10}$ **2**

d) Does the series below have a limiting sum ? Justify your answer. **2**

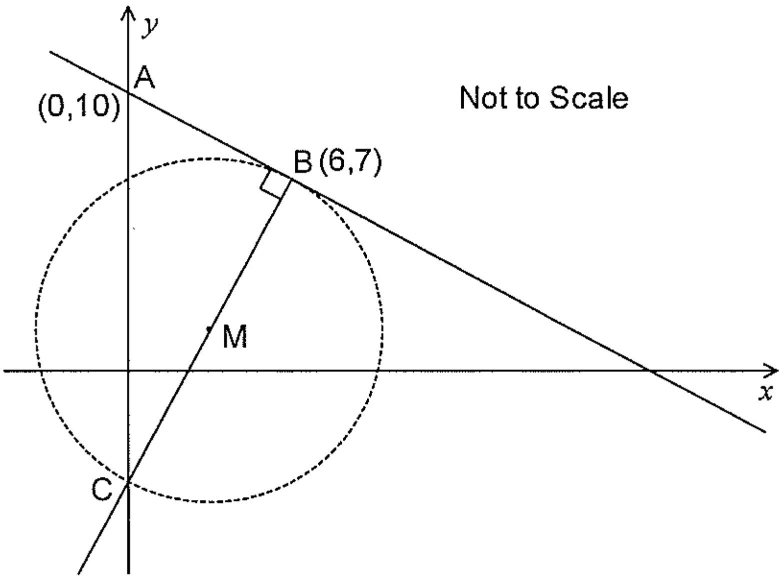
$$2 - 3 + \frac{9}{2} - \frac{27}{4} \ldots\ldots\ldots\ldots$$

e) Differentiate $\frac{e^{2x}}{x}$ **2**

f) Without sketching when is the curve $y = x^3 - 6x^2 + 9x + 2$ increasing? **2**

Question 2 **(12 marks) - Start on a new page** **Marks**

a)



(i) Find the gradient of AB. **1**

(ii) Show the equation of the line l perpendicular to AB passing through B is given by **2**

$$y = 2x - 5.$$

(iii) The line l cuts the y axis at C. Find the co-ordinates of C. **1**

(iv) M is the midpoint of BC. Find the co-ordinates of M. **1**

(v) Find the equation of the circle which passes through B and C with centre M. **2**

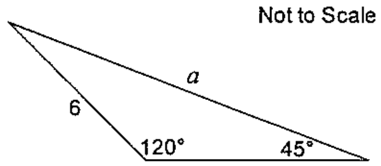
(vi) Another line $y = mx + b$ perpendicular to AB is a tangent to the circle in (v). If $b > 0$ find the point of intersection of this tangent to the circle. **1**

b) (i) Fred weighed 130 kg and lost 1.2 kg every week for a period of time. How much did Fred weigh after n weeks? **1**

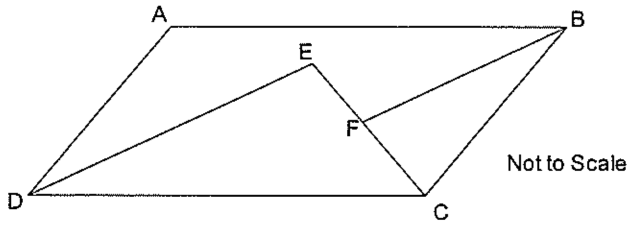
(ii) Dana started losing weight at the same time as Fred. She weighed 120.1 kg and lost 0.9 kg each week for a period of time. After how many weeks did Fred and Dana weigh the same? **2**

(iii) What percentage of Fred's body weight had he then lost? **1**

Question 3 **(12 marks) - Start a new page** **Marks**

a) 

Find the exact value of a in the diagram above. **2**

b) 

ABCD is a parallelogram. DE bisects $\angle ADC$, EC bisects $\angle DCB$ and BF bisects $\angle ABC$

(i) Prove ΔDEC ||| ΔBFC **3**

(ii) Show that $\angle DEC = 90°$ **2**

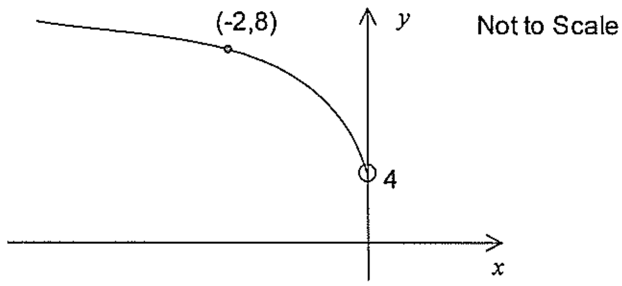
(iii) If DE $=6cm$, EC$=10\ cm$ and EF= FC, find the length of BC. **2**

c) If $log_a 3 = x$ and $log_a 2 = y$ express $log_a 18$ in terms of x and y. **1**

d) If α and β are the roots of $3x^2 - 2x + 6 = 0$ find without solving : **2**

$$\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha}$$

Question 4 **(12 marks) - Start a new page**

a) 

Above is the portion of the curve $y = f(x)$.

(i) If the function is odd sketch the entire graph of $y = f(x)$ on your own paper. **1**

(ii) State the range of the function. **1**

(iii) If $f(-2) + f(-3) = 17$ find $f(3)$. **2**

b) (i) Find $\int \sqrt{6x+1}\, dx$ **2**

(ii) Evaluate $\int_0^1 4e^{2x}\, dx$ **3**

c) Find the equation of the tangent to the curve $y = 2\cos \pi x$ at $x = \frac{1}{6}$ **3**

Question 5 **(12 marks) - Start a new page** **Marks**

a) Consider the parabola $x^2 = 4(y-5)$.

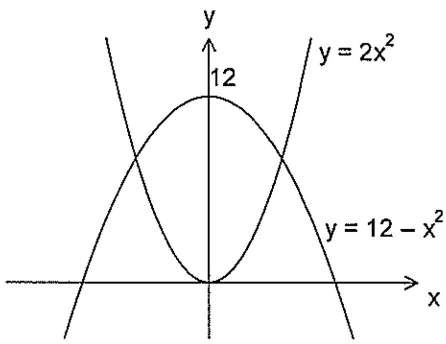
(i) Write down the co-ordinates of the vertex. **1**

(ii) What are the co-ordinates of the focus? **1**

(iii) Sketch the parabola $x^2 = 4(y-5)$ **1**

(iv) Calculate the area enclosed by the parabola and the line $y = 6$. **3**

b)



(i) Show that the points of intersection of the two curves are $(-2,8)$ and $(2,8)$. **1**

(ii) The area enclosed by the two curves is rotated about the y axis.
Find the volume of the solid generated. **5**

Question 6 **(12 marks) - Start a new page**

a) (i) Two dice are rolled and the lowest number of the two dice is recorded. **2**
What is the probability that a 2 is recorded when the two dice are rolled?

(ii) Two dice are rolled again and the numbers on both dice differ by 3. **2**
What is the probability that a 2 is recorded?

b) (i) Complete the table below for $y = \sqrt{\sin x}$ *1*

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y			0.841		

(ii) Hence estimate $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx$ using the trapezoidal rule with 4 strips. *3*

c) (i) Show that $\frac{d}{dx}(\operatorname{cosec} x) = -\cot x \operatorname{cosec} x$ **2**

(ii) Hence evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot x \operatorname{cosec} x \, dx$ **2**

Question 7 **(12 marks) - Start a new page** **Marks**

a) A farm harvested 3000 tonnes of wheat in the year 2000.
Each year the amount harvested is 4% more than the previous year.

(i) How much will be harvested in 2020? **2**

(ii) How much will be harvested from 2000 to 2020? **2**

b) A particle moving along the x axis starts at the origin. At time t seconds the particle has a displacement x metres from the origin and is travelling with a velocity of $v\ ms^{-1}$.
The displacement is given by

$$x = 4t - 6\ln(t+1).$$

(i) Find an expression for v the velocity of the particle. **1**

(ii) Find the initial velocity of the particle. **1**

(iii) Find when the particle comes to rest. **1**

(iv) Find the distance travelled by the particle in the first 3 seconds. **3**

(v) When is the acceleration of the particle positive? **2**

Question 8 **(12 marks) - Start a new page**

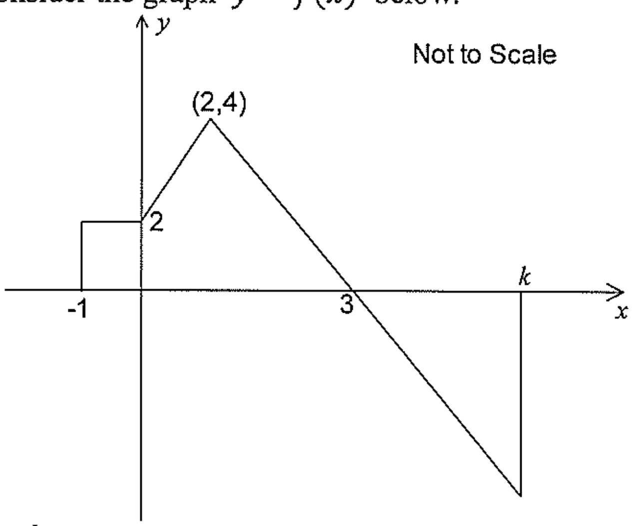
a) (i) Show that $\frac{d}{dx}[(3-x)(x-1)^3] = 2(x-1)^2(5-2x)$ **2**

(ii) Given $y = (3-x)(x-1)^3$ state the intercepts. **1**

(iii) Find the stationary points and determine their nature. **3**

(iv) Sketch the curve $y = (3-x)(x-1)^3$. **2**

b) Consider the graph $y = f(x)$ below. **4**



If $\int_{-1}^{k} f(x)\,dx = 0$ find the value of k.

Question 9 (12 marks) - Start a new page **Marks**

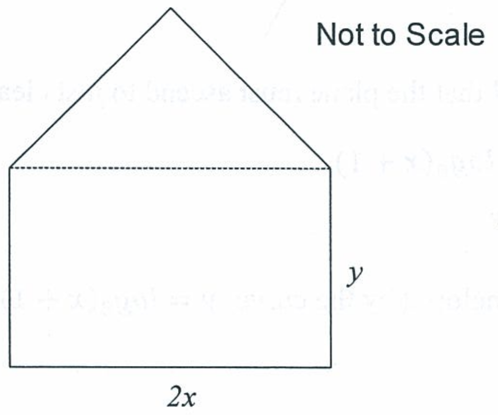
a) A drug is used to control a medical condition. It is known that the quantity Q of the drug remaining in the body after t hours satisfies the equation

$$Q = Q_0 e^{-kt} \quad \text{where } Q_0 \text{ and } k \text{ are constants.}$$

(i) An initial dose is administered and 4 hours later half the original dose remains. Find the value of k. **2**

(ii) What percentage of the initial dose remains after 6 hours? **2**

b) A window is made up of a rectangle surmounted by an equilateral triangle with dimensions as shown. The perimeter of the window is 18 metres.



(i) Show that the area (A) of the window is given by **2**

$$A = 18x - x^2(6 - \sqrt{3})$$

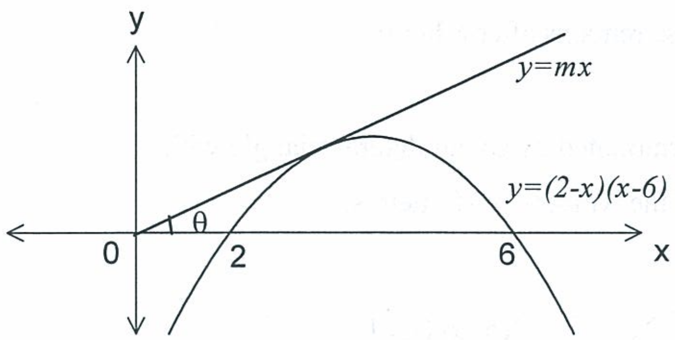
(ii) Hence find the dimensions of the window that would allow the maximum amount of light to enter the window. Give your answer to the nearest centimetre. **4**

c) If $f(x) = x + \frac{1}{x}$ show that $f(x).f\left(x + \frac{1}{x}\right) = f(x^2) + 3$ **2**

Question 10 (12 marks) - Start a new page

a) Solve $4\tan x \sin x + 3\sin x - \cos x = 0$ for $0° \leq x \leq 360°$ **3**

b) The line $y = mx$ represents the flight path of a plane which has just taken off from the airport at 0. The parabola $y = (2 - x)(x - 6)$ represents a hill that a plane must fly over. **4**



Not to Scale

Find the angle of elevation θ that the plane must ascend to just clear the hill.

c) (i) Sketch the graph $y = log_8(x + 1)$ **1**

(ii) Differentiate $x = 8^y$ **1**

(iii) Hence find the area enclosed by the curve $y = log_8(x + 1)$, the x axis and the line $x = 7$. **3**

End of Exam

1a) $|3x-2|<1$

$3x-2<1$ $-3x+2<1$

$x<1$ $-3x<-1$

$x>\frac{1}{3}$ ①

$\therefore$ Sol'n $\frac{1}{3}<x<1$ ①

AW① for either $x<1$ or $x>\frac{1}{3}$

b) $A=3$ ① $\frac{2\pi}{n}=\pi \Rightarrow n=2$ ①

c) $\frac{2}{(\sqrt{10}+3)}\times\frac{(\sqrt{10}-3)}{(\sqrt{10}-3)}=\frac{2\sqrt{10}-6}{10-9}$

$= -6+2\sqrt{10}$

$\Rightarrow a=-6$ ① $b=2$ ①

AW① for rationalising correctly

AW① for error but a & b correct from previous working (CPW)

d) $r=\frac{T_2}{T_1}=\frac{-3}{2}$ ①

Since $-\frac{3}{2} \not< |1|$ then ① the series does not have a limiting sum (Must conclude correctly)

e) $\frac{d}{dx}\left(\frac{e^{2x}}{x}\right)=\frac{2xe^{2x}-e^{2x}}{x^2}$ ②

$=\frac{e^{2x}(2x-1)}{x^2}$ → Not necessary

AW① for $\frac{d}{dx}(e^{2x})=2e^{2x}$

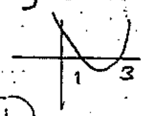
AW① for correct use of formula with 1 error.

f) y increases when $y'>0$

ie $3x^2-12x+9>0$

ie $3(x^2-4x+3)>0$ ①

$3(x-1)(x-3)>0$



$\therefore x<1$ or $x>3$ ①

2. (i) $m_{AB}=\frac{10-7}{0-6}=-\frac{1}{2}$ ①

(ii) $\perp m=2$ ①

$y-7=2(x-6)$

$y=2x-12+7$

$y=2x-5$ ①

(iii) at C $x=0$ ①

$\therefore y=-5$ $C(0,-5)$

AW① for just $y=-5$.

(iv) $M\left(\frac{0+6}{2},\frac{-5+7}{2}\right)=(3,1)$ ①

(v) $MB=\sqrt{(6-3)^2+(7-1)^2}$

$=\sqrt{45}$ ①

eqn $(x-3)^2+(y-1)^2=45$ ①

(vi) To find the point from M move ← 6 units ↑ 3 ∴ point is $(-3,4)$. ①

b) (i) Fred $=130-1.2n$ ①

(ii) $130-1.2n=120.1-0.9n$ ①

$9.9=0.3n$

$n=33$ ①

(iii) Fred weighed $130-1.2\times 33$

$=90.4$ kg

% loss $=\frac{39.6}{130}\times 100\%$

$=30.46\%$ ①

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3. a) $\frac{a}{\sin 120°}=\frac{6}{\sin 45°}$

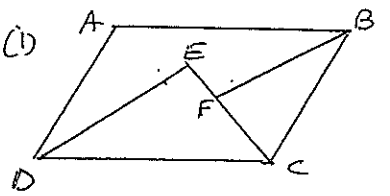
$a=\frac{6^3\cdot\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}$

$=3\sqrt{6}$

AW① for correct sub'n into sine rule.

AW① for correct exact values & wrong final answer

b) (i)



Let $\angle ADE=x°$

$\therefore \angle EDC=x°$ (DE bisects $\angle ADC$)

$\therefore \angle ABC=2x°$ (opposite ∠'s of par'm)

$\therefore$ FBC $=x°$ (BF bisects $\angle ABC$)

let $\angle ECD=y°$

$\therefore \angle ECB=y°$ (EC bisects ...

$\therefore \triangle DEC \,|||\, \triangle BFC$ (AA)

AW① for using fact that intervals bisect angles.

AW① for opposite ∠'s of par'm =

AW① for correct similarity test.

(ii) $\angle ADC+\angle BCD=180°$ (Cointerior ∠'s on || lines)

$\therefore 2x+2y=180°$

$\therefore x+y=90°$ ①

hence $\angle DEC=90°$ (Angle Sum of Δ) ①

(iii) $\frac{EC}{FC}=\frac{DE}{BF}$ (sides in ratio in ||| Δ's) ← not essential

ie $\frac{10}{5}=\frac{6}{BF}$

$\therefore BF=3$ ①

$\therefore BC^2=BF^2+FC^2$

$BC=\sqrt{3^2+5^2}$

$=\sqrt{34}$ ①

c) $\log_a 3=x$ $\log_a 2=y$

$\therefore \log_a 18=2\log_a 3+\log_a 2$ ①

$= 2x + y$

d) $3x^2-2x+6=0$

$\alpha+\beta=\frac{2}{3}$ $\alpha\beta=2$ ①

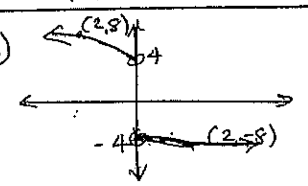
$\therefore \frac{1}{\alpha^2\beta}+\frac{1}{\beta^2\alpha}=\frac{\beta+\alpha}{\alpha^2\beta^2}$

$=\frac{2}{3}\div 2^2$

$=\frac{1}{6}$ ①

AW① if error in $\alpha+\beta$ or $\alpha\beta$ but CPW

4 a) (i)



(ii) Range $y\geq 4$ or $y<-4$

(iii) $f(-2)+f(-3)=17$

then $f(2)+f(3)=-17$ ①

$\therefore -8+f(3)=-17$

$\therefore f(3)=-9$ ①

b (i) $\int(6x+1)^{\frac{1}{2}}dx$ ①

$=\frac{(6x+1)^{\frac{3}{2}}}{\frac{3}{2}\times 6}+C$ ①

$=\frac{1}{9}(6x+1)^{3/2}+C$ (ignore c)

b(ii) $\int_0^1 4e^{2x}\,dx$

$= 2\left[e^{2x}\right]_0^1$ ①

$= 2(e^2 - e^0)$ ①

$= 2(e^2 - 1)$ ①

Aw① for incorrect ∫ but CPW

c) $y = 2\cos \pi x$

at $x = \frac{1}{6}$ $y = 2\cos\frac{\pi}{6} \Rightarrow y = \sqrt{3}$ ①

$y' = -2\pi \sin \pi x$

at $x = \frac{1}{6}$ $y' = -2\pi \sin\frac{\pi}{6}$
$= -\pi$ ①

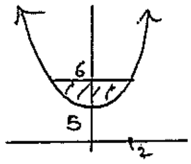
$\therefore$ Eq'n $y - \sqrt{3} = -\pi\left(x - \frac{1}{6}\right)$
$y = -\pi x + \frac{\pi}{6} + \sqrt{3}$ ①

5 a) $x^2 = 4(y-5)$

(i) Vertex $(0,5)$ ①

(ii) $4a = 4$ $a = 1$
$\therefore S(0,6)$ ①

(iii)



(iv) when $y = 6$ $x = \pm 2$

$x^2 = 4(y-5) \Rightarrow y = \frac{x^2}{4} + 5$

$\therefore$ Area $= 2\int_0^2 6 - \left(\frac{x^2}{4} + 5\right) dx$
$= 2\int_0^2 1 - \frac{x^2}{4}\, dx$ ①

$= 2\left[x - \frac{x^3}{12}\right]_0^2$ ①

$= 2\left[\left(2 - \frac{8}{12}\right) - 0\right]$

$= \frac{8}{3}$ ①

b) $12 - x^2 = 2x^2$
$\therefore 3x^2 = 12$
$x = \pm 2$
$y = 2(\pm 2)^2 \to y = 8$

$\therefore$ pts $(2,8)$ $(-2,8)$

(ii) $y = 2x^2 \to x^2 = \frac{y}{2}$

$y = 12 - x^2 \to x^2 = 12 - y$

$\therefore$ Volume $=$
$\pi\int_0^8 \frac{y}{2}\,dy + \pi\int_8^{12} 12 - y\, dy$

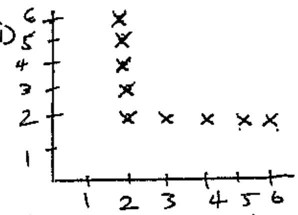
① for correct bounds
① for correct substitution for x
① for summing volumes

$= \pi\left[\frac{y^2}{4}\right]_0^8 + \pi\left[12y - \frac{y^2}{2}\right]_8^{12}$ ①

$= \pi\left[(16 - 0)\right] + \pi\left[(144 - 72) - (96 - 32)\right]$

$= 16\pi + 8\pi$ ①
$= 24\pi$

6 a) (i)



x denotes 2 as lowest score

$\therefore P(\text{2 recorded}) = \frac{9}{36}$ → ①
① $= \frac{1}{4}$

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6 a)(ii) Sample Space
$(1,4)$ $(2,5)$ $(3,6)$ or $(4,1)$
$(5,2)$ $(6,3)$ ①

$\therefore P(\text{recording } 2) = \frac{2}{6} = \frac{1}{3}$ ①

(i)

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0	0.619	0.841	0.961	1

①

$y = \sqrt{\sin x}$

(ii) Area $= \frac{\frac{\pi}{8}}{2}\left[0 + 1 + 2(0.619 + .841 + .961)\right]$ ① ①

$= 1.147$ → ①

c (i) $\frac{d}{dx}(\sin x)^{-1} = -1(\sin x)^{-2}\cos x$ ①

① $\begin{cases} = \frac{-\cos x}{\sin^2 x} \\ = -\cot x \operatorname{cosec} x \end{cases}$

(ii) $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cot x \operatorname{cosec} x$

$= -(\operatorname{cosec} x)_{\frac{\pi}{6}}^{\frac{\pi}{4}}$ ①

$= -\left[\operatorname{cosec}\frac{\pi}{4} - \operatorname{cosec}\frac{\pi}{6}\right]$

$= -(\sqrt{2} - 2)$

$= 2 - \sqrt{2}$ (0.585) ①

7 a) (i) $3000 \times 1.04^{20} = 6573.4t$
① ①

(ii) Total $= 3000 + 3000 \times 1.04^1 + \dots$
$\dots + 3000 \times 1.04^{20}$ ①

$S_n = \frac{3000(1.04^{21} - 1)}{1.04 - 1}$ ①

$= 95907.6\,t$

b) $x = 4t - 6\ln(t+1)$

(i) $v = 4 - \frac{6}{t+1}$ ①

(ii) when $t = 0$ $v = -2$ m/s ①

(iii) $v = 0$ $4 = \frac{6}{t+1}$
$t + 1 = \frac{3}{2}$
$t = \frac{1}{2}$ ①

(iv) when $t = 0$ $x = 0$
① $t = \frac{1}{2}$ $x = 2 - 6\ln\frac{3}{2}$
$= (-0.433)$

when $t = 3$ $x = 12 - 6\ln 4$
(3.682) ①

$\therefore$ Distance $= 2 \times |-0.433| + 3.682$
$= 4.548$ ①

(v) $a = \frac{d}{dt}\left(4 - 6(t+1)^{-1}\right)$

$= \frac{6}{(t+1)^2}$ ①

① $\begin{cases} \text{since } (t+1)^2 > 0 \text{ for all } t \\ \text{then } \frac{6}{(t+1)^2} > 0 \text{ for all } t \\ \therefore \text{acc'n is always positive} \end{cases}$

8 (i) $\frac{d}{dx}\left((3-x)(x-1)^3\right)$ ①

$= -1(x-1)^3 + 3(x-1)^2(3-x)$

$= (x-1)^2\left[-(x-1) + 3(3-x)\right]$

$= (x-1)^2\left[-x + 1 + 9 - 3x\right]$

$= (x-1)^2(10 - 4x)$ ①

$= 2(x-1)^2(5 - 2x)$

(ii) at $y = 0$ $(3-x)(x-1)^3 = 0$
$\therefore x = 3, 1$
when $x = 0$ $y = -3$ ①

(iii) st pts $2(x-1)^2(5-2x) = 0$

① $\begin{cases} x = 1 & , \frac{5}{2} \\ y = 0 & y = \frac{27}{} \end{cases}$

Test.

Nature using y'

$x = 1$ (1)

x	0	1	2
y'	10 $\nearrow$	0 $\leftrightarrow$	2 $\nearrow$

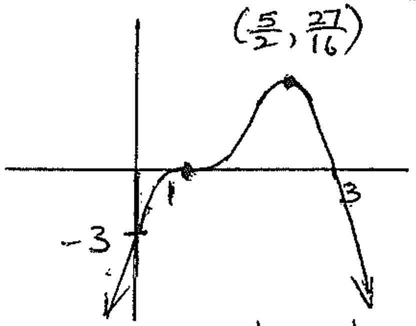
∴ Horizontal pt. of inflex.

$x = 2\frac{1}{2}$ (1)

x	2	$2\frac{1}{2}$	3
y'	2 $\nearrow$	0 $\leftrightarrow$	-2 $\searrow$

∴ Max t.p.

(iv)



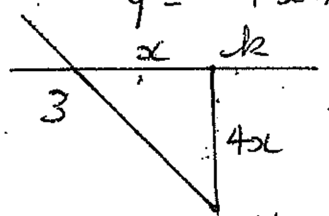
(2)

Must show intercepts & st. pts.

b) Area above x axis $= 10$ unit^2 (1)

eq'n of line through (2,4) (3,0)

$y = -4x + 12$ (1)



$\frac{1}{2} \times 4x^2 = 10$ (1)

$x^2 = 5$

$x = \pm\sqrt{5}$

$x > 0 \quad x = \sqrt{5}$

∴ $k = 3 + \sqrt{5}$ (1)

9 a) (i) $Q = Q_0 e^{-kt}$

when $t = 4$ $Q = \frac{Q_0}{2}$

∴ $\frac{Q_0}{2} = Q_0 e^{-4k}$ (1)

$\frac{1}{2} = e^{-4k}$

∴ $\ln \frac{1}{2} = -4k \ln e$

∴ $k = \frac{\ln \frac{1}{2}}{-4}$

$= 0.173...$ (1)

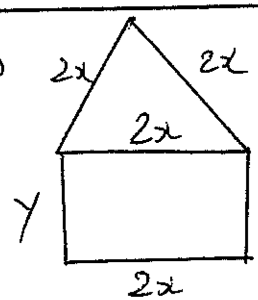
(ii) when $t = 6$

$Q = Q_0 e^{-6(0.173...)}$ (1)

$= Q_0 \times 0.3535...$

∴ 35.4% of the original dose remains (1)

b) (i)



$A = 2xy + \frac{1}{2}.(2x)^2 \sin 60$

$= 2xy + 2x^2.\frac{\sqrt{3}}{2}$ (1)

$A = 2xy + x^2\sqrt{3}$

but $6x + 2y = 18$

∴ $y = 9 - 3x$

∴ $A = 2x(9 - 3x) + x^2\sqrt{3}$

$= 18x - 6x^2 + x^2\sqrt{3}$

$= 18x - x^2(6 - \sqrt{3})$ (1)

c) $\frac{dA}{dx} = 18 - 2(6 - \sqrt{3})x = 0$ (1)

when $x = \frac{18}{2(6 - \sqrt{3})}$

$= \frac{9}{6 - \sqrt{3}}$ (1)

test $A'' = -2(6 - \sqrt{3}) < 0$

∴ max t.p. (1)

$x = 211$ cm.

∴ $2x = 422$ cm

$y = 900 - 3(211)$

$= 267$ cm. (1)

c) $f(x) = x + \frac{1}{x}$

$f(x + \frac{1}{x}) = x + \frac{1}{x} + \frac{1}{(x + \frac{1}{x})}$

$f(x).f(x + \frac{1}{x}) = (x + \frac{1}{x})\left[(x + \frac{1}{x}) + \frac{1}{(x + \frac{1}{x})}\right]$ (1)

$= \left(x + \frac{1}{x}\right)^2 + 1$

$= x^2 + \frac{1}{x^2} + 2 + 1$

$= x^2 + \frac{1}{x^2} + 3$ (1)

$= f(x^2) + 3.$

10 a) $4\tan x \sin x + 3\sin x - \cos x = 0$

$4\frac{\sin^2 x}{\cos x} + 3\sin x - \cos x = 0$ (1)

$4\sin^2 x + 3\sin x \cos x - \cos^2 x = 0$

$(4\sin x - \cos x)(\sin x + \cos x) = 0$

$\therefore 4\sin x = \cos x \quad \sin x = -\cos x$ (1)

$\therefore \tan x = \frac{1}{4} \quad \tan x = -1$

$x = 14°2', 194°2', 135°, 315°$ (1)

b) Need pt of intersection

ie $mx = (2-x)(x-6)$

$mx = -x^2 + 8x - 12$ (1)

$\therefore 0 = -x^2 + x(8-m) - 12$

Since $y = mx$ is a tangent then only 1 point of intersection

$\therefore \Delta = 0$

ie $(8-m)^2 - 4 \times -1 \times -12 = 0$

$64 - 16m + m^2 - 48 = 0$

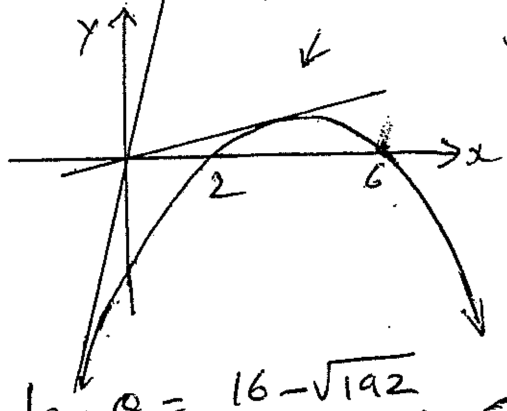
$m^2 - 16m + 16 = 0$

$m = \frac{16 \pm \sqrt{(-16)^2 - 4.16}}{2}$

(1) $m = \frac{16 \pm \sqrt{192}}{2}$

$\therefore m = \frac{16 + \sqrt{192}}{2}, \quad m = \frac{16 - \sqrt{192}}{2}$

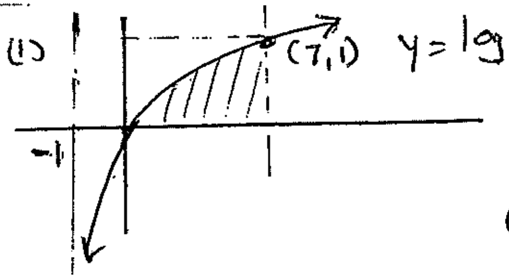
The required gradient is the smallest value of m since steeper gradient will intersect below x axis (1)



$\therefore \tan\theta = \frac{16 - \sqrt{192}}{2}$ (1)

$\theta = 46°59'$

10 c) (i)

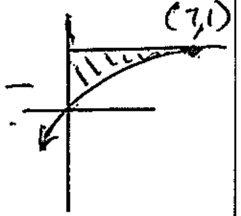


(1)

(ii) $x = 8^y$

$\frac{dx}{dy} = 8^y \ln 8$ (1)

(iii) Area = Rectangle −



$y = \log_8(x+1)$

$x + 1 = 8^y$

$x = 8^y - 1$

$\therefore \text{Area} = 7 \times 1 - \int_0^1 8^y - 1$ (1)

$= 7 - \left[\frac{8^y}{\ln 8} - y\right]_0^1$ (1)

$= 7 - \left[\left(\frac{8}{\ln 8} - 1\right) - \left(\frac{1}{\ln 8} - 0\right)\right]$

$= 7 - \frac{8}{\ln 8} + 1 + \frac{1}{\ln 8}$

$= 8 - \frac{7}{\ln 8}$ (4.634) (1)