BAULKHAM HILLS HIGH SCHOOL

TRIAL 2013

YEAR 12 TASK 4

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 180 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-16
- Marks may be deducted for careless or badly arranged work
- All the diagrams are not to scale

Total marks – 100

Exam consists of 10 pages.

This paper consists of TWO sections.

<u>Section 1</u> – Page 2 and 3 (10 marks)
Questions 1-10

- Attempt Question 1-10
- Allow about 15 minutes for this section

<u>Section II</u> – Pages 4-9 (90 marks)

- Attempt questions 11-16
- Allow about 2 hours and 45 minutes for this section

Table of Standard Integrals is on page 10

Section I - 10 marks

Use the multiple choice answer sheet for question 1-10
Allow about 15 minutes for this section.

1. Evaluate to three significant figures $\frac{4.67\times\sin\ 28^{\circ}}{\sqrt{4.6\times10^6}}$

(A) 1.02

(B) 0.06

(C) 2.89×10^7

(D) 1.02×10^{-3}

2. The value of the limit $\lim_{x\to10} \dfrac{x^2-100}{x-10}$

(A) undefined

(B) 0

(C) 8

(D) 20

3. Solve the equation $2x - 5 = \frac{x+3}{2}$

(A) $x = \frac{-7}{3}$

(B) $x = \frac{-2}{3}$

(C) $x = \frac{8}{3}$

(D) $x = \frac{13}{3}$

4. The first term of an arithmetic progression is 3, and eleventh term is 23. The n^{th} term is

(A) $T_n = 3 + 23(n-1)$

(B) $T_n = 23 + 10(n-1)$

(C) $T_n = 3 + 2(n-1)$

(D) $T_n = 3 + 10(n-1)$

5. Given that $\cos x = 0.5$ and $0^{\circ} < x < 90^{\circ}$, which of the following has the greatest value?

(A) $\cos^2 x$

(B) $\sin x$

(C) $\tan x$

(D) 0.75

6. Given that $f(x) = x^2 + x$, find the values of b for which $f''(b) = f(b)$

(A) $b = 2$ and 1

(B) $b = -1$ and 2

(C) $b = -2$ and -1

(D) $b = -2$ and 1

7. If $\log_{10} 7 = a$ then $\log_{10}\left(\frac{1}{70}\right)$ is equal to

(A) $-(1 + a)$

(B) $(1 + a)^{-1}$

(C) $\frac{a}{10}$

(D) $\frac{1}{10a}$

8. The number of animals P on an island, at time t is given by $P = 7000e^{-kt}$, where k is a positive constant. Over time the number of animals on the island is

(A) increasing exponentially.

(B) decreasing exponentially.

(C) increasing at a constant rate

(D) decreasing at a constant rate.

9. If $z = 1 - y^3$ and $y = 1 - x$, then $z =$

(A) $x^3 + 3x^2 + 3x$

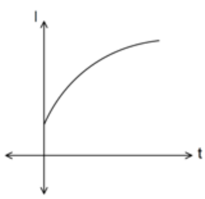
(B) $x^3 - 3x^2 + 3x$

(C) $x^3 - 3x^2 - 3x$

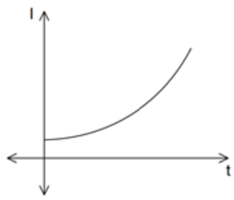
(D) $x^3 + 3x^2 - 3x$

10. Interest rates are increasing at an decreasing rate . Which of the following graphs represents the above statement.

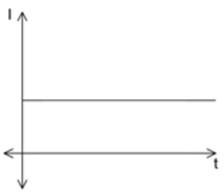
(A)



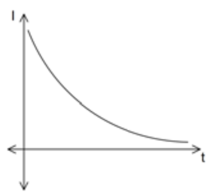
(B)



(C)



(D)



End of Section I

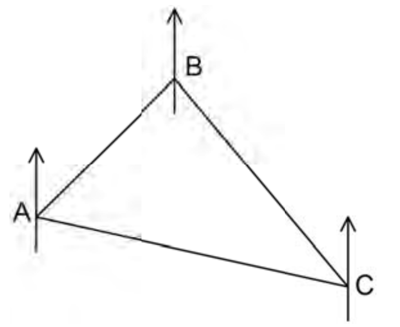
Section II – Extended Response

Attempt questions 11-16.
Allow about 2 hours and 45 minutes for this section.
Answer each question on a SEPARATE PAGE. Clearly indicate question number.
Each piece of paper must show your BOS#.
In Question 11-16, your responses should include relevant mathematical reasoning and/or calculation

Question 11 (15 marks)		**Marks**
a)	Solve for x $$\frac{x+6}{2} \leq x$$	**2**
b)	Express $\frac{3+\sqrt{2}}{6+\sqrt{2}}$ with rational denominator.	**2**
c)	Factorise $3x^2 - 48y^2$	**2**
d)	Solve for x $\lvert 2x-5 \rvert = 3$	**2**
e)	Find the value of *k* for which $(k-2)x^2 - 2kx - 1 = 0$ has real and distinct roots.	**3**
f)	If α and β are the roots of $2x^2 - 3x + 6 = 0$ then write the value of :	
	(i) $\alpha + \beta$	**1**
	(ii) $\alpha\beta$	**1**
	(iii) $(2+\alpha)(2+\beta)$	**2**
	End of Question 11	

Question 12 (15 marks)

a) A bush walker walks from Point A on a bearing of 030° for 2.4km to Point B. He changes direction at B to a bearing of 145° to avoid a swamp and follows this course for 3.6km to Point C.



(i) Copy the diagram in your booklet and mark on it all the given information.

(ii) Calculate the distance from Point A to Point C (to one decimal place). **2**

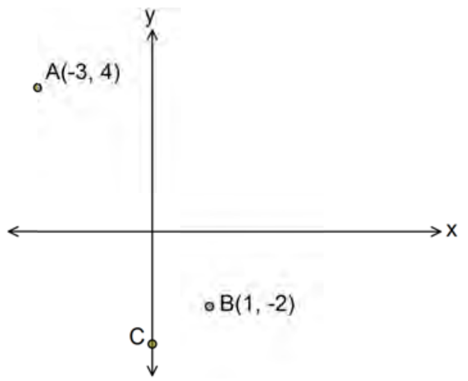
(iii) What is the bearing of Point C from Point A? **2**

b) Differentiate the following functions with respect to x

(i) $\cos^2 3x$ **2**

(ii) $\ln\left(\frac{1-x}{1+x}\right)$ **2**

c)



(i) Show that the equation through A(-3, 4) and B(1, -2) is $3x + 2y + 1 = 0$. **1**

(ii) Find the perpendicular distance from C(0, -3) to the line AB. **2**

(iii) Hence find the area of ΔABC. **2**

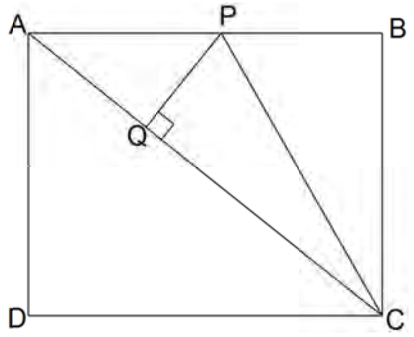
d) Find the value of $\tan x$ when $\tan^2 x + \sec^2 x = 9$ **2**

End of Question 12

Question 14 (15 marks) **Marks**

a) Tangents of the parabola $y = x^2 + ax - 3$ at $x = 1$ and $x = 0$ are known to be perpendicular. Find the value of a. **2**

b) In the diagram ABCD is a square. PC bisects $\angle ACB$ and Q is the foot of the perpendicular from P to AC.



(i) Copy the diagram into your booklet

(ii) Prove that $\Delta PBC \equiv \Delta PQC$. **3**

(iii) Hence if $BC = x$cm and $PB = \frac{1}{3}AB$, prove that the area of ΔAPQ is $\frac{x^2}{6}$ cm^2. **3**

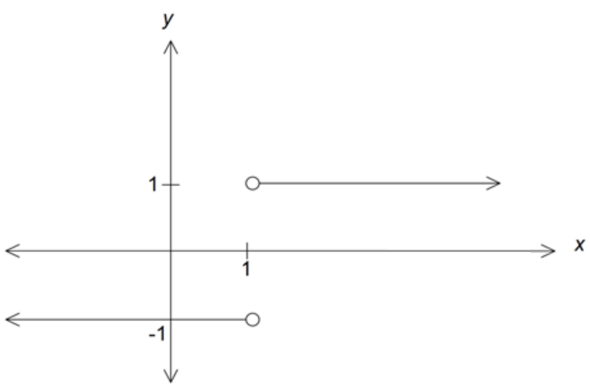
c) A train travels from Oxford to Cambridge. It's velocity $\dot{x}$ after leaving Oxford is given by $\dot{x} = \frac{1}{5}t(4-t)$ where x is measured in kilometres and time in minutes. Find

(i) The time taken to travel to Cambridge. **1**

(i) The maximum velocity attained. **2**

(ii) The distance between Oxford and Cambridge. **2**

d) Sketch a possible function which could have the gradient function as graphed below. **2**

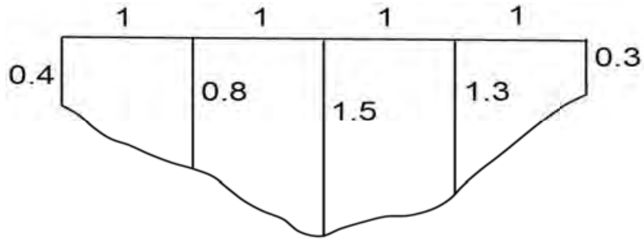


End of Question 14

Question 13 (15 marks)

a)	Find (i) $\int \frac{2}{3x} - e^{-3x}\,dx$	**2**
	(ii) $\int_0^{\frac{\pi}{6}} \sec^2 2x\,dx$	**2**
b)	Find the equation of the tangent to the curve $y = 3\sin\frac{x}{2}$ at $x = \frac{3\pi}{2}$	**3**
c)	For the function $y = x^3 - 6x^2 + 9x + 1$ find the	
	(i) Stationary points and determine their nature.	**3**
	(ii) Coordinates of any point of inflexion.	**2**
	(iii) Hence sketch the curve for $-1 \leq x \leq 5$	**3**
	End of Question 13	

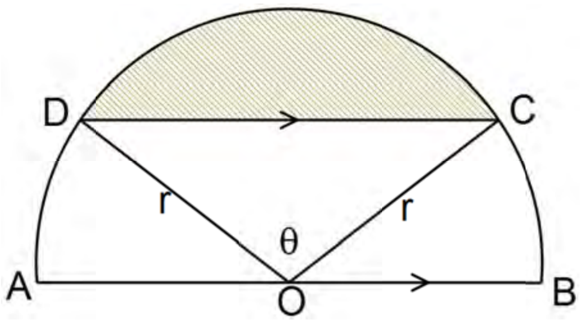
Question 15 (15 marks)		**Marks**
a)	The depth of water in the cross section of a creek 4m wide was measured and recorded in the table below.	



(i) By applying Simpson's Rule with the five depth measurements, find the cross sectional area of the creek. **2**

(ii) If the water is flowing at a rate 0.25ms^{-1}, calculate the volume of water which passes this point in one day. **1**

b) The shaded segment has an area one half of the semicircle with centre O and radius r. $CD \parallel AB$.



Prove that $\frac{\pi}{2} = \theta - \sin\theta$. **3**

c) The population of a country given by $P = P_o e^{kt}$, is increasing at a rate proportional to its size. The population was 9 million in 1954 and 20 million in 2004.

(i) Find the value of P_0 and k **3**

(ii) At what rate is the population increasing after 10 years?? **2**

(iii) In what year will the population reach 50 million? **2**

d) If $y^2 = (2m - y)(2n - y)$ show that $\frac{1}{m}, \frac{1}{y}, \frac{1}{n}$ is an arithmetic sequence. **2**

End of Question 15

Question 16 (15 marks) | **Marks**

a) Kelvin borrows \$200 000 from his bank. Interest is compounded monthly at 0.425% per month. A_n is the amount owed after n payments, \$M is the amount of the monthly instalments and the loan is repaid after n months.

(i) Show that $A_2 = 200\,000(1.00425)^2 - M(1.00425) - M$ **1**

(ii) Show that the monthly repayment, $\$M$ is given by **3**

$$M = \frac{200\,000(1.00425)^n(0.00425)}{1.00425^n - 1}$$

(iii) Find the amount of the monthly instalments if Kelvin agrees to repay the loan over 30 years. **1**

(iv) How much will Kelvin pay in total after 30 years? **1**

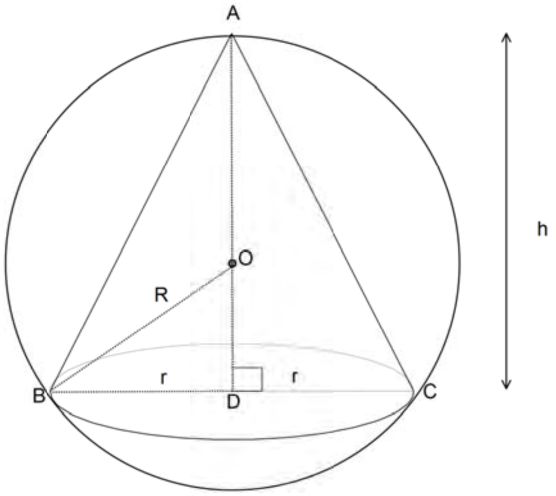
(v) If Kelvin instead decided to pay monthly instalment of \$1331 from the beginning of the loan, how long will he take to repay the loan? **2**

b) If $\sin x \neq \pm 1$ show that

$$1 + \sin^2 x + \sin^4 x + \sin^6 x + \cdots = \sec^2 x$$ **2**

c) (i) If r is the radius of the cone show that $r^2 = 2hR - h^2$ **1**

(ii) Show that volume of the cone that can be inscribed in a sphere of radius R is given by
$V = \frac{1}{3}\pi(2h^2R - h^3)$ where h is the height of the inscribed cone. **1**



(ii) Show that the volume of the largest cone is $\frac{8}{27}$ of the volume of the sphere. **3**

End of Exam

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \quad if \quad n < 0$$

$$\int \frac{1}{x} dx = \ln x, \qquad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln(x + \sqrt{x^2 - a^2}), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln(x + \sqrt{x^2 + a^2})$$

NOTE: $\ln x = \log_e x, \quad x > 0$

M.C

Q1 D Q2 D Q3 D Q4 C Q5 C
Q6 D Q7 A Q8 B Q9 B Q10 A

Q11 a) $\frac{x+6}{2} \le x$

$x+6 \le 2x$

$6 \le x$ — (2) with working.

$x \ge 6$

b) $\frac{3+\sqrt{2}}{6+\sqrt{2}} \times \frac{6-\sqrt{2}}{6-\sqrt{2}}$ — (1)

$\frac{18-3\sqrt{2}+6\sqrt{2}-2}{36-2}$

$\frac{16+3\sqrt{2}}{34}$ — (1)

c) $3(x^2-16y^2)$ — (1)

$3(x-4y)(x+4y)$ — (1)

d) $2x-5=3$ | $2x-5=-3$

$2x=8$ | $2x=2$

$x=4$ (1) | $x=1$ — (1)

e) For real & distinct roots

$\Delta > 0$

$4k^2-4\times-1\times(k-2) > 0$ — (1)

$4k^2+4k-8>0$

$k^2+k-2>0$ — (1)

$(k+2)(k-1)>0$

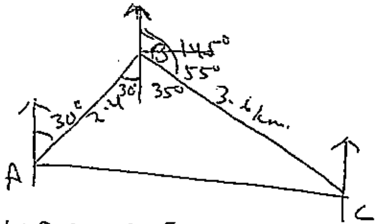
$k>1$ or $k<-2$ — (1)

f) (i) $\alpha+\beta = \frac{-b}{a} = \frac{3}{2}$ — (1)

(ii) $\alpha\beta = \frac{c}{a} = \frac{6}{2} = 3$ — (1)

(iii) $4 + 2(\alpha+\beta)+\alpha\beta = 4+2\times\frac{3}{2}+3 = 10$ — (2)

Q12a



(i) $\angle CBA = 65°$ — (1)

$AC^2 = (2.4)^2+(3.6)^2-2\times2.4\times3.6\cos65°$ — (1)

$AC = 3.4$ km

(ii) $\frac{\sin A}{3.6} = \frac{\sin 65°}{3.4}$ — (1)

$\sin A = \frac{3.6\times\sin65°}{3.4}$

$A = 73°39'$ or $74°$

∴ Bearing of C from A $= 30+73°39'$
$= 103°39'$ — (1)
or $107°$

b) (i) $y = \cos^2 3x$

$y' = 2\cos3x \times -\sin3x\times3$

$= -6\sin3x\cos3x$ (2)

−1 for any error.

(ii) $y = \ln(1-x) - \ln(1+x)$ — (1)

$y' = \frac{1}{1-x}\times-1 - \frac{1}{1+x}$

$= \frac{-1}{1-x} - \frac{1}{1+x}$ — (1) $= \frac{-2}{1-x^2}$

g) (i) $m_{AB} = \frac{-2-4}{1+3} = -\frac{3}{2}$

$y+2 = \frac{-3}{2}(x-1)$ — (1)

$2y+4 = -3x+3$

$3x+2y+1=0$.

(ii) $D = \frac{|3\times0+2\times-3+1|}{\sqrt{9+3}}$ — (1) — (1)

$= \frac{5}{\sqrt{13}}$

(iii) $d_{AB} = \sqrt{4^2+36} = \sqrt{52}$ — (1)

$A_r = \frac{1}{2}\times\frac{5}{\sqrt{13}}\times\sqrt{52}$

$= \frac{1}{2}\times\frac{5}{\sqrt{13}}\times\sqrt{13}\times2$ (1)

$= 5\ u^2$

d) $\tan^2x+1+\tan^2x = 9$ — (1)

$2\tan^2 x = 8$

$\tan x = \pm2$ — (1)

Q13 a)(i) $\int \frac{2}{3x} - e^{-3x}\,dx$

$= \frac{2}{3}\ln x + \frac{e^{-3x}}{3} + C$ (1) (1)

(−1) if no C

(ii) $\int_0^{\pi/6} \sec^2 2x\,dx = \frac{1}{2}\tan 2x\Big]_0^{\pi/6}$ — (1)

$= \frac{1}{2}\tan\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ — (1)

i) $y = 3\cos\frac{x}{2} \times \frac{1}{2}$

$m_T = \frac{3}{2} \times \cos\frac{3\pi}{4} = -\frac{3}{2\sqrt{2}}$ — (1)

$y = 3 \times \sin\frac{3\pi}{4} = \frac{3}{\sqrt{2}}$ — (1)

eq. of tangent is

$y - \frac{3}{\sqrt{2}} = -\frac{3}{2\sqrt{2}}\left(x - \frac{3\pi}{2}\right)$ — (1)

$2\sqrt{2}y - 6 = -3x + \frac{9\pi}{2}$

$3x + 2\sqrt{2}y - 6 - 9\pi/2 = 0.$

$y = x^3 - 6x^2 + 9x + 1$

$y' = 3x^2 - 12x + 9$

For stat pts $y' = 0$

$3(x^2 - 4x + 3) = 0$

$(x-3)(x-1) = 0$

$x = 3, 1$
$y = 1, 5$ } — (1)

coordinates of stats pts are $(3,1), (1,5)$

$y'' = 6x - 12$

at $x = 1$

$y'' = 6 - 12 = -6 < 0$ — (1)

∴ max. at $(1,5)$

at $x = 3$

$y'' = 18 - 12 = 6 > 0$ — (1)

∴ min at $(3,1)$

(ii) For P.O.I

$y'' = \frac{d^2y}{dx^2} = 0$

$6x - 12 = 0$

$x = 2$ — (1)

x	1	2	3
y''	-6	0	6

— (1)

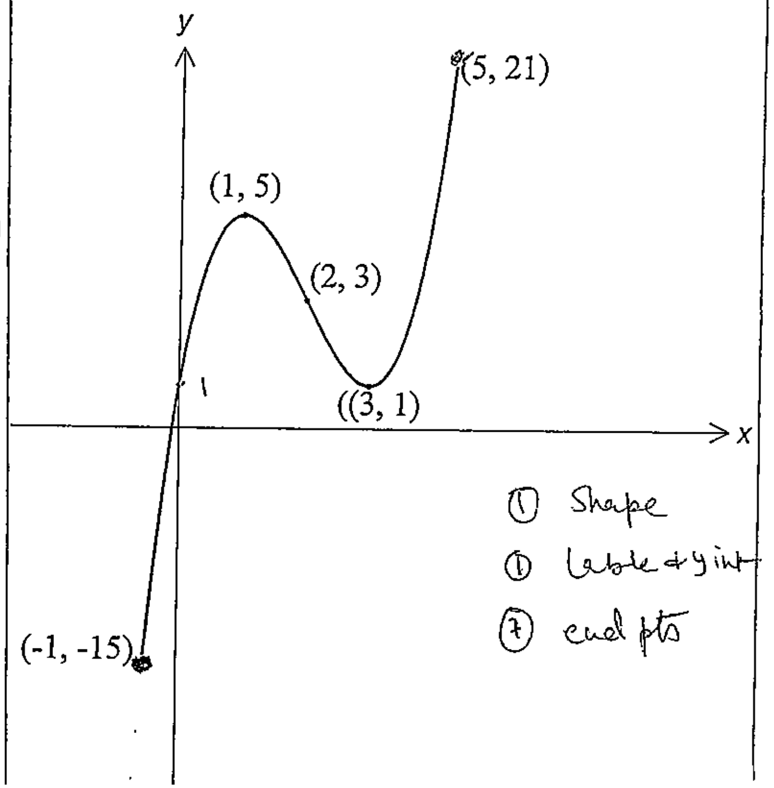
∴ concavity changes. Hence $(2,3)$ is a P.O.I

(iii) For $-1 \le x \le 5$

when $x = -1$, $y = -15$

$x = 5$ $y = 21$

∴ end pts are $(-1,-15)$ $(5,21)$



(1) Shape

(1) lable & y int

(1) end pts

Q14 a) $y' = 2x + a$

at $x = 1$
$m_1 = 2 + a$
at $x = 0$
$m_2 = a$ } (1)

Since tangents are ⊥lar.

∴ $m_1 \times m_2 = -1$

$(2+a) \times a = -1$

$a^2 + 2a + 1 = 0$

$(a+1)^2 = 0$ — (1)

$a = -1$

b) (ii) In ΔPBC & ΔPQC — (1)

$\angle PBC = \angle PQC = 90°$ (given, ∠s in a sq)

$\angle BCP = \angle QCP$ (PC bisects ∠ACB) — (1)

AC is common

∴ $\Delta PBC \equiv \Delta PQC$ (AAS) — (1)

(iii) $AB = BC = x$ (sides of sq)

∴ $PB = \frac{x}{3}$

Ar. $\Delta PBC = \frac{1}{2} x \times \frac{1}{3} x$

$= \frac{x^2}{6}$ — (1)

∴ Ar $\Delta PQC = \frac{x^2}{6}$ (∵ $\Delta PBC \equiv \Delta PQC$)

Ar $\Delta ABC = \frac{1}{2} x \times x = \frac{1}{2}x^2$ — (1)

∴ Ar $\Delta APQ = \frac{x^2}{6} - 2 \times \frac{x^2}{6}$ — (1)

$= \frac{x^2}{6}$ cm^2

c) (i) When $\dot{x} = 0$

$\frac{1}{5}t(4-t) = 0$

$t = 0$ or 4

time = 4 min. — (1)

(ii) $v = \dot{x} = \frac{4}{5}t - \frac{1}{5}t^2$

$\frac{dv}{dt} = \ddot{x} = \frac{4}{5} - \frac{2}{5}t$

$\frac{d^2v}{dt^2} = -\frac{2}{5}$

max. vel. when $\frac{dv}{dt} = 0$

$\frac{4}{5} - \frac{2}{5}t = 0$ — (1)

$t = 2$

Since $\frac{d^2v}{dt^2} = -\frac{2}{5} < 0$ for all t, $t = 2$ gives max. vel. } (1)

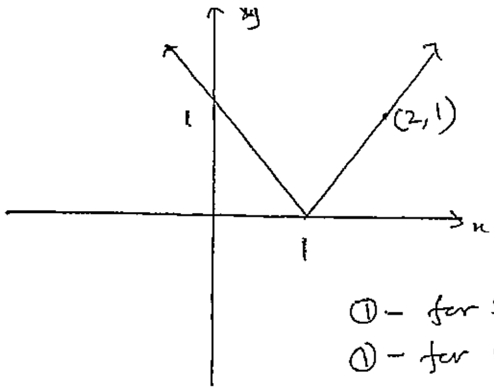
max. vel $= \frac{2}{5}(4-2) = \frac{4}{5}$ km/m.

(iii) $x = \int_0^4 \left(\frac{4}{5}t - \frac{1}{5}t^2\right) dt$ — (1)

$= \left[\frac{2t^2}{5} - \frac{1}{15}t^3\right]_0^4$

$= \frac{32}{5} - \frac{64}{15} = 2\frac{2}{15}$ km. — (1)

d)



(1) – for shape

(1) – for y = 1, any pt on the other branch should

Q15 a) (i) $A = \frac{1}{3}\left[0.4 + 4 \times 0.8 + 2 \times 1.5 + 4 \times 1.3 + 0.3\right]$ — (1)

$= 4.03$ — (1)

(ii) Vol. $= 4.03 \times 0.25 \times 60 \times 60 \times 24$

$= 87120\ m^3$ — (1)

b) Ar of Seg CDE $= \frac{1}{4}\pi r^2$ — (1)

Ar. of Δ DOC $= \frac{1}{2}r^2 \sin\theta$ — (1)

Ar. of sector DOC $= \frac{1}{2}r^2\theta$

$\therefore$ Ar of Seg CDE

= Ar. of sector − Ar of Δ

$\frac{1}{4}\pi r^2 = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$] (1)

$\frac{1}{2}\pi = \theta - \sin\theta$

c) (i) Initially $t = 0$, $P = 9\,000\,000$ — (1)

$\therefore 9000000 = P_0e^0 = P_0$

(ii) When $t = 2004 - 1954 = 50$

$P = 2 \times 10^7$

$\therefore 2 \times 10^7 = 9 \times 10^6 e^{50k}$

$\frac{20}{9} = e^{50k}$

$\ln\left(\frac{20}{9}\right) = 50k$ — (1)

$k = \frac{1}{50}\ln\left(\frac{20}{9}\right) = 0.015970$

(ii) $\frac{dP}{dt} = k.P_0e^{kt}$

$= kP$

$= \frac{1}{50}\ln\left(\frac{20}{9}\right) \times e^{\frac{1}{50}\times\ln\left(\frac{20}{9}\right)}$ — (1)

$= 168620$ per year — (1)

(iii) $P = 9 \times e^{\frac{1}{50}\ln\left(\frac{20}{9}\right) \times 60}$ — (1)

$= 23.4632$ millions

(iv) $50 = 9 \times e^{kt}$

$\frac{50}{9} = e^{kt}$

$\ln\left(\frac{50}{9}\right) = kt$

$t = \frac{1}{k} \times \ln\left(\frac{50}{9}\right)$

(1) $= \frac{\ln\left(\frac{50}{9}\right)}{\frac{1}{50}\ln\left(\frac{20}{9}\right)} = 107.375$

million in year 2061 — (1)

15d If $y^2 = (2m-y)(2n-y)$

then $y^2 = 4mn - 2my - 2ny + y^2$

$4mn - 2my - 2ny = 0$

$2mn = my + ny$

$\div$ mny

$\frac{2}{y} = \frac{1}{n} + \frac{1}{m}$ —(1)

$\frac{1}{y} + \frac{1}{y} = \frac{1}{n} + \frac{1}{m}$

$\frac{1}{y} - \frac{1}{m} = \frac{1}{n} - \frac{1}{y}$ —(1)

$\therefore \frac{1}{m}, \frac{1}{y}, \frac{1}{n}$ are an arithmetic sequence.

3(i) $P = \$200000$

$r = 0.425$

$A_1 = 200000(1.00425) - M$

$A_2 = A_1(1.00425) - M$

$= [200000(1.00425) - M] \times (1.00425) - M$ (1)

$= 200000(1.00425)^2 - M(1.00425) - M$

(ii) $A_3 = A_2(1.00425) - M$

$= [200000(1.00425)^2 - M(1+1.00425)] \times (1.00425) - M$

$= 200000(1.00425)^3 - M(1+1.00425 + (1.00425)^2)$

$A_n = 200000(1.00425)^n - M(1+1.00425 + \cdots (1.00425)^{n-1})$ —(1)

$A_n = 200000(1.00425)^n - M\left(\frac{1.00425^n - 1}{0.00425}\right)$

G.P with a=1

r=1.00425

If the loan is paid after n months

$A_n = 0$ —(1)

$M\left(\frac{1.00425^n - 1}{0.00425}\right) = 200000(1.00425)^n$ (1)

$M = \frac{200000(1.00425)^n(0.00425)}{1.00425^n - 1}$

(iii) 30 years = 30×12 months = 360 installments

$M = \frac{200000(1.00425)^{360}(0.00425)}{1.00425^{360} - 1}$ (1)

$= \$1085.90$

(iv) Total repaid $= 360 \times 1085.90$

$= 390924$ (1)

(v) $1331 = \frac{200000(1.00425)^n \times 0.0042}{1.00425^n - 1}$

$1331(1.00425)^n - 1331 = 850(1.004$

$(1331 - 850)(1.00425)^n = 1331$ (1)

$(1.00425)^n = 2.767151\cdots$

$n \ln(1.00425) = \ln(2.76715\cdots)$

$n = \frac{\ln(2.76715\cdots)}{\ln(1.00425)}$ (1)

$= 239.99\cdots = 240$ months

(2 marks with correct ans & working)

(b) $1 + \sin^2 x + \sin^4 x + \sin^6 x - - - -$

$a = 1 \quad r = \sin^2 x$

$-1 \le \sin x \le 1$ for all values of x

$\sin x \ne \pm 1$

$\therefore -1 < \sin x < 1$ (1)

$0 \le \sin^2 x < 1$

$\therefore$ limiting sum exists.

$S_\infty = \frac{a}{1-r} = \frac{1}{1-\sin^2 x} = \frac{1}{\cos^2 x} = \sec^2 x.$ (1)

I) (i) Given R is the radius of sphere & h the height HD of inscribed cone.

$OD = AD - OA = h - R$

$BD^2 = OB^2 - OD^2$ — (1)

$r^2 = h^2 - (h-R)^2 = (2hR - h^2)$

(ii) $V_{cone} = \frac{1}{3}\pi r^2 h$ — (1)

$= \frac{1}{3}\pi(2hR - h^2) \times h$

$= \frac{1}{3}\pi 2h^2R - h^3$

(iii) $\frac{dV}{dh} = \frac{\pi}{3} \times (\cdot hR - 3h^2)$

For max or min $\frac{dV}{dh} = 0$

$\frac{1}{3}\pi(4hR - 3h^2) = 0$

$h(4R - 3h) = 0$

$h = 0$ or $h = \frac{4R}{3}$ — (1)

↓ not possible

$\frac{d^2V}{dh^2} = \frac{1}{3}\pi(4R - 6h)$

at $h = \frac{4R}{3}$

$\frac{d^2V}{dh^2} = \frac{1}{3}\pi\left(4R - 6 \times \frac{4R}{3}\right)$

$= -\frac{4\pi R}{3} < 0$

∴ Vol. is max. at $h = \frac{4R}{3}$ } (1)

∴ Vol. of the largest cone

$\frac{1}{3}\pi\left(2 \times \frac{16R^2}{9} \times R - \frac{64R^3}{27}\right)$

$= \frac{1}{3}\pi\left(\frac{32R^3}{9} - \frac{64R^3}{27}\right)$

$= \frac{32\pi R^3}{81}$

(1) mark with workin

$V_{sphere} = \frac{4}{3}\pi R^3$

∴ $\frac{\text{V of largest cone}}{\text{V of sphere}} = \frac{\frac{32\pi R^3}{81}}{\frac{4}{3}\pi R^3}$

$= 8/27$

∴ V. of the largest cone is $\frac{8}{27}$ V. of the sphere