BAULKHAM HILLS HIGH SCHOOL

2017 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics

General Instructions
- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions 11–16, show relevant mathematical reasoning and/or calculations
- Marks may be deducted for careless or badly arranged work

Total marks – 100
Exam consists of 10 pages.

This paper consists of TWO sections.

Section 1 – Page 2-4 (10 marks)

- Attempt Question 1-10
- Allow about **15** minutes for this section

Section II – Pages 4-10 (90 marks)

- Attempt questions 11-16
- Allow about **2 hours and 45** minutes for this section

Section I

10 marks

Attempt questions 1-10
Allow about 15 minutes for this section.
Use the multiple choice answer page for questions 1-10

1. Simplify $\frac{x^2-4}{x^3-8}$

(A) $\frac{x-2}{x^2-2x+4}$

(B) $\frac{1}{x+2}$

(C) $\frac{x+2}{x^2+4x+4}$

(D) $\frac{x+2}{x^2+2x+4}$

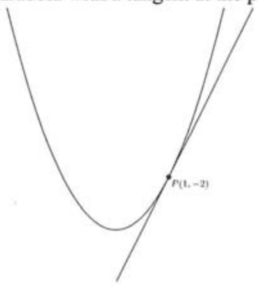
2. A yacht sailed directly from A to B on a bearing of 196°T. To sail from B back to A, the bearing should be:

(A) 016°T
(B) 074°T
(C) 164°T
(D) 196°T

3. Which of the following defines the domain of the function $f(x) = \frac{1}{\sqrt{4-x^2}}$?

(A) $x < -2$ or $x > 2$
(B) $x < -2$
(C) $x > 2$
(D) $-2 < x < 2$

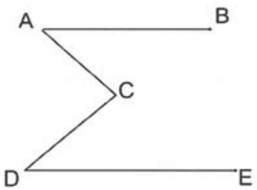
4. The diagram shows a concave up parabola with a tangent at the point P(1,-2).



Which of the following could be the equation of the normal at P?

(A) $x - 3y + 5 = 0$
(B) $2x - 3y + 1 = 0$
(C) $x + 3y + 5 = 0$
(D) $x + 3y - 5 = 0$

5. In the diagram, AB and DE are parallel, $\angle BAC = 30°$ and $\angle CDE = 55°$.



What is the size of $\angle ACD$?

(A) 45°
(B) 65°
(C) 85°
(D) 95°

6. What is the derivative of $\frac{e^x}{e^x+1}$?

(A) $\ln(e^x + 1)$
(B) $2\ln(e^x + 1)$
(C) $\frac{2e^{2x}+e^x}{(e^x+1)^2}$
(D) $\frac{e^x}{(e^x+1)^2}$

7. The quadratic equation $3x^2 - 5x + 1 = 0$ has roots α and β.
Which of the following statements is true?

(A) $2\alpha\beta = -\frac{4}{3}$
(B) $\alpha^2 + \beta^2 = \frac{13}{9}$
(C) $2\alpha + 2\beta = \frac{10}{3}$
(D) $\alpha^2\beta^2 = \frac{2}{9}$

8. In ΔABC, AB=5m, AC=8m and $\angle ACB = 40°$. If BC=d metres, which equation could be solved to find the length of BC?

(A) $d^2 = 5^2 + 8^2 - 2 \times 5 \times 8sin40°$
(B) $d^2 = 5^2 + 8^2 - 2 \times 5 \times 8cos40°$
(C) $5^2 = 8^2 + d^2 - 2d \times 8sin40°$
(D) $5^2 = 8^2 + d^2 - 2d \times 8cos40°$

9. The values of x for which the geometric series $2 + 4x + 8x^2 + \cdots$ has a limiting sum are:

(A) $x < \frac{1}{2}$
(B) $x \leq \frac{1}{2}$
(C) $-\frac{1}{2} < x < \frac{1}{2}$
(D) $-\frac{1}{2} \leq x \leq \frac{1}{2}$

10. What is the number of solutions for the equation $x = 10\sin x$?

(A) 4
(B) 5
(C) 6
(D) 7

END OF SECTION I

Section II

90 marks

Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section
Answer each question on the appropriate page in the writing booklet.
In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Start on the appropriate page in the answer booklet

(a) Factorise $2x^2 + x - 28$ **2**

(b) Solve $|x - 1| = 2x - 3$ **2**

(c) Find $\int \sqrt{3x - 4}\, dx$ **2**

(d) If $\tan\theta = \frac{5}{8}$ and $\sin\theta < 0$ find the exact value of $\operatorname{cosec}\theta$. **2**

(e) The points $A(4,12)$, $B(8,6)$ and $C(-4,k)$ are collinear. Find the value of k. **2**

(f) The quadratic equation $2x^2 - (4p + 1)x + 2p^2 - 1 = 0$ has two roots which are equal in magnitude but opposite in sign. Find the value of p. **2**

(g) Find the equation of the curve $y = f(x)$ given that $y = 5x - 7$ is a tangent to this curve, and its gradient function is $f'(x) = 4x - 3$. **3**

Question 12 (15 marks) Start on the appropriate page in the answer booklet

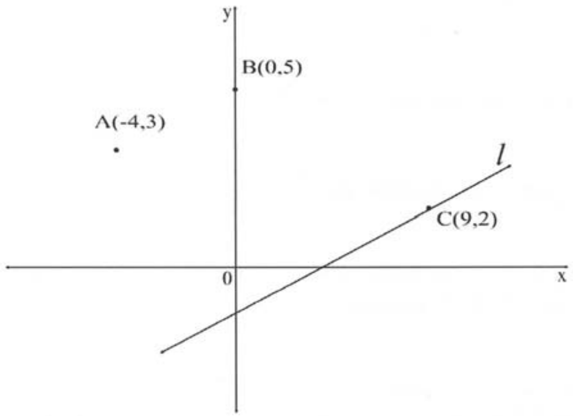
(a) Differentiate with respect to x:

(i) $(x^2 + 4)^5$ **2**

(ii) $e^x \log_e x$ **2**

(iii) $\frac{\sin x}{x-1}$ **2**

(b) The diagram shows the points $A(-4,3)$, $B(0,5)$ and $C(9,2)$, and the line l which passes through C and is parallel to AB.



Copy the diagram into your answer booklet.

(i) Find the length AB. **1**

(ii) Show that the equation of line l is $x - 2y - 5 = 0$ **2**

(iii) Find the coordinates of point D, where line l meets the x-axis. **1**

(iv) Prove that $ABCD$ is a parallelogram. **2**

(v) Find the perpendicular distance from point B to line l. **2**

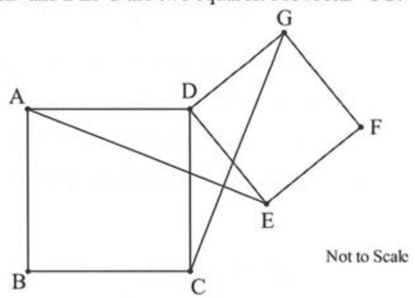
(vi) Hence or otherwise, find the area of parallelogram $ABCD$ **1**

Question 13 (15 marks) Start on the appropriate page in the answer booklet

(a) Evaluate $\int_1^3 \frac{5x}{x^2+1}\,dx$ **2**

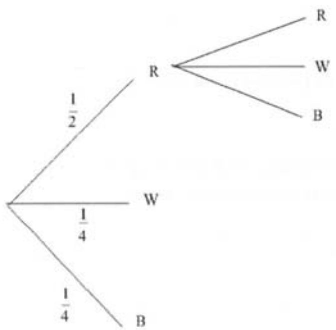
(b) For which values of k does the equation $3x^2 + kx + 5 = 0$ have no real solutions? **3**

(c) In the diagram below, *ABCD* and *DEFG* are two squares. Prove *AE=CG*. **3**



(d) A jar contains 2 red, 1 black and 1 white marble only. Ken randomly selects two marbles in succession (without replacement), and places them in a bag.

(i) Copy and complete the probability tree to show all possible outcomes. **2**



(ii) Find the probability that both of the selected marbles are red. **1**

(ii) If one of the marbles falls out of the bag and we see that it is white, what is the probability that the marble still in the bag is red? **1**

(e) Evaluate $\sum_{n=1}^{20} (3^n + 2n - 1)$ **3**

Question 14 (15 marks) Start on the appropriate page in the answer booklet

(a) Given $\log_3 2 = p$ and $\log_3 5 = q$, find in terms of p and q:

(i) $log_3 12.5$ **1**

(ii) $log_5 2$ **1**

(b) The velocity of a particle is given by $v = 1 - 2\cos t$ for $0 \le t \le 2\pi$, where v is measured in metres per second and t is measured in seconds.

(i) At what times during this period is the particle at rest? **2**

(ii) What is the maximum velocity of the particle during this period? **2**

(iii) Sketch the graph of v as a function of t for $0 \le t \le 2\pi$ **2**

(iv) Calculate the total distance travelled by the particle between $t = 0$ and $t = \pi$ **3**

(c) For the parabola $x^2 - 10x - 16y - 7 = 0$, find :

(i) the coordinates of the vertex. **2**

(ii) the equation of the directrix. **1**

(iii) the coordinates of the focus. **1**

Question 15 (15 marks) Start on the appropriate page in the answer booklet

(a) Radium is a radioactive substance which decays over time, with a half-life of 1600 years. (Note: the half-life of a substance is the time taken for it to decay to half its mass).

The mass M remaining at time t years is given by:

$$M = M_0 e^{-kt}$$

where M_0 and k are constants.

(i) Evaluate k, expressing your answer correct to 3 significant figures. **2**

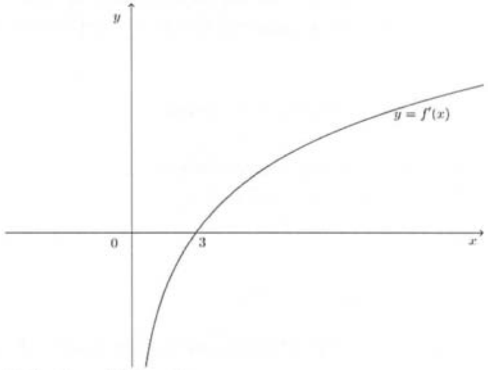
(ii) If a piece of radium in a laboratory has a mass of 200 grams, what would have been its mass 1000 years ago? Answer correct to the nearest gram. **2**

(b) Prove that $(1 - cos\theta)(1 + sec\theta) = sin\theta tan\theta$ **2**

Question 15 continues on the next page.

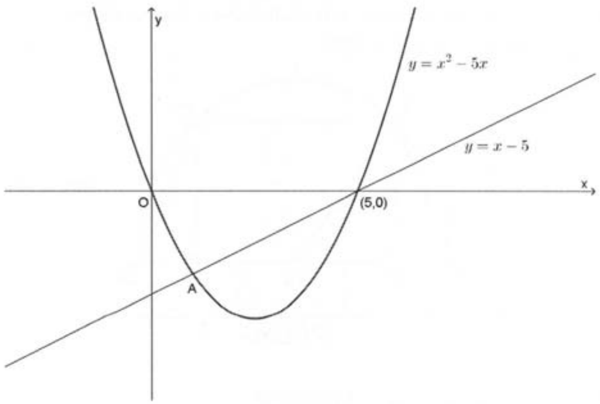
Question 15 (Continued)

(c) A sketch of a gradient function $y = f'(x)$ is shown.



Sketch the curve $y = f(x)$, given $f(3) = 0$. **3**

(d) The graphs of $y = x - 5$ and $y = x^2 - 5x$ intersect at the points (5,0) and A, as shown in the diagram.



(i) Show that A has coordinates (1,-4) **1**

(ii) Find the area between the line $y = x - 5$ and the curve $y = x^2 - 5x$ **3**

(e) A sector of a circle is enclosed by two radii of length 10cm and an arc of length 14cm. Find the area of the sector. **2**

Question 16 (15 marks) Start on the appropriate page in the answer booklet

(a) Karina borrows \$80 000 to start a business. She plans to repay the loan in equal monthly instalments of \$$M$ at 0.5% per month reducible interest. Interest is calculated and charged just before each repayment.

Let A_n= amount owing after n repayments (in dollars)

(i) Show that the amount owing after 3 repayments is
$A_3 = 80000(1.005)^3 - M(1.005^2 + 1.005 + 1)$ **2**

(ii) Show that

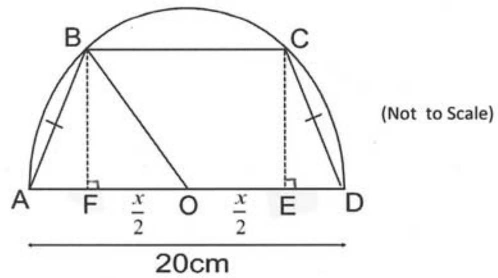
$A_n = 1.005^n(80000 - 200M) + 200M$ **2**

(iii) If she wishes to repay the loan in 10 years, calculate the amount of each instalment. **2**

(iv) If she can only repay \$700 per month, how long will it take her to repay the loan? Answer correct to the nearest year. **2**

(b) Solve $\sin(x + \frac{\pi}{6}) = \frac{\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$ **2**

(c) $ABCD$ is a trapezium inscribed in a semicircle of diameter 20cm, as shown. AB=CD and O is the centre of the semicircle.



(i) If $EO = OF = \frac{x}{2}$, show that $BF = \frac{1}{2}\sqrt{400 - x^2}$ **1**

(ii) Show that the area $A\ cm^2$ of the trapezium ABCD is given by $A = \frac{1}{4}(x + 20)\sqrt{400 - x^2}$ **1**

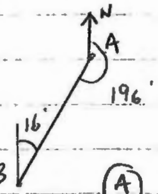
(iii) Hence find the length of BC such that the area of the trapezium $ABCD$ is a maximum **3**

End of exam

SOLUTIONS:

2U Trial 2017 — BHHS

1. $\dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)}$ D

2.


A

3. $4-x^2>0$

$4>x^2$

$x^2<4$

$-2<x<2$ D

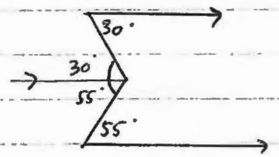
4. $m_{\text{tang.}}>0 \quad \therefore m_{\text{Norm}}<0$

$\therefore$ C or D

but must pass through $(1,-2)$

Test C: $1-6+5=0$

Test D: $1-6-5\neq 0 \quad \therefore$ C

5.


$30+55=85°$ C

6. $\dfrac{d}{dx}\left(\dfrac{e^x}{e^x+1}\right)$

$=\dfrac{(e^x+1)e^x-e^x.e^x}{(e^x+1)^2}$

$=\dfrac{\cancel{e^{2x}}+e^x-\cancel{e^{2x}}}{(e^x+1)^2}$ D

7. $\alpha+\beta=\dfrac{5}{3}$, $\alpha\beta=\dfrac{1}{3}$

From A, $\alpha\beta=-2/3$. NO.

From B, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\dfrac{25}{9}-\dfrac{2}{3}$

$=\dfrac{19}{9} \neq 13/9$

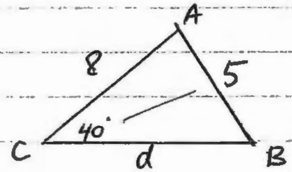
From C, $2\alpha+2\beta=2(\alpha+\beta)$

$=2\left(\dfrac{5}{3}\right)$

$=\dfrac{10}{3}$ yes

From D, $\alpha^2\beta^2=\left(\dfrac{1}{3}\right)^2=\dfrac{1}{9}$

$\therefore$ C

8.


$5^2=8^2+d^2-2\times 8\times d\cos 40°$ D

9. $r=2x \quad |r|<1$

$-1<2x<1$

$-\frac{1}{2}<x<\frac{1}{2}$ C

10. 7 (by graphing) D

Section II.

Question 11

a) $(2x-7)(x+4)$

2 – correct answer
[1 – signs incorrect eg $(2x+7)(x-4)$]

b) $|x-1|=2x-3$

$x-1=2x-3$ or $-x+1=2x-3$

$2=x$ $\qquad -2=3x$

(Test : OK) $\qquad x=-2/3$

(Test : fails) RHS $=-\frac{4}{3}-3<0$

$\therefore x=2$ only

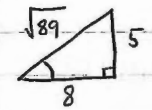
2 – correct solution/testing.
[1 – leaving both]

c) $\int\sqrt{3x-4}\,.dx$

$=\dfrac{(3x-4)^{3/2}}{3/2\times 3}+c$

$=\dfrac{2}{9}\left(\sqrt{3x-4}\right)^3+c$ or $\dfrac{2}{9}(3x-4)^{3/2}+c$

2 — correct answer.

d)


$\operatorname{cosec}\theta=-\dfrac{\sqrt{89}}{5}$

2 – correct answer
[1 – finding $\sqrt{89}$]

e) $m_{AB}=m_{BC}$ $\qquad \dfrac{6-12}{8-4}=\dfrac{k-6}{-4-8}$

$\dfrac{-3}{2}=\dfrac{k-6}{-12}$

$k-6=18 \qquad \therefore k=24$

[1 – equating gradients]
2 – correct solution

f) Let roots $=\alpha, -\alpha$ $\qquad \alpha+(-\alpha)=\dfrac{4p+1}{2}$

$0=\dfrac{4p+1}{2}$

$p=-\dfrac{1}{4}$

[1 – writing $\frac{4p+1}{2}$ for sum]
2 – correct solution.

g) $f'(x) = 4x - 3 = 5$, at the tangent

$4x = 8$

$x = 2,\ y = 5(2) - 7 = 3$ 1 - finding $x=2$

$\therefore$ Pt of contact is $(2,3)$

$f(x) = 2x^2 - 3x + c$ (by integration)   1 - finding primitive

$3 = 2(2)^2 - 3(2) + c$

$3 = 8 - 6 + c \quad \therefore c = 1$

$\therefore f(x) = 2x^2 - 3x + 1$ 1 - answer.

Question 12.

a) i) $\frac{d}{dx}\left((x^2+4)^5\right) = 5(x^2+4)^4 \cdot 2x$ 1   1

$= 10x(x^2+4)^4$

ii) $\frac{d}{dx}\left(e^x \cdot \log_e x\right) = uv' + vu'$ $\begin{bmatrix} u = e^x & u' = e^x \\ v = \log_e x & v' = \frac{1}{x} \end{bmatrix}$

$= e^x \cdot \frac{1}{x} + \log_e x \cdot e^x$

$= e^x\left(\frac{1}{x} + \log_e x\right)$ 2 - correct soln [1 - attempt prod. rule]

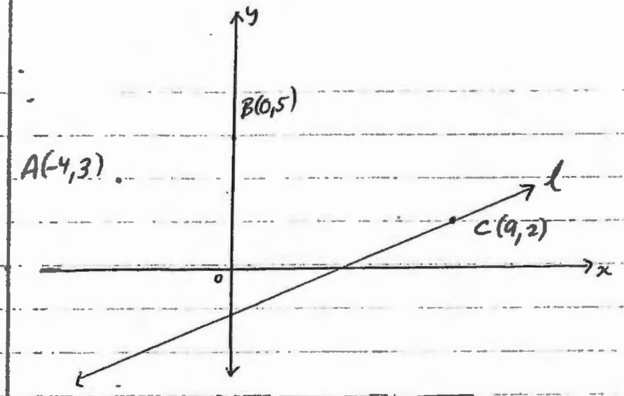
iii) $\frac{d}{dx}\left(\frac{\sin x}{x-1}\right)$ $\begin{bmatrix} u = \sin x & u' = \cos x \\ v = x - 1 & v' = 1 \end{bmatrix}$

$= \frac{vu' - uv'}{v^2}$

$= \frac{(x-1)\cdot \cos x - \sin x \cdot 1}{(x-1)^2}$

$= \frac{x\cos x - \cos x - \sin x}{(x-1)^2}$ 2 - correct soln. [1 - attempt quot. rule]

b)



(i) $AB = \sqrt{(5-3)^2 + (0+4)^2}$

$= \sqrt{20}$ or $2\sqrt{5}$ units.   1

(ii) $m_{AB} = \frac{5-3}{0+4} = \frac{1}{2}$ — 1

l has $m = \frac{1}{2}$ and through $C(9,2)$

$y - 2 = \frac{1}{2}(x - 9)$ — 1

$2y - 4 = x - 9$

$x - 2y - 5 = 0$

(iii) At D: $y = 0 \quad x - 2(0) - 5 = 0,\ x = 5$ — 1

$\therefore D(5,0)$

(iv) $AB \parallel CD$ (given).

$CD = \sqrt{(9-5)^2 + (2-0)^2} = \sqrt{20} = AB$ — 1

$\therefore$ ABCD is a parallelogram, since a pair of sides are equal and parallel. — 1

v) $B(0,5)$, $l: x - 2y - 5 = 0$

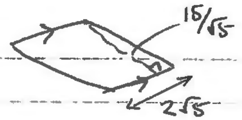
$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2+B^2}}\right|$

$= \left|\frac{0 - 2(5) - 5}{\sqrt{1^2 + (-2)^2}}\right|$ — 1

$= 15 \quad 3\sqrt{}$

vi) Area $= bh$

$= 2\sqrt{5} \times \dfrac{15}{\sqrt{5}}$



$= \underline{30 \text{ units}^2}$ — 1

Question 13.

a) $\displaystyle\int_1^3 \frac{5x}{x^2+1}\,dx$

$= \dfrac{5}{2}\displaystyle\int_1^3 \frac{2x}{x^2+1}\,dx$

$= \dfrac{5}{2}\Big[\ln(x^2+1)\Big]_1^3$ 1 – correct primitive

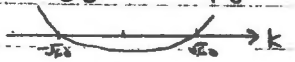
$= \underline{\dfrac{5}{2}\left(\ln 10 - \ln 2\right)}$ 1 – correct substitution into found primitive.

or $\underline{\dfrac{5}{2}\ln 5}$.

b) $\Delta = b^2 - 4ac$

$= k^2 - 4.3.5$ ← 1

$= k^2 - 60 < 0$ if no real solutions ← 1



$-\sqrt{60} < k < \sqrt{60}$ ← 1

c) In ΔADE, ΔDCG:

$AD = DC$ (sides of square)

$\angle ADC = \angle GDE$ ($\angle$s of square)

$\angle ADE = \angle ADC + \angle CDE$

$\angle GDC = \angle GDE + \angle CDE$

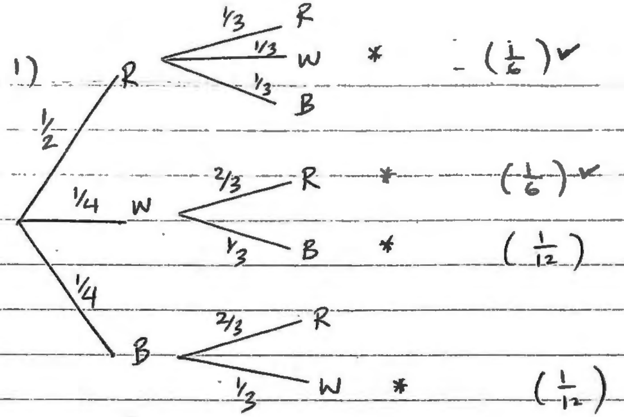
$\therefore \underline{\angle ADE = \angle GDC}$ (adding equal $\angle$s) ← 1

$DE = DG$ (sides of square)

$\therefore \Delta ADE \equiv \Delta DCG$ (SAS) ← 1

$\therefore AE = CG$ (matching sides of congruent Δs equal) ← 1

d) i)



2 – fully correct

[1 – partially correct]

ii) $P(RR) = \dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{6}$ ← 1

iii) $P(\text{1 red, given 1 white}) = \dfrac{1/3}{1/2}$ ← outcomes labelled ✓ ← outcomes labelled *

$= \dfrac{2}{3}$ ← 1

e) $\displaystyle\sum_{n=1}^{20} (3^n + 2n - 1)$

$= \displaystyle\sum_{n=1}^{20} 3^n + (2n-1)$ ← 1 split + recognise AP, GP

$= (3 + 9 + 27 + \dots) + (1 + 3 + 5 + 7 + \dots + 39)$

$= \dfrac{a(r^n - 1)}{r-1} + \dfrac{n}{2}(a + l)$

↖ sum of geom. series ↖ sum of arith. series

$= \dfrac{3(3^{20} - 1)}{3 - 1} + \dfrac{20}{2}(1 + 39)$ ← 1 sum of GP ← 1 sum of AP

$= 5230176600 + 400$

$= 5230177000$

Question 14

a) i) $\log_3 12.5 = \log_3 \frac{5^2}{2}$

$= 2\log_3 5 - \log_3 2$

$= 2q - p. \longleftarrow 1$

ii) $\log_5 2 = \dfrac{\log_3 2}{\log_3 5} = \dfrac{p}{q} \longleftarrow 1$

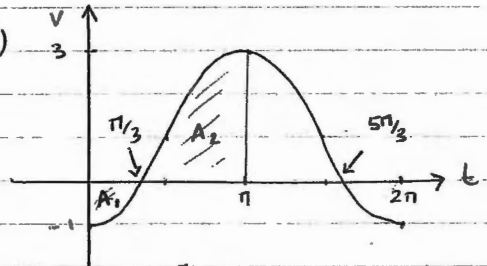
b) i) $v = 0$ when $1 - 2\cos t = 0$

$\cos t = \frac{1}{2} \longleftarrow 1$

$t = \frac{\pi}{3}, \frac{5\pi}{3}$ sec $\longleftarrow 1$

ii) Max $v = 1 - 2(-1) = 3$ m/s $\longleftarrow 1$

iii)



2 - correct graph.

iv) $A_1 = \left| \int_0^{\pi/3} 1 - 2\cos t \, . \, dt \right|$

$= \left| \left[t - 2\sin t \right]_0^{\pi/3} \right|$

$= \left| \frac{\pi}{3} - 2\left(\frac{\sqrt{3}}{2}\right) - (0 - 0) \right| \longleftarrow 1$

$= \sqrt{3} - \frac{\pi}{3}$

$A_2 = \int_{\pi/3}^{\pi} 1 - 2\cos t \, . \, dt$

$= \left[t - 2\sin t \right]_{\pi/3}^{\pi}$

$= \pi - 2(0) - \left(\frac{\pi}{3} - 2\left(\frac{\sqrt{3}}{2}\right) \right)$

$\therefore$ Distance travelled $= \sqrt{3} - \frac{\pi}{3} + \frac{2\pi}{3} + \sqrt{3}$

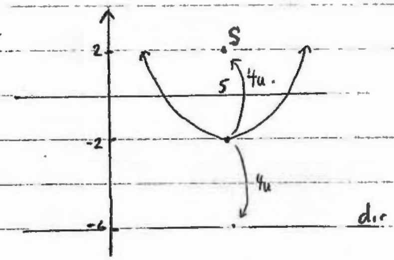
$= 2\sqrt{3} + \frac{\pi}{3}$ cm $\longleftarrow 1$

c) i) $x^2 - 10x + 25 = 16y + \cancel{7}^{32} + 25 \longleftarrow 1$ (completing the sq.)

$(x-5)^2 = 16(y+2)$

vertex $(5, -2)$. $\longleftarrow 1$

~~ii)~~



$4a = 16 \therefore a = 4$

ii) Directrix $y = -6 \longleftarrow 1$

iii) Focus $(5, 2)$. $\longleftarrow 1$

Question 15.

a) i) When $t = 1600$, $m = \frac{1}{2}m_0$

$\frac{1}{2}\cancel{m_0} = \cancel{m_0} e^{-1600k} \longleftarrow 1$

$-1600k = \ln \frac{1}{2}$

$k = 0.000433 \longleftarrow 1$ (with correct rounding)

ii) When $t = 1000$, $m = 200$:

$200 = m_0 e^{-1000k} \longleftarrow 1$

$200 = m_0 \times 0.6484$

$m_0 = 308$ g. $\longleftarrow 1$

b) $LHS = (1-\cos\theta)(1+\sec\theta)$

$= 1 + \sec\theta - \cos\theta - \cos\theta . \sec\theta$

$= 1 + \sec\theta - \cos\theta - \cancel{\cos\theta} . \frac{1}{\cancel{\cos\theta}}$

$= \sec\theta - \cos\theta$ $\longleftarrow 1$

$= \frac{1-\cos^2\theta}{\cos\theta}$

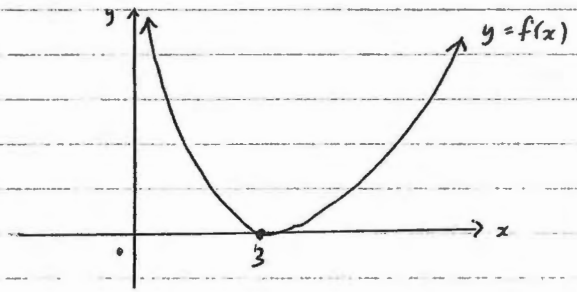
$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta} . \sin\theta$ $\longleftarrow 1$

$= \tan\theta . \sin\theta$

$= RHS.$

c)



1 - through (3,0)
1 - all conc. up
1 - asymp.

d) i) At A : $x^2 - 5x = x - 5$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$\cancel{x=5}$ or $x = 1$ $\longleftarrow 1$

↑ this is the x-int

$y = 1 - 5 = -4$

$\therefore A(1,-4)$

ii) $A = \int_1^5 x - 5 - (x^2 - 5x) \; dx$

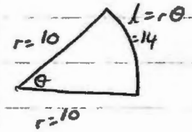
$= \int_1^5 -x^2 + 6x - 5 \; dx$ $\longleftarrow 1$

$= \left[-\frac{x^3}{3} + 3x^2 - 5x\right]_1^5$ $\longleftarrow 1$

$= \left(-\frac{125}{3} + 75 - 25\right) - \left(-\frac{1}{3} + 3 - 5\right)$

$= 10\frac{2}{3}$ units^2 $\longleftarrow 1$

f)



$A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} r(r\theta)$ $\longleftarrow 1$

$= \frac{1}{2} \times 10 \times 14$

$= 70 \text{ cm}^2$ $\longleftarrow 1$

OR $l = r\theta \quad \therefore 10\theta = 14$

$\theta = 1.4$ $\leftarrow 1$

Then $A = \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times 10^2 \times 1.4$

$= 70 \text{ cm}^2$ $\leftarrow 1$

v) If $M = 700$ and $A_n = 0$:

$$(80\,000 - 200 \times 700) \times 1.005^n + 200 \times 700 = 0 \longleftarrow 1$$

$$1.005^n = \frac{-140\,000}{-60\,000} = \frac{7}{3}$$

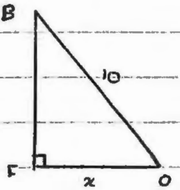
$$n \ln 1.005^n = \ln \frac{7}{3}$$

$$n = \frac{\ln 7/3}{\ln 1.005} \doteqdot \underline{170 \text{ months}} \longleftarrow 1$$

) $\sin\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ $\quad \left(\begin{array}{l} 0 \le x \le 2\pi \\ \frac{\pi}{6} \le x + \frac{\pi}{6} \le \frac{13\pi}{6} \end{array}\right)$

$$x + \frac{\pi}{6} = \frac{\pi}{3}, \frac{2\pi}{3} \longleftarrow 1$$

$$\underline{x = \frac{\pi}{6}, \frac{\pi}{2}} \longleftarrow 1$$

c) i) 

$$BF^2 = 10^2 - \left(\frac{x}{2}\right)^2$$

$$= 100 - \frac{x^2}{4}$$

$$= \frac{400 - x^2}{4} \longleftarrow 1$$

$$BF = \sqrt{\frac{400 - x^2}{4}} = \frac{1}{2} \cdot \sqrt{400 - x^2}$$

ii) Area $= \frac{1}{2}(a+b)h$ $\quad \left\{\begin{array}{l} a = x \\ b = 20 \\ h = BF \end{array}\right.$

$$= \frac{1}{2}(x+20) \cdot \frac{1}{2} \cdot \sqrt{400 - x^2} \longleftarrow 1$$

$$= \frac{1}{4}(x+20) \cdot \sqrt{400 - x^2}$$

iii) For max area, $A' = 0$ and $A'' < 0$

$$A' = \frac{1}{4}\left((x+20) \cdot \frac{-x}{\sqrt{400 - x^2}} + \sqrt{400 - x^2} \cdot 1\right)$$

$= 0$ when:

$$\frac{x(x+20)}{\sqrt{400 - x^2}} = \sqrt{400 - x^2}$$

$$\left.\begin{array}{l} x^2 + 20x = 400 - x^2 \\ 2x^2 + 20x - 400 = 0 \end{array}\right\} \longleftarrow 1$$

$$x^2 + 10x - 200 = 0$$

$$(x+20)(x-10) = 0$$

$$\cancel{x = -20} \text{ or } \underline{x = 10} \longleftarrow 1$$

A'' is rather 'messy', so will use A' to test

x	9	10	11
A'	$\frac{1}{4}(3.24)$	0	$\frac{1}{4}(-3.7)$
	(+ ,	0 ,	−)

 $\longleftarrow 1$

$\therefore$ Max area when $x = 10$

Question 16

a) i) $A_1 = 80000(1.005) - m$

$A_2 = A_1 \times 1.005 - m$

$= 80000(1.005)^2 - m(1.005) - m \longleftarrow 1$

$= 80000(1.005)^2 - m(1.005 + 1)$

$A_3 = A_2 \times 1.005 - m$

$= 80000(1.005)^3 - m(1.005^2 + 1.005) - m \longleftarrow 1$

$= 80000(1.005)^3 - m(1.005^2 + 1.005 + 1)$

ii) Continuing the pattern

$A_n = 80000(1.005)^n - m(1.005^{n-1} + 1.005^{n-2} + \dots + 1.005 + 1)$

$= 80000(1.005)^n - m \times \dfrac{a(r^n - 1)}{r - 1}$ where $a = 1$, $r = 1.005$, n terms.

$= 80000(1.005)^n - m \times \dfrac{1(1.005^n - 1)}{0.005} \longleftarrow 1$

$= 80000(1.005)^n - m \times 200(1.005^n - 1)$

$= 80000(1.005)^n - 200m(1.005)^n + 200m \longleftarrow 1$

$= (80000 - 200m) \times 1.005^n + 200m$

iii) If $n = 120$ and $A_{120} = 0$:

$(80000 - 200m) \times 1.005^{120} + 200m = 0$

$80000(1.005)^{120} - (200m) \times 1.005^{120} + 200m = 0$

$80000(1.005)^{120} = 200m(1.005^{120} - 1) \longleftarrow 1$

$145551.74 = 200m \times 0.8194$

$m = \$888.16 \longleftarrow 1$