Baulkham Hills High School

2018

Higher School Certificate
Trial Examination

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference Sheet is provided at the end of this paper.
- In questions 11-16 show relevant mathematical reasoning and/or calculations.

Total Marks 100

Section I 10 marks

- Attempt questions 1 – 10
- Allow about 15 minutes for this section

Section II 90 marks

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section.

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section.
Use the multiple choice answer sheet for questions 1-10

---

1. The 40^{th} term of the sequence $-4, 1, 6, \ldots$ is:

(A) 25

(B) 61

(C) 155

(D) 191

2. Which of the following expressions is equivalent to $\log_2 7$?

(A) $\frac{\ln 7}{\ln 2}$

(B) $\frac{\ln 2}{\ln 7}$

(C) $2\ln 7$

(D) $7\ln 2$

3. When expressed in terms of π, what is $450°$ in radians?

(A) $\frac{9\pi}{4}$

(B) 3π

(C) $\frac{7\pi}{5}$

(D) $\frac{5\pi}{2}$

4. Evaluate $\int_0^2 dx$.

(A) 0

(B) 1

(C) 2

(D) None of the above

5. Which of the following could not be the probability of a particular event?

(A) $\frac{2}{3}$

(B) 60%

(C) 0.2378

(D) $\frac{7}{5}$

6. Which of the following statements must be true if a particle is stationary?

(A) The particle's displace is zero

(B) The particle's velocity is zero

(C) The particle's acceleration is zero

(D) All of the above

7. Sixty tickets are sold in a raffle. There are two prizes. Harry buys five tickets. Which expression gives the probability that Harry wins both prizes?

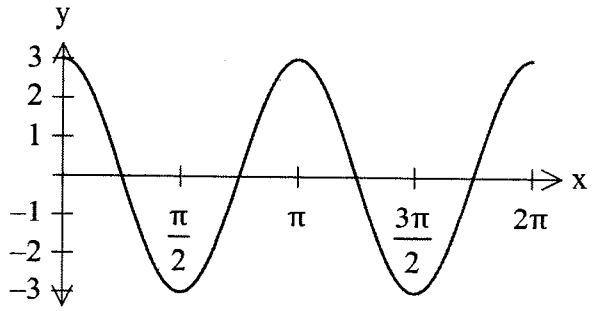
(A) $\frac{5}{60}+\frac{4}{59}$

(B) $\frac{5}{60}+\frac{4}{60}$

(C) $\frac{5}{60}\times\frac{4}{59}$

(D) $\frac{5}{60}\times\frac{4}{60}$

8. The diagram shows a sketch of the graph of $y = f(x)$, for $0 \leq x \leq 2\pi$.



Which of the following equations best describes this curve?

(A) $y = 3\cos 2x$

(B) $y = 2\cos 3x$

(C) $y = 3\cos\frac{x}{2}$

(D) $y = 2\cos\frac{x}{3}$

9. What are the solutions of $\cos 2x = \frac{1}{2}$ for $-\pi \leq x \leq \pi$?

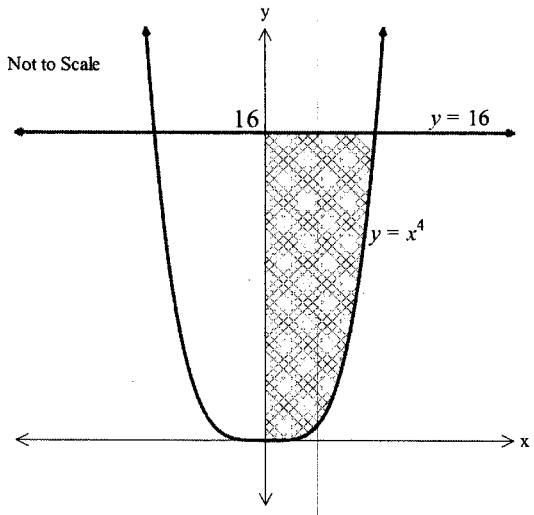
(A) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{-5\pi}{6}, \frac{-\pi}{6}$

(B) $x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{-11\pi}{12}, \frac{-\pi}{12}$

(C) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

(D) $x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}$

10. The region in the diagram is bounded by the curve $y = x^4$, the y-axis and the line $y = 16$.



Which of the following expressions is correct for the volume of the solid of revolution when this region is rotated about the y-axis?

(A) $\pi \int_0^2 x^8 dx$

(B) $\pi \int_0^{16} x^8 dx$

(C) $\pi \int_0^2 \sqrt{y}\, dy$

(D) $\pi \int_0^{16} \sqrt{y}\, dy$

END OF SECTION I

Section II

90 marks
Attempt questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in the **appropriate page of the writing booklet**. Extra sheets of writing paper are available.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

QUESTION 11 (15 marks) – Start on the appropriate page in your booklet.

a) Evaluate $\sqrt{\pi^2 + 5}$ to two decimal places. **1**

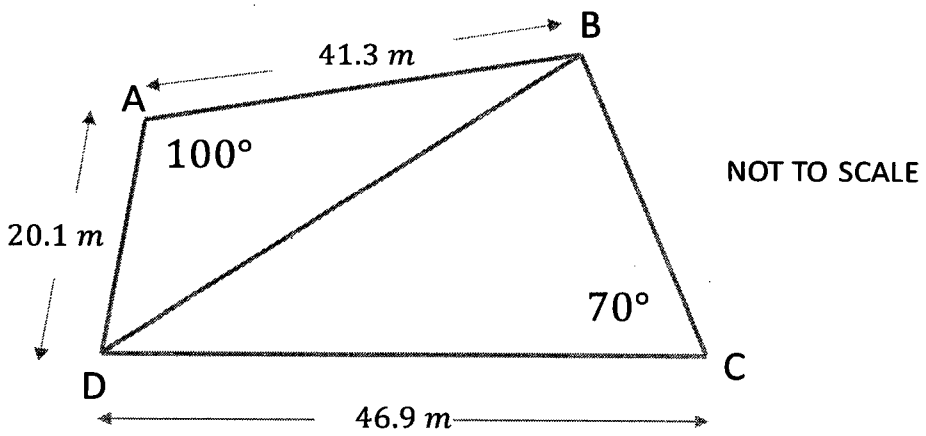
b) Simplify $\frac{x}{x^2-4} - \frac{2}{x-2}$. **2**

c) Solve $|2x - 3| = 4 - 3x$. **3**

d) Consider the expression $(2\sqrt{3} - 1)(3\sqrt{3} + 2) = p + q\sqrt{3}$. Given that p and q are rational numbers, find the values of p and q. **2**

e) Find all the solutions to $4^x - 17 \times 2^x = -16$. **3**

f)



In the above figure, $ABCD$ is a quadrilateral in which $AB = 41.3\ m$, $AD = 20.1\ m$, $DC = 46.9\ m$, $\angle A = 100°$ and $\angle C = 70°$.

(i) Find the length of BD, giving your answer to two decimal places. **2**

(ii) Find $\angle DBC$, correct to the nearest degree. **2**

End of Question 11

QUESTION 12 (15 marks) – Start on the appropriate page in your booklet.

a) Differentiate with respect to x:

(i) $y = x^2 \ln 4x$. **2**

(ii) $y = \frac{\sin 5x}{x^2}$. **2**

b) Find $\int (1 + \sec^2 x)dx$ **1**

c) Evaluate the limiting sum of the series

$\frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \cdots$ **2**

d) Let α *and* β be the roots of $2x^2 - 4x - 2 = 0$.

(i) State the value of $\alpha\beta$. **1**

(ii) Find $\frac{5}{\alpha} + \frac{5}{\beta}$. **2**

(iii) Find $\alpha^3 + \beta^3$. **2**

e) The number of bacteria in a culture is given by $N = Ae^{kt}$. If 6000 bacteria increase to 9000 after 8 hours,

(i) find k, correct to three significant figures. **2**

(ii) find the number of bacteria after 2 days. **1**

End of Question 12

QUESTION 13 (15 marks) – Start on the appropriate page in your booklet.

a) Given the equation of the parabola $y = x^2 + 6x + 6$

(i) Find the co-ordinates of the vertex. **1**

(ii) Find the co-ordinates of the focus. **1**

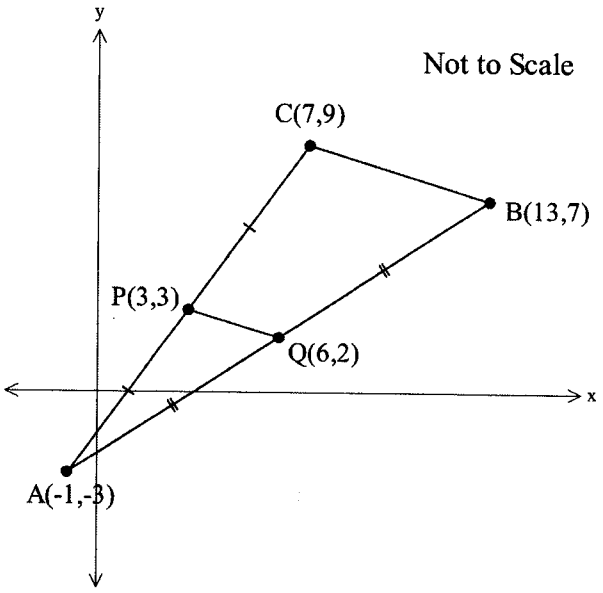
b) If $f'(x) = 6x^2 + 5x - 1$ and $f(-1) = 5$, find an expression for $f(x)$. **2**

c) Consider the function $f(x) = |x - 4|$.

(i) Sketch the function. **1**

(ii) Hence, or otherwise, evaluate $\int_0^6 |x - 4| dx$ **2**

d) In the diagram A, B and C are the points $(-1, -3)$, $(13, 7)$ and $(7, 9)$ respectively. The points $P(3, 3)$ *and* $Q(6, 2)$ are the mid points of AC and AB respectively.



(i) Show that the equation of the line PQ is $x + 3y - 12 = 0$. **2**

(ii) Prove that ΔABC is similar to ΔAQP. **3**

(iii) Find the perpendicular distance of point A to the line PQ. **2**

(iv) Hence find the area of the ΔAQP. **1**

End of Question 13

QUESTION 14 (15 marks) – Start on the appropriate page in your booklet.

a) Find the value of m for which the equation $(m-1)x^2 + 3x - 3 = 0$ has one root twice the other. **3**

b) The line $y = mx + b$ is a tangent to the curve $y = x^2 + 4x + 2$ at $x = -3$.

(i) Find the value of m. **1**

(ii) Find the angle of inclination that the tangent makes with the positive x-axis. **2**

c) A sum of \$5000 is deposited at the start of each year in an account that earns 8% per annum, compounded annually.

(i) Show that the amount accrued by the end of the third year can be expressed as

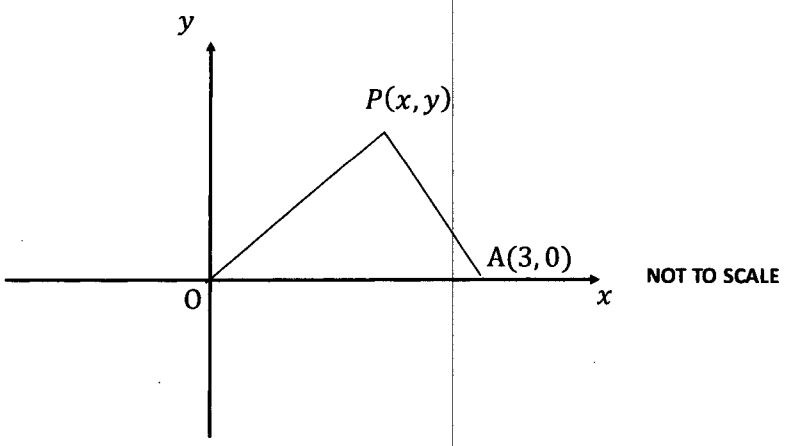
$$A_3 = 5000(1.08 + 1.08^2 + 1.08^3)$$ **1**

(ii) Show that the amount accrued after n years is given by

$$A_n = 67\,500(1.08^n - 1)$$ **1**

(iii) Find the total value of the investment at the end of the 15^{th} year, correct to the nearest dollar. **1**

d) Consider the diagram below.

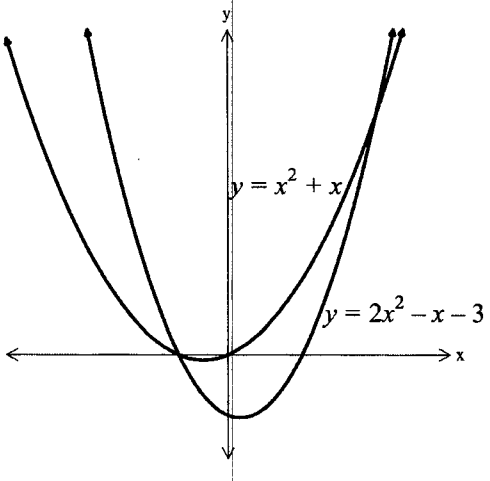


Show that the equation of the locus of all points P, such that $OP \perp AP$ is $x^2 - 3x + y^2 = 0$. **2**

Question 14 continues on the following page

QUESTION 14 (continued)

e) Consider the two parabolas as shown below:



(i) Show that the x values at the points of intersection of $y = x^2 + x$ and $y = 2x^2 - x - 3$, are $x = -1$ and $x = 3$. **1**

(ii) Find the area enclosed between the two curves. **3**

End of Question 14

QUESTION 15 (15 marks) – Start on the appropriate page in your booklet.

a) If $6x^2 - 11 \equiv A(x+2)^2 + Bx + C$ find the values of A, B and C. **2**

b) Using Simpson's rule, approximate the answer to $\int_1^5 \frac{dx}{x^2+1}$, using four sub-intervals. **3**

c) Water was poured into a tank for 30 hours until it was full. At any time, t hours, the volume, V litres, of water in the tank was given by $V = 2(20t - t^2 + 100)$

(i) How much water was in the tank initially? **1**

(ii) At what rate was water poured into the tank at the end of 5 hours? **2**

d) Consider the function $f(x) = xe^{-x}$.

(i) Show that $f'(x) = e^{-x}(1-x)$. **1**

(ii) Show that $f''(x) = e^{-x}(x-2)$. **1**

(iii) Find the coordinates of any stationary points and determine their nature. **2**

(iv) Find the coordinates of the point of inflexion. **1**

(v) By considering the values of the function as x approaches infinity, sketch $y = f(x)$, indicating all important features found in part (iii) and (iv). **2**

End of Question 15

QUESTION 16 (15 marks) – Start on the appropriate page in your booklet.

a) Show that

i) $\frac{4}{x+2} - \frac{2}{x-3} = \frac{2x-16}{x^2-x-6}$ **1**

ii) Hence find $\int \frac{2x-16}{x^2-x-6}\,dx$ **2**

b) An airline company marks the price of a flight at \$400, less a group discount based on the number of bookings made. The price R dollars for each person in the group of x people is $R = 400 - 0.5x$. The cost of running the flight is a fixed cost of \$5000 plus \$150 per person.

(i) Show that the profit on a flight of a group of x people is $(250x - 0.5x^2 - 5000)$ dollars. **2**

(ii) Hence find the required group size to gain the maximum profit and find this profit. **3**

c) Two particles A and B are moving along the x-axis. Their displacements from the origin are given by: $x = -\frac{1}{\pi}(1 + cos\pi t)$ and $x = t^2 - 4t$ respectively.

i) Express the velocities of the two particles in terms of time. **2**

ii) On the same diagram sketch the two velocities. **2**

iii) Use that sketch to show that the particles have the same non-zero velocities at two occasions by marking t_1 *and* t_2. **1**

iv) Show that the distance travelled by the second particle between t_1 and t_2 is: $({t_1}^2 + {t_2}^2) - 4(t_1 + t_2) + 8$ **2**

End of Examination

Solutions – Trial – 2unit 2018.

Section one

1. (D) $T_{40} = 5 \times 40 - 9$

2. (A)

3. (D) $450 = 360^\circ + 90^\circ$

radians $= 2\pi + \frac{\pi}{2}$

$= \frac{5\pi}{2}$

4. (C) $\int_0^2 dx = [x]_0^2$

$= 2$

5. (D) $0 \leq P(E) \leq 1$

6. (B)

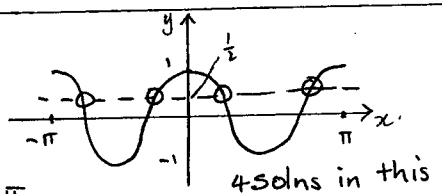
7. (C)

8. (A)

9. (A) $2x = \frac{\pi}{3}$

$x = \frac{\pi}{6}$

$\therefore \frac{\pi}{6}, \frac{5\pi}{6}, -\frac{5\pi}{6}, -\frac{\pi}{6}$.



4 solns in this domain.

10. (D) $V = \pi \int_a^b x^2 \, dy$

$= \pi \int_0^{16} \sqrt{y} \, dy$

1 mark each.

Section two.

Question 11. (15 marks)

	mark.	comment
a) $3.856.. = 3.86$ (2dp)	1	correct solution.
b) $\frac{x}{x^2-4} - \frac{2}{x-2} = \frac{x-2(x+2)}{(x-2)(x+2)}$ $= \frac{-x-4}{(x-2)(x+2)}$	2	2 – correct solution 1 – established correct first line
c) $\lvert 2x-3 \rvert = 4-3x$ $2x-3 = 4-3x$ $\quad 2x-3 = -4+3x$ $x = \frac{7}{5}$ $\quad x = 1$ test: $\lvert 2\times\frac{7}{5} - 3 \rvert = 4 - 3\times\frac{7}{5}$ $\quad \lvert 2\times 1 - 3 \rvert = 4 - 3\times 1$ $\lvert -\frac{1}{5} \rvert \neq -\frac{1}{5}$ $\quad \lvert -1 \rvert = 1$ no soln. $\quad \therefore x = 1$ is the soln.	3	(3) – correctly solves and tests $\therefore x = 1$ (2) correctly solves – no test. $\therefore x = \frac{7}{5}, 1$ (1) evidence of two solns.
d) $(2\sqrt{3}-1)(3\sqrt{3}+2) = p + q\sqrt{3}$ $\therefore 6\times 3 + 4\sqrt{3} - 3\sqrt{3} - 2$ $= 16 + \sqrt{3}$ $\quad \therefore p = 16$ and $q = 1$	2	(2) correctly finds both values. (1) finds either value.
e) $4^x - 17 \times 2^x + 16 = 0$ let $m = 2^x$ $\therefore m^2 - 17m + 16 = 0$ $(m-16)(m-1) = 0$ $m = 1, 16$ $\therefore 2^x = 16$ $\quad 2^x = 1$ $= 2^4$ $\quad = 2^0$ $\therefore x = 4$ $\quad x = 0$ soln is $x = 0$ or 4	3.	(3) – a correct solution (any method) (2) attempts to solve with 2 solns. (1) one correct solution. (1) incorrect substitution completing to 2 solns.

Quest. 11. continued

f. i) $BD^2 = AD^2 + AB^2 - 2 \times AD \times AB \times \cos\theta$

$= 20.1^2 + 41.3^2 - 2 \times 20.1 \times 41.3 \times \cos 100$

$= 2398.0011\ldots$

$BD = 48.9693\ldots$

$= 48.97$ (2dp) m.

mark	comment
2.	② – correct answer – ignore dec. places. ① sets up the cosine rule correctly.

ii) $\frac{\sin D\hat{B}C}{DC} = \frac{\sin C}{BD}$

$\sin D\hat{B}C = \frac{46.9 \times \sin 70}{48.969\ldots}$

$= 0.899\ldots$

$D\hat{B}C = 64.155\ldots$

$\doteq 64^\circ$

mark	comment
2.	② correct soln to nearest degree ① sets up the sine rule correctly.

Question 12 (15 marks)

a) i) $\frac{dy}{dx} = 2x \ln 4x + x^2 \times \frac{4}{4x}$

$= 2x \ln 4x + x.$

mark	comment
2.	② correct soln. ① attempts product rule showing $x^2 \times \frac{4}{4x}$

ii) $y = \frac{\sin 5x}{x^2}$

$\frac{dy}{dx} = \frac{x^2 \times 5\cos 5x - \sin 5x \times 2x}{x^4}$

$= \frac{5x\cos 5x - 2\sin 5x}{x^3}$

mark	comment
2	② correct soln ① attempts quotient rule.

b) $\int 1 + \sec^2 x\, dx = x + \tan x + c$

mark	comment
1	① correct soln ignore '+c'

c) $a = \frac{3}{4} \quad r = \frac{1}{4}$

$\lim S = \frac{a}{1-r}$

$= \frac{\frac{3}{4}}{1 - \frac{1}{4}}$

$= 1$

mark	comment
2	② finding lims correctly ① identifying $r = \frac{1}{4}$.

quest. 12. cont.

d) $2x^2 - 4x - 2 = 0$

i) $\alpha\beta = \frac{c}{a}$

$= \frac{-2}{2} = -1$

mark	comment
1	correct value

ii) $\frac{5}{\alpha} + \frac{5}{\beta} = \frac{5(\alpha+\beta)}{\alpha\beta}$ $\qquad \alpha+\beta = \frac{-b}{a}$

$= \frac{5 \times 2}{-1}$ $\qquad = \frac{4}{2} = 2$

$= -10$

mark	comment
2	② correct value ① evidence of correct method. ① finds $\alpha+\beta = 2$.

iii) $\alpha^3 + \beta^3 = (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= (\alpha+\beta)((\alpha+\beta)^2 - 3\alpha\beta)$

$= 2(2^2 - 3 \times -1)$

$= 14$

mark	comment
2	② correct value ① evidence of factorising cubic

e) $N = Ae^{kt}$

(i) $t = 0 \quad N = 6000 = Ae^0$

$\therefore A = 6000$

$N = 6000e^{kt}$

$t = 8 \quad 9000 = 6000e^{8k}$

$\frac{3}{2} = e^{8k}$

$\ln \frac{3}{2} = 8k \ln e$

$k = \frac{\ln 1.5}{8}$

$= 0.05068\ldots$

$= 0.0507$ (3 sig fig)

mark	comment
2	② correct k to 3 sig fig ① evidence evaluating k from correct A.

ii) 2 days = 48 hrs

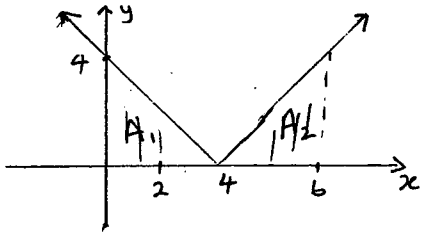
$N = 6000e^{48k}$

$= 68343.75$

$\doteq 68343$ bacteria

mark	comment
1	① correct answer – approx. ok.

Question 13 (15 marks)

	mark	comment
a) $y = x^2 + 6x + 6$		
i) $x^2 + 6x + 9 = y + 9 - 6$	1	① correct vertex
$(x+3)^2 = y + 3$		
vertex: $(-3, -3)$		
ii) focus: $(-3, -2\frac{3}{4})$ as $4a = 1$, $a = \frac{1}{4}$.	1	① correct focus
b) $f'(x) = 6x^2 + 5x - 1$		② correctly finds $f(x)$
$f(x) = 2x^3 + \frac{5}{2}x^2 - x + c$	2	① forgets 'c'
now $f(-1) = -2 + \frac{5}{2} + 1 + c = 5$		
$\therefore c = \frac{7}{2}$		
$f(x) = 2x^3 + \frac{5}{2}x^2 - x + \frac{7}{2}$.		
c) $f(x) = \lvert x - 4 \rvert$		
i) 	1	must use ruler! and show $y = 4$, $x = 4$
ii) $\int_0^6 \lvert x-4 \rvert \, dx = A_1 + A_2$		② correct Area $= 10u^2$
$= \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times 2 \times 2$	2	① evidence of splitting the area up. and correctly finds one Area
$= 10\ u^2$		
d) i) $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$		② evidence of method. - can sub points in but must show both work.
$= \frac{2-3}{6-3}$	2	
$= -\frac{1}{3}$		① finds gradient
$y - y_1 = m(x - x_1)$ where (x_1, y_1) is $(6, 2)$		
$y - 2 = -\frac{1}{3}(x - 6)$		
$x + 3y - 12 = 0$		

	mark	comment
quest 13 cont.		
d) ii) Aim: to prove $\triangle ABC \parallel\parallel \triangle AQP$		③ clearly structured proof. - "included" stated
Proof: In $\triangle ABC$ and $\triangle AQP$		② - final reason poorly stated but 'body' supports proof. - ratio not clearly defined
$\hat{A}$ is common	3	① made an attempt - $\hat{A}$ is common
$\frac{AP}{AB} = \frac{1}{2}$ (P is the midpt - given)		
$\frac{AQ}{AC} = \frac{1}{2}$ (Q is midpt - given)		
$\therefore \triangle ABC \parallel\parallel \triangle AQP$ (two sides in ratio included $\angle$ equal)		
iii) perp $d = \frac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2 + b^2}}$ $A(\overset{x_1}{-1}, \overset{y_1}{-3})$		② correct answer
line: $x + 3y - 12 = 0$	2	① correct substitution
$= \frac{\lvert 1 \times -1 + 3 \times -3 - 12 \rvert}{\sqrt{1+9}}$		
$= \frac{\lvert -22 \rvert}{\sqrt{10}}$		
$= \frac{22}{\sqrt{10}}$ or $\frac{11\sqrt{10}}{5}$ units.		
iv. $d_{PQ} = \sqrt{(3-6)^2 + (3-2)^2}$		correct result. (no problem if units missing)
$= \sqrt{10}$.	1	
$A = \frac{1}{2} \times \frac{22}{\sqrt{10}} \times \sqrt{10}$		
$= 11\ u^2$		

Question 14 (15 marks).

a) $(m-1)x^2 + 3x - 3 = 0$

let the roots be α and 2α

$\therefore$ sum: $\alpha + 2\alpha = -\frac{b}{a}$

$$3\alpha = \frac{-3}{m-1} \quad \text{---} \quad (1)$$

product: $\alpha \times 2\alpha = \frac{c}{a}$

$$2\alpha^2 = \frac{-3}{m-1} \quad \text{---} \quad (2)$$

rearrange (1)

$$\alpha = \frac{-1}{m-1}$$

sub into (2)

$$2\left(\frac{-1}{m-1}\right)^2 = \frac{-3}{m-1}$$

$$2 = -3(m-1)$$

$$m = \frac{1}{3}$$

mark	comment
3	(3) correct answer. - may use another method. (2) sum and product clearly shown and a substitution made. (1) sum & product of roots attempted

b) $y = mx + b$ $\quad y = x^2 + 4x + 2$ $\quad$ at $x = -3$

i) $\frac{dy}{dx} = 2x + 4$ $\quad m = 2x - 3 + 4$

$= -2$

mark	comment
1	correct value (any method)

ii) $\tan\theta = m$

$= -2$

$\theta = 180 - 63.43$

$= 116.56$

$\doteq 117^\circ$ (nearest degree)

mark	comment
2.	(2) correct answer. - doesn't need to be nearest degree (1) states $\tan\theta = m = -2$

quest 14 cont:

c) \$5000 $\quad r = 0.08$

$A = P \times (1+r)$

i) $A_1 = 5000(1+0.08)$

End of yr 2.

$A = A_1(1.08) + A_2$

$= 5000(1.08)^2 + 5000(1.08)$

$= 5000(1.08 + 1.08^2)$

end of yr 3

$A = 5000(1.08 + 1.08^2 + 1.08^3)$

mark	comment
1.	must clearly show development from 1st to 2nd to 3rd yr.

ii) $A_n = 5000(\underbrace{1.08 + 1.08^2 + 1.08^3 + \cdots 1.08^n}_{\text{G.P.}})$

$a = 1.08 \quad r = 1.08$

$$\therefore A_n = 5000 \times \frac{1.08(1.08^n - 1)}{1.08 - 1}$$

$$= 67500(1.08^n - 1)$$

mark	comment
1	clearly show working towards (i.e. 2nd last line!)

iii) when $n = 15$

$A_{15} = 67500(1.08^{15} - 1)$

$\doteq$ \$146621.42

$=$ \$146621 (nearest dollar)

mark	comment
1	accept not rounded

d) $m_{OP} = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m_{PA} = \frac{y-0}{x-3}$

$= \frac{y-0}{x-0}$ $\quad = \frac{y}{x-3}$

$= \frac{y}{x}$

now $OP \perp AP \rightarrow m_{OP} = -\frac{1}{m_{PA}}$

$$\frac{y}{x} = \frac{-(x-3)}{y}$$

$$y^2 = -x^2 + 3x$$

$$x^2 - 3x + y^2 = 0$$

mark	comment
2	(2) clear working showing $m_1 = -\frac{1}{m_2}$ (1) finds gradients.

quest 14 cont.

e) i) $y = x^2 + x = 2x^2 - x - 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = -1, 3$

1 — comments: - can sub both points in or solve

ii) $A = \int_{-1}^{3} x^2 + x - (2x^2 - x - 3)\,dx$

$= \int_{-1}^{3} (-x^2 + 2x + 3)\,dx$

$= \left[-\frac{x^3}{3} + x^2 + 3x\right]_{-1}^{3}$

$= (-9 + 9 + 9) - (\frac{1}{3} + 1 - 3)$

$= \frac{32}{3}\ u^2$

3 — comments: ③ complete solution. ② sets up integrals and integrates - makes calculation error. ② wrong way round for integrals but reaches an answer. ① clearly 2 errors but method correct.

Question 15 (15 marks)

a) $6x^2 - 11 \equiv A(x+2)^2 + Bx + C$

RHS $= Ax^2 + 4Ax + 4A + Bx + C$

$= Ax^2 + x(4A + B) + 4A + C$

now

$6 = A$ $\quad 0 = 4A + B$ $\quad -11 = 4A + C$

$0 = 24 + B$ $\quad -11 = 24 + C$

$B = -24$ $\quad C = -35$

$\therefore A = 6,\ B = -24\ \ C = -35$

2. — comments: ② correct solution ① 2 solutions correct

b) $\int_1^5 \frac{dx}{x^2+1}$

x	1	2	3	4	5
$f(x)$	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{17}$	$\frac{1}{26}$

$\int_1^5 \frac{dx}{x^2+1} = \frac{h}{3}\{1st + last + 4 even + 2 \times odd\}$

$= \frac{1}{3}\left\{\frac{1}{2} + \frac{1}{26} + 4\left(\frac{1}{5} + \frac{1}{17}\right) + 2 \times \frac{1}{10}\right\}$

$= \frac{1}{3} \times \frac{392}{221}$

$= 0.5912\ldots$

3 — comments: ③ correct solution ② apply simpson's rule correctly ① significant progress towards solution

Quest 15 cont.

c) $V = 2(20t - t^2 + 100)$

i) $t = 0$

$\therefore V = 2(0 + 100)$

$= 200\ l$

1 — comment: correct answer - ignore units

ii) $V = 2(20t - t^2 + 100)$

$\frac{dV}{dt} = 2(20 - 2t)$

at $t = 5^-$

$\frac{dV}{dt} = 2(20 - 10)$

$= 20\ l/h$

2 — comments: ② correct solution including units ① correct derivative or ① incorrect derivative but sub in $t = 5^-$.

d) $f(x) = x e^{-x}$

i) $f'(x) = e^{-x} - x e^{-x}$

$= e^{-x}(1 - x)$

1 — comment: shows product rule clearly.

ii) $f''(x) = -e^{-x} + x e^{-x} - e^{-x}$

$= x e^{-x} - 2 e^{-x}$

$= e^{-x}(x - 2)$

1 — comment: shows first line correctly.

iii) stationary pts $f'(x) = 0$

$\therefore f'(x) = e^{-x}(1 - x) = 0$

since $e^{-x} > 0$ $\quad 1 - x = 0$

$x = 1$ $\quad y = e^{-1}$ $\quad (1, e^{-1})$

test table or 2nd derivative.

$f''(x) = e^{-1}(1 - 2)$

$= -e^{-1} < 0$ $\quad \therefore$ max at $x = 1$

2 — comments: ② correctly finds pt and tests. ① wrong pt and tests. ① states $e^{-x} > 0 \therefore x =$

iv) Point of inflexion: $f''(x) = 0$

$f''(x) = e^{-x}(x - 2) = 0$

since $e^{-x} > 0$ $\quad \therefore x - 2 = 0$

$x = 2$ $\quad y = 2e^{-2}$ $\quad (2, 2e^{-2})$

test: — good practise!

$f''(1.5) = e^{-1.5}(1.5 - 2)$

$= -0.111\ldots < 0$

$f''(2.5) = e^{-2.5}(2.5 - 2)$

$= 0.04\ldots > 0$

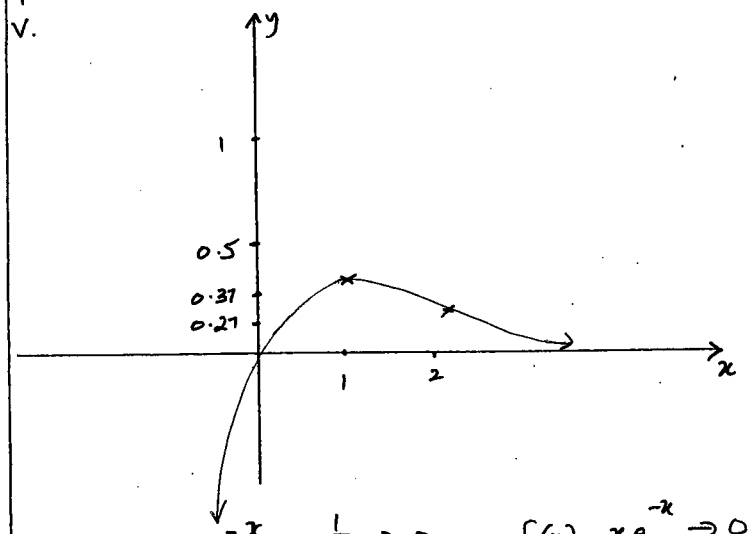
a change in concavity

$\therefore (2, 2e^{-2})$ is a point of inflexion

1 — comment: finds point giving x and y

Quest 15 cont.

v.



as $x \to \infty$ $e^{-x} = \frac{1}{e^x} \to 0$ $\therefore f(x) = xe^{-x} \to 0$

at $x = -3$ $xe^{-(-3)} = -60$

mark	comment
2	② detail clearly shown - including approaching x axis. ① stat pt & inflexion pt - smooth curve shown.

Question 16 (15 marks)

a) i) $\frac{4}{x+2} - \frac{2}{x-3} = \frac{2x-16}{x^2-x-6}$

LHS $= \frac{4}{x+2} - \frac{2}{x-3}$

$= \frac{4x-12-2(x+2)}{(x+2)(x-3)}$

$= \frac{2x-16}{x^2-x-6}$ as required.

mark	comment
1	clearly shows

ii) $\int \frac{2x-16}{x^2-x-6}\,dx = \int \frac{4}{x+2} - \frac{2}{x-3}\,dx$

$= 4\ln(x+2) - 2\ln(x-3) + C$

or $= \ln\frac{(x+2)^4}{(x-3)^2} + C$

mark	comment
2	② correct form of soln. ① correct split for integration. ignore +c.

Quest 16 cont.

b) Price = \$400 / person x = no. of people

i) $R = 400 - 0.5x$ ⟶ for each person

Cost = \$5000 + \$150 × x

= 5000 + 150x

Profit = Price − cost Price = R × x

$= (400 - 0.5x)x - (5000 + 150x)$

$= 400x - 0.5x^2 - 5000 - 150x$

$= 250x - 0.5x^2 - 5000$

mark	comment
2	② clearly explained ① Price = R × x shown.

ii) $P = 250x - 0.5x^2 - 5000$

$\frac{dP}{dx} = 250 - 2 \times 0.5x$

$= 250 - x = 0$

$x = 250$ people

test:

$\frac{d^2P}{dx^2} = -1 < 0$ $\therefore$ max

$\therefore$ maximum group size is 250 people.

Profit $= 250 \times 250 - 0.5 \times 250^2 - 5000$

$= \$26250$.

mark	comment
3	③ correct solution ② solves $\frac{dP}{dx}$ and tests ② solves $\frac{dP}{dx}$ and finds profit. ① finds x

c. A: $x = -\frac{1}{\pi}(1 + \cos \pi t)$ B: $x = t^2 - 4t$.

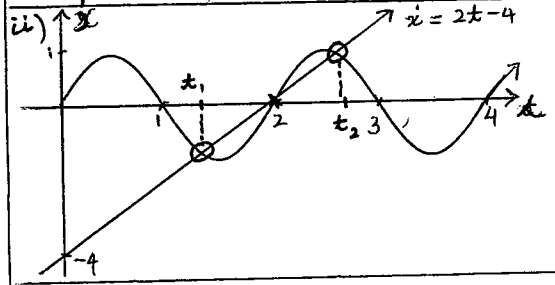
i) velocity

$\dot{x} = -\frac{1}{\pi}(-\pi \sin \pi t)$ $\dot{x} = 2t - 4$

$= \sin \pi t$

mark	comment
2	② both correct ① one correct.

ii)



mark	comment
2	② both accurate ① one only accurate.

iii) t_1, t_2

mark	comment
1	t_1, t_2 clearly indicated

quest 16 cont.

mark	comment
2	② correct soln. ① correctly splits area.

iv) $\text{Dist} = \left| \int_{t_1}^{2} 2t-4 \, dt \right| + \int_{2}^{t_2} 2t-4 \, dt$

$= \left| \left[t^2-4t \right]_{t_1}^{2} \right| + \left[t^2-4t \right]_{2}^{t_2}$

$= \left| 4-8-(t_1^2-4t_1) \right| + \left((t_2)^2-4t_2-4+8 \right)$

$= \left| -(4+t_1^2-4t_1) \right| + t_2^2-4t_2+4$

$= 4+t_1^2-4t_1+t_2^2-4t_2+4$

$= (t_1^2+t_2^2)-4(t_1+t_2)+8$ as required